Supplementary Material

Shannon and von Neumann entropies of multi-qubit Schrödinger's cat states

Nathan D. Jansen, Matthew Loucks, Scott Gilbert, Corbin Fleming-Dittenber, Julia Egbert, and Katharine L. C. Hunt, Department of Chemistry, Michigan State University, East Lansing, Michigan, USA 48824

CP-ART-11-2021-005255

Computational time scaling for state tomography

The computational time required for tomography grows exponentially with the number of qubits, because 3^n circuits are needed for quantum state tomography on an n-qubit cat state. We have examined the timing of runs with 2-5 entangled qubits on the IBM quantum computers lima, belem, and manila. Timing data are provided in Tables S.1-S.3. All currently available timing data are listed in the tables. The times in the first five data rows come from the more recent runs, with 100 circuits per job, rather than 75. Tomography on a 4-qubit cat state, which requires 81 circuits, can be now completed in a single job, rather than two. Tomography on a 5-qubit cat state, which requires 243 circuits, can be completed in three jobs, rather than four. Timing data in the subsequent rows comes from our earlier runs, which were limited to 75 circuits per job. These typically ran longer, for the 4- and 5-qubit tomography.

Table S.1. Times for quantum tomography of cat states on lima. The "time in system" is listed in seconds, depending on the number of entangled qubits, n.

n = 2	n = 3	n = 4	n = 5
5.2	11.1	28.1	82.7
6.2	11.2	27.3	83.9
5.2	11.1	27.5	85.4
5.2	11.4	27.4	83.0
5.1	11.0	27.3	85.4
5.2	10.8	30.8	86.6
5.3	12.4	31.2	87.3
5.2	10.9	31.7	89.5
6.3	10.9	30.8	87.3
5.2	10.9	30.6	91.8
		31.4	89.5
		37.1	88.6
Average of times in top five data rows (s)			
5.38	11.16	27.52	84.08

Table S.2. Times for quantum tomography of cat states on belem. The "time in system" is listed in seconds, depending on the number of entangled qubits, n.

n = 2	n = 3	n = 4	n = 5
5.2	11.4	26.9	82.3
5.2	11.1	27.1	84.4
6.3	11.0	27.9	83.3
5.4	11.5	26.9	84.6
5.3	11.1	28.0	84.3
5.1	11.2	30.6	87.0
5.1	10.9	32.8	86.7
5.1	11.1	30.5	86.5
5.1	10.9	30.3	86.5
5.1	10.9	30.7	88.8
Average of times in the top five data rows (s)			
5.48	11.22	27.36	83.78

Table S.3. Times for quantum tomography of cat states on manila. The "time in system" is listed in seconds, depending on the number of entangled qubits, n.

n = 2	n = 3	n = 4	n = 5
7.6	12.7	28.3	88.9
7.7	12.6	29.1	85.2
7.6	12.7	28.3	86.0
7.6	12.6	28.1	85.7
7.6	12.9	28.3	85.9
7.9	13.0	28.1	85.4
8.7	13.1	28.6	84.9
8.1	13.1	28.2	84.8
7.8	13.1	28.2	85.3
7.9	12.9	28.8	85.5
8.5	13.1	35.5	95.7
8.4	13.3	35.3	95.5
	13.1		
	13.2		
	13.2		
Average of times in the top five data rows (s)			
7.62	12.70	28.42	86.34

We have used the average timing data from the more recent runs to find fits to the "time in system" as a function of the number of qubits. We note that the "Running" time and the "Total completion time" are both longer than the "time in system."

The times for n-qubit tomography are fit well by the form $k_1 3^n + k_2$, where k_1 and k_2 are constants. The constant k_2 appears to reflect the overhead associated with the set-up time for the runs. Its inclusion improves the fit to the required times for 2- and 3-qubit cat states, but it is small compared with $k_1 3^n$ for the 4- and 5-qubit cat states. The values for k_1 and k_2 that we obtained with the FindFit function in Mathematica are listed in Table S.4.

Table S.4. Parameters k_1 and k_2 in the fits of time in system for quantum state tomography of n-qubit cat states on IBM quantum computers.

Computer	k_1	k_2
lima	0.33738	1.67071
belem	0.33572	1.74476
manila	0.33912	3.24905

The parameters k_1 is quite similar for all three quantum computers; k_2 is quite similar for lima and belem, close to twice as large on manila.

Figures F.1-F.3 show the fit to the average times, along with data points from the five most recent runs shown in red. Data points from the earlier runs, with 75 circuits per job, are shown in green. All of the points lie fairly close to the functional fits.

