A General Strategy for Designing Two-dimensional High-efficient Layered Thermoelectric Materials

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Figure S1. Side and top views of three possible stacking models (AA, AB1 and AB2) for bilayer (2L) Bi$_2$Se$_3$. Red and blue balls stand for Bi and Se atoms, respectively.

Figure S2. Side and top views of three possible stacking models (AA, AB1 and AB2) for 2D BiBi$_2$Se$_3$. Red and blue balls stand for Bi and Se atoms, respectively.
Figure S3. Calculated electronic band structures of 2D BiBi$_2$Se$_3$ with AB1 and AB2 stacking patterns at PBE+SOC level. The electronic band structure of 2D BiBi$_2$Se$_3$ with AB1 stacking pattern (two staggered Se layers) appears to be slightly non-degenerate due to the reduction of crystal symmetry.

Figure S4. Electronic localization function (ELF) of Bi monolayer. Calculated ELF results show that the p$_z$-orbital occupied lone pairs are covering the Bi ML. The red balls represent Bi atoms. The isosurface value is set to 0.75.
Figure S5. Phonon dispersion relations of 2D BiBi$_2$X$_3$ (X=S/Se/Te). There is no imaginary phonon mode appears in the whole Brillouin zone, which indicates that the 2D BiBi$_2$X$_3$ are dynamically stable.

Figure S6. Variation of total energy as a function of time at 300 K. AIMD simulation in the NVT ensemble lasts for 8 ps with a time step of 2fs. The total energies of 2D BiBi$_2$X$_3$ (X=S/Se/Te) fluctuate around a constant value with a considerably small fluctuation magnitude.
Figure S7. Calculated partial charge density of Bi$_2$X$_3$ (X=S/Se/Te) monolayers at CBM (a) and VBM (b), respectively. Red, light blue, blue, and light purple colors represent Bi, Te, Se, and S, respectively.

Figure S8. Calculated fat-band representation of 2D BiBi$_2$Se$_3$ consisting of 11 atomic layers. Red, green, blue, and orange stand for s-, px-, py-, and pz-orbital contribution, respectively.
Figure S9. A schematic of decreasing the interlayer distance between the Bi ML and the two quintuplet Bi$_2$Se$_3$ layers to the stable BiBi$_2$Se$_3$ consisting of 11 atomic layers. $d_0$ and $\Delta d_{B-BS}$ represent the vdW gap of the 2D stable BiBi$_2$Se$_3$ and the variation of interlayer distance between the Bi ML and the two quintuplet Bi$_2$Se$_3$ layers, respectively. Red and blue balls stand for Bi and Se atoms, respectively.
Figure S10. Calculated electronic bands of Bi ML, two quintuplet Bi$_2$Se$_3$ layers (2L) and 2D BiBi$_2$Se$_3$ with a series of different $\Delta d_{\text{B-BS}}$ at PBE+SOC level. The red and blue dots stand for the contribution from Bi ML and Bi$_2$Se$_3$ 2L, respectively. We monotonously decrease the interlayer distance between the Bi ML and the two quintuplet Bi$_2$Se$_3$ layers, and obtain a series of corresponding electronic band structures, as shown in Fig. 3. We noticed from Fig. 3 that both Bi ML and two quintuplet Bi$_2$Se$_3$ layers maintain individual and complete electronic band structure until the variation ($\Delta d_{\text{B-BS}}$) of interlayer distance is decreased to 3Å. Interestingly, the electronic band structures of Bi ML and two quintuplet Bi$_2$Se$_3$ layers start to appear an obvious hybridization at the $\Delta d_{\text{B-BS}}$ of 2Å (Fig. 3e), and this hybridization is getting stronger and stronger with the further decreasing of $\Delta d_{\text{B-BS}}$. At the $\Delta d$ of 0.8Å, the reconstructed electronic band structure is formed. We think that this is an amazing electronic band structure evolving process, which is also likely to occur in other similar multilayer structures.
Figure S11. Calculated carrier mobility ($\mu$) of Bi intercalated and non-intercalated 2L Bi$_2$Se$_3$ and Bi$_2$Te$_2$S.

Figure S12. Calculated electronic thermal conductivity relative to relaxation time ($\kappa_e/\tau$) and electronic thermal conductivity ($\kappa_e$) of 2L Bi$_2$Se$_3$/Bi$_2$Te$_2$S and 2D Bi ML inserted Bi$_2$Se$_3$/Bi$_2$Te$_2$S crystals as a function of carrier concentration ($n$) at 300 K. The units for $\kappa_e/\tau$ and $\kappa_e$ are Wm$^{-1}$K$^{-1}$s$^{-1}$ and Wm$^{-1}$K$^{-1}$, respectively.
**Figure S13.** Position of Bi1, X1, X2, and X3 atoms in 2D Bi$_3$X$_3$ (X=S/Se/Te) crystal. Red, light blue, blue, and light purple colors represent Bi, Te, Se, and S, respectively.