Figure F.1. Fit of the average times in system T(n) for the five most recent runs on lima, to the form $T(n) = k_1 3^n + k_2$ as a function of the number of qubits n. Individual data points from the five most recent runs are plotted in red and data from the remaining runs are plotted in green.

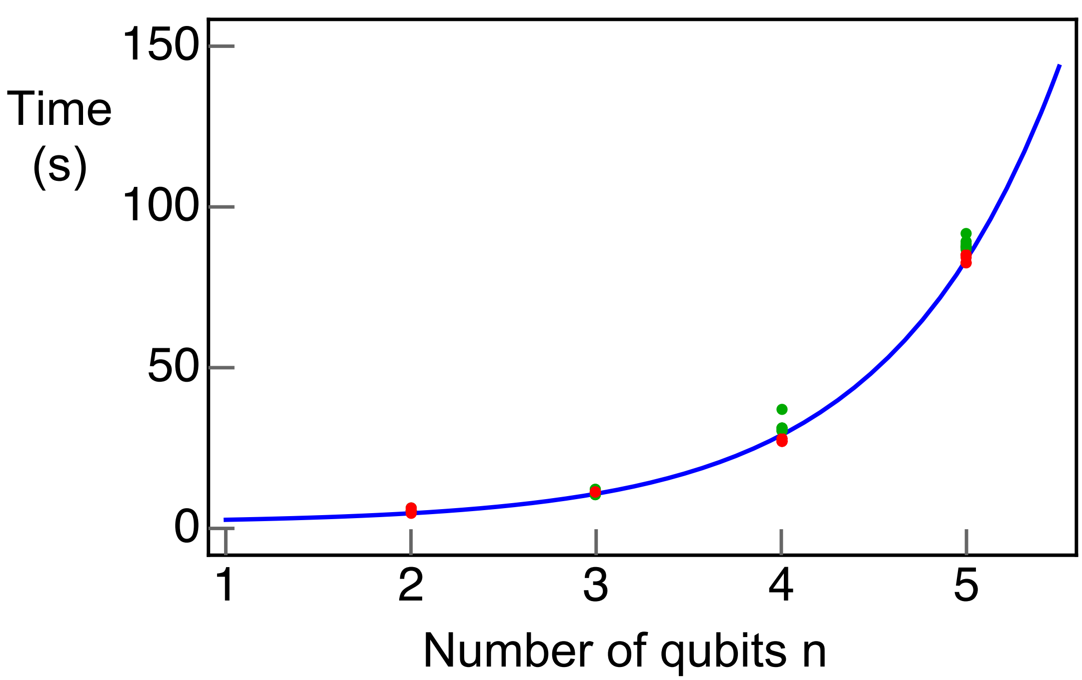


Figure F.2. Fit of the average times in system T(n) for the five most recent runs on belem, to the form $T(n) = k_1 3^n + k_2$, as a function of the number of qubits n. Individual data points from the five most recent runs are plotted in red and data from the remaining runs are plotted in green.

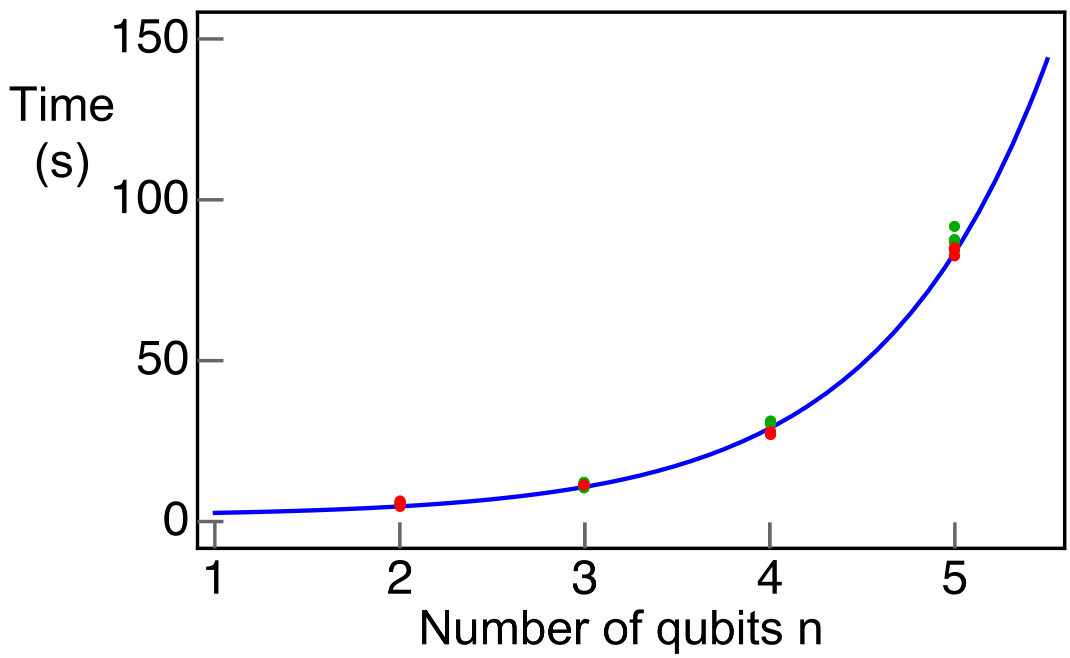
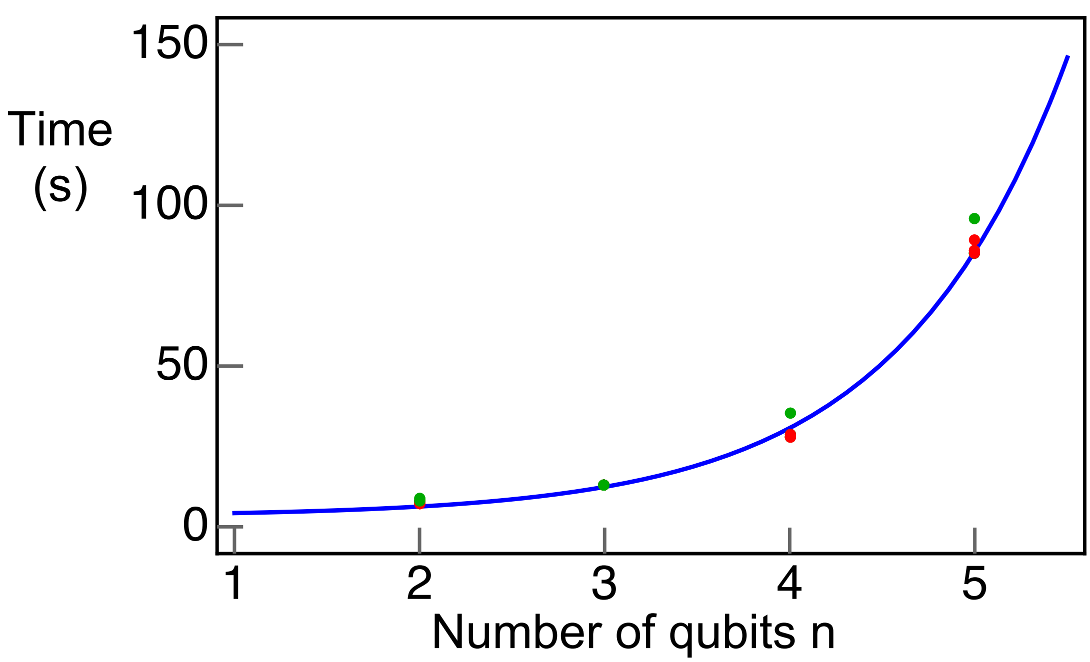


Figure F.3. Fit of the average times in system T(n) for the five most recent runs on manila, to the form $T(n) = k_1 3^n + k_2$, as a function of the number of qubits n. Individual data points from the five most recent runs are plotted in red and data from the remaining runs are plotted in green.



As expected, tomography on 4- and 5-qubit cat states took slightly longer for the older runs, with 75 circuits per job rather than 100.

If the same functional form of the time requirements continues to hold for additional qubits, then it appears that results could be obtained reasonably for more than five entangled qubits, if considered on this basis alone. For example, based on the fit to the timing data from lima, with 10 qubits, the "time in system" is predicted to be about 5.5 hours. Due to the exponential growth, the time requirements become unwieldy for the larger quantum computers, though. For a 16-qubit cat state with the same system-time function as lima, about 168 days would be required; for a 27-qubit cat state, this would jump to ~81,525 years; for a 65-qubit cat state, $\sim 1.101 \cdot 10^{23}$ years; and for a 127-qubit cat state, $\sim 4.202 \cdot 10^{52}$ years. In brief, it is clear why IBM would not allow state tomography for all the qubits that are currently available. There may be additional engineering or access considerations that account for the IBM limit of 5 qubits.