**Figure S14.** Normal modes with localized vibrations of Bi middle bilayer in BiBi$_2$X$_3$ (X=S/Se/Te) in the low-frequency A-region (marked in pink color in Fig. 5a). Red, light blue, blue, and light purple colors represent Bi, Te, Se, and S, respectively.
Figure S15. Calculated thermoelectric (TE) properties of 2D BiBi$_2$Te$_2$S consisting of 11 atomic layers (1L) and bulk BiBi$_2$Te$_2$S at the optimum concentration ($n$) where the maximum PF occurs in $n$-dependent PF, respectively. The units for $S$, $\sigma$, $S^2\sigma$, $\kappa_e$, and $\kappa_L$ are $\mu V^{-1}K^{-1}$, $10^5\Omega^{-1}m^{-1}$, $10^{-3}Wm^{-1}K^{-2}$, $Wm^{-1}K^{-1}$, and $Wm^{-1}K^{-1}$, respectively.

Figure S16. Calculated electronic band structures of 2D BiBi$_2$Te$_2$S consisting of 11 atomic layers (1L) and bulk BiBi$_2$Te$_2$S at HSE+SOC level, respectively.
Figure S17. Calculated electronic band structures of (a) 2D BiBi$_2$Te$_2$S consisting of 11 atomic layers (1L), 2D BiBi$_2$Te$_2$S consisting of 22 atomic layers (2L), and (c) bulk BiBi$_2$Te$_2$S at PBE+SOC level, respectively.
Table S1 Calculated total energies (units in eV) of 2L Bi$_2$Se$_3$ and 2D BiBi$_2$Se$_3$ with different possible stacking models.

<table>
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<tr>
<th>Stacking models</th>
<th>AA</th>
<th>AB1</th>
<th>AB2</th>
<th>AB3</th>
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</thead>
<tbody>
<tr>
<td>2L Bi$_2$Se$_3$</td>
<td>-46.658</td>
<td>-46.711</td>
<td>-46.282</td>
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<tr>
<td>2D BiBi$_2$Se$_3$</td>
<td>-57.034</td>
<td>-57.179</td>
<td>-57.289</td>
<td>-56.778</td>
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Cohesive energy calculation

The cohesive energy ($E_{coh}$) of 2D BiBi$_2$X$_3$ (X=S/Se/Te), 2L Bi$_2$X$_3$ (X=S/Se/Te), and Bi monolayer (ML) can be calculated by

$$E_{coh}(A_xB_y) = \frac{1}{x+y}(xE^A_{atom} + yE^B_{atom} - E_{tot}),$$

where $x$ and $y$ are the number of A and B atom in unit cell, respectively, and $E^A_{atom}$, $E^B_{atom}$, and $E_{tot}$ are the total energies of A atom, B atom, and 2D $A_xB_y$ unit cell, respectively. The calculated cohesive energies of 2D BiBi$_2$X$_3$ (X=S/Se/Te), 2L Bi$_2$X$_3$ (X=S/Se/Te), and Bi ML are listed in Table S2.

Binding energy calculation

In order to further access the possibility to synthesize these multilayer structures, we have calculated the binding energy ($E_b$) of 2D BiBi$_2$X$_3$ (X=S/Se/Te) and 2L Bi$_2$X$_3$ via the formula:

$$E_b = E_A + E_B - E_t,$$

where $E_A$ and $E_B$ are total energies of 2D components forming the multilayer structures, and $E_t$ is the total energy of multilayer structures.

Table S2 Calculated cohesive energy ($E_{coh}$) of 2D BiBi$_2$X$_3$ (X=S/Se/Te), 2L Bi$_2$X$_3$ (X=S/Se/Te), and Bi ML, respectively. Calculated binding energy ($E_b$) of 2D BiBi$_2$X$_3$ (X=S/Se/Te) and 2L Bi$_2$X$_3$ (X=S/Se/Te), respectively. The units are eV for both $E_{coh}$ and $E_b$. 

<table>
<thead>
<tr>
<th>Systems</th>
<th>BiBi$_2$Se$_3$</th>
<th>BiBi$_2$Te$_2$S</th>
<th>BiBi$_2$Te$_2$Se</th>
<th>Bi$_2$Se$_3$</th>
<th>Bi$_2$Te$_2$S</th>
<th>Bi$_2$Te$_2$Se</th>
<th>Bi</th>
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<tbody>
<tr>
<td>$E_{coh}$</td>
<td>2.947</td>
<td>2.382</td>
<td>2.240</td>
<td>2.887</td>
<td>2.771</td>
<td>2.673</td>
<td>1.939</td>
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<tr>
<td>$E_{b}$</td>
<td>2.605</td>
<td>2.675</td>
<td>2.642</td>
<td>2.219</td>
<td>2.293</td>
<td>2.420</td>
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