Computational time scaling for finding the algorithm-specific quantum volume, based on the Shannon entropy

Calculating the algorithm-specific quantum volume is always faster than calculating the standard quantum volume if other conditions are held constant, because the latter requires at least 100 different circuits. The first step in finding the quantum volume involves a set of runs to determine the Shannon entropy of measurement outcomes. This is followed by brief post-processing to determine the per cents of "heavy" outputs and then to determine the probability that the percentage of heavy outputs is greater than two-thirds.

In analyzing the scaling of the algorithm-specific quantum volume with the number of qubits, we have determined the "time in system" for runs on bogota, to find the Shannon entropy of the distribution measurement outcomes, with 2-5 entangled qubits. To offer a balanced comparison with the times for tomography, we have determined the times for 9, 27, 81, and 243 identical circuits (as would be needed for tomography on 2-5 qubits). The results for "time in system" are listed in Table S.5, for various numbers of circuits, 1024 or 8192 shots, and qubits. We used 1024 shots in the quantum state tomography runs.

Table S. 5. "Time in system" for calculation of the Shannon entropy of the distribution over measurement outcomes, for different numbers of shots and circuits.

Number of circuits	Number of shots	Number of qubits	Time in system (s)
9	1024	2	8.7
9	1024	3	9.6
9	1024	4	8.8
9	1024	5	8.9
9	8192	2	26.3
9	8192	3	26.3
9	8192	4	26.2
9	8192	5	26.4

Number of circuits	Number of shots	Number of qubits	Time in system (s)
27	1024	2	13.8
27	1024	3	14.7
27	1024	4	13.8
27	1024	5	14.4
27	8192	2	66.9
27	8192	3	66.2
27	8192	4	66.5
27	8192	5	67.1
81	1024	2	29.0
81	1024	3	28.8
81	1024	4	30.6
81	1024	5	29.3
81	8192	2	184.6
81	8192	3	186.7
81	8192	4	186.8
81	8192	5	188.1
243	1024	2	89.0
243	1024	3	88.8
243	1024	4	89.4
243	1024	5	90.9
243	8192	2	555.8
243	8192	3	556.1
243	8192	4	557.0
243	8192	5	558.8

The required times are virtually independent of the number of qubits, for the range of qubit numbers from 2 to 5. Significantly, the times shown in Table S.5 for 9 circuits of 1024 shots are quite similar to the times for quantum state tomography of 2 qubits. For the runs with 1024 shots, the times in Table S.5 are also quite similar for 27 circuits vs. tomography for 3 qubits, 81 circuits vs. tomography for 4 qubits, and 243 circuits vs. tomography for 5 qubits. This makes it clear that the time required for quantum state tomography is exponential in the number of qubits n, because the number of circuits needed to carry out the tomography is 3^n, not because extra time is required for operations on more qubits. The "time in system" (in seconds) scales as $s_1 c + s_2$ in the number of circuits c. For 1024 shots, $s_1 = 0.3472$ and $s_2 = 4.287$; for 8192 shots, $s_1 = 2.269$ and $s_2 = 4.880$.

Interestingly, the time does not scale linearly with the number of shots. For example, the average time required for 9 circuits with 1024 shots per circuit is 9.0 s, while the average time required for 9 circuits with 8192 shots per circuit is 26.3 s. If the time required scaled linearly in the number of shots, then 8192 shots would require 8 times the time needed for 1024 shots. Instead the time increase is smaller, with factors of 2.92 going from 1024 shots to 8192 shots for 9 circuits; 4.70 going from 1024 shots to 8192 shots for 27 circuits; 6.34 going from 1024 shots to 8192 shots for 81 circuits; and 6.22 going from 1024 shots to 8192 shots for 243 circuits.

For scaling of the algorithm-specific calculations of quantum volume as the number of qubits increases, the relation between the number of qubits and the number of circuits needed is therefore the key. From Table 5 (in the paper), the percentages of heavy outputs appear to be reasonably well determined with 25 runs. The z-score for the 2/3 in the case with 5 qubits is 1.733 after 25 runs and 1.728 after 75 runs. In a case where the z-score is quite close to the cut-off for the 97.5% confidence interval, additional runs may be needed, but here there is little difference; and the z-scores in the cases with 2, 3, and 4 qubits are well above that needed for the 97.5% confidence level.

It is not yet clear how the number of circuits required to determine the percentage of heavy outputs will be affected by a large increase in the number of qubits, but in the range from 2 to 5 qubits, 25 circuits appear to be sufficient for the algorithm-specific quantum volume of the cat states.