1	- Electronic Supplementary Information -
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3	Engineering deformation-free
4	plastic spiral inertial microfluidic system
5	for CHO cell clarification in biomanufacturing
6	
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Fig. S1. Analysis of channel deformation in the MDDS device for blood separation based on the
confocal imaging. Height profiles of the cross-section at each loop under various input flow rate
conditions. MDDS device, multi-dimensional double spiral device.



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Fig. S2. Analysis of channel deformation in the PDMS spiral device for CHO cell retention based 37 on numerical simulation. (a) Channel configuration of the CHO retention device. (b) A 3D profile 38 39 of the overall channel deformation; for clearer visualization, the 3D deformation profile was amplified with a scale factor of 25. (c) The channel deformation at each loop in a cross-sectional 40 41 view; the solid black line represents the initial channel outline (scale bar: 400 µm). Profiles of the 42 deformation ratio at each loop under (d) various input flow rate conditions, (e) various Young's 43 modulus conditions, and (f) various PDMS thickness conditions, obtained by the numerical 44 simulation; in (e), the confocal imaging result at the optimal flow rate of 10 mL/min was overlaid 45 for comparison. The deformation ratio was defined by '(change of the cross-section area)/(original 46 cross-sectional area)'. All the numerical simulation results were obtained at default parametric conditions of the input flow rate (10 mL/min), Young's modulus (2.25 MPa), and PDMS thickness 47

48 (5 mm) except when the parameter becomes a variable. CHO cell, Chinese Hamster Ovary cell;49 IW, inner wall; OW, outer wall.

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51 The channel deformation in the CHO retention device was also analyzed by the numerical 52 simulation and confocal imaging methods. As shown in Fig. S2a, the second spiral device has a conventional single spiral configuration but has a specific dimension that was optimized for 53 54 retention (or removal) of CHO cells in the size range of 10-20 µm. The device has a trapezoidal 55 cross-section with 1,500 µm in width and 180 and 110 µm in height for the inner-and outer-wall sides, respectively, with eight loops.<sup>1–3</sup> The three parameters of 1) input flow rate (default: 10.0 56 mL/min), 2) Young's modulus of PDMS (default: 2.25 MPa), and 3) PDMS thickness (default: 5 57 mm) were modulated in the numerical simulation. Fig. S2b shows the overall channel deformation 58 3D profile of the device, and Fig. S2c shows the channel deformation at each loop in a cross-59 60 sectional view (the solid black line represents the initial channel outline), under the default 61 conditions. Similar to the first device, the channel deformation was highest ( $\sim 70\%$  of deformation ratio from the initial dimension) at the inlet and decreased as going to the outlet according to the 62 pressure decrease. Fig. S2d shows the profiles of the deformation ratio at each loop depending on 63 the input flow rate. Due to the increased pressure drop, the deformation ratios for the entire channel 64 65 regions increase as the input flow rate increases (total pressure drop in the entire channel:  $\sim 1.36$ 66  $\times 10^5$ Pa, under the default parameter conditions). Fig. S2e shows the variation of the channel deformation profile depending on the PDMS modulus in the range of 1.5 to 3.0 MPa with the step 67 size of 0.25 MPa. The results clearly showed that the deformation ratio significantly decreases as 68 the modulus increases (up to 25% difference between 1.5 and 3.0 MPa conditions at the inlet region 69 which has the highest deformation). In the simulation, the input flow is applied to the inlet surface 70

71 as a boundary condition while the real input flow is infused into the device through an external 72 tubing, so the inlet surface keeps its initial shape and dimension without deformation as shown in 73 Fig. S2b. The constraint suppresses the channel deformation near the inlet region but becomes 74 negligible as going away from the inlet. Due to the smaller applied pressure in the inlet region of 75 the second spiral device compared to the first device, the suppression effect was more significant so that the deformation ratio at the channel of 'loop0' shows a lower value than the channel of 76 77 'loop1' (Fig. S2e). We also analyzed the channel deformation of the second spiral device 78 depending on the PDMS thickness (Fig. S2f). Similar to the first device, the results represented 79 that the effect of the PDMS thickness is negligible unless it becomes thin enough (<1 mm) to be comparable to the channel height. 80



Fig. S3. Analysis of channel deformation in the PDMS spiral device for CHO cell retention based on the confocal imaging. (a) Conversion and analysis process of the confocal cross-sectional image at the inlet region of the device using a MATLAB code. (b) Profiles of the deformation ratio at each loop under various input flow rate conditions, where the deformation ratio was defined by (change of the cross-section area)/(original cross-sectional area)'. (c) Height profiles of the crosssection at each loop under various input flow rate conditions. MDDS device, multi-dimensional double spiral device. CHO cell, Chinese Hamster Ovary cell.

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Fig. S3a,c show the image conversion process and the height profiles at the cross-section of the inlet region of the second spiral device, respectively. Similar to the first spiral device, the confocal imaging results clearly showed the channel deformation from its original shape (represented by the solid black line) and increase of the deformation with increasing the input flow rate (Fig. S3c). Fig. S3b shows the profiles of the deformation ratio at each loop depending on the

95 input flow rate. The profiles are very similar to the results from the numerical simulation (Fig. 96 S2d) and clearly show the decrease of the deformation ratio as going from the inlet to the outlet 97 and also the increase of the deformation ratio as increasing the input flow rate. Based on the comparison between the deformation profiles obtained from the numerical simulation and the 98 99 confocal imaging in Fig. S2e, we found that the second spiral device showed also their best match 100 at 2.25-MPa Young's modulus, which verifies that the same curing condition results in the same 101 mechanical property of PDMS. From the numerical simulation result at Young's modulus of 2.25 102 MPa and the optimal flow rate of 10.0 mL/min, the average deformation ratio of the entire spiral 103 channel in the MDDS device was 36.3%.



104

105 Fig. S4. The average deformation ratio of the MDDS and CHO cell retention devices depending 106 on the theoretically calculated dimensionless number  $(\Delta p w_0 / E h_0)$ , where  $\Delta p$  represents the 107 pressure drop applied to the device which was calculated by ignoring channel deformation,  $w_0$ 108 and  $h_0$  denote the original channel width and height, respectively, and *E* means Young's 109 modulus).

110

111 One can theoretically calculate the dimensionless number ( $\Delta p w_0 / E h_0$ ) to access the channel 112 deformation. The following equations were used to calculate the hydraulic resistance and the 113 pressure drop applied to the PDMS spiral devices:

114 
$$\Delta p = R \times Q \tag{S1}$$

115 
$$R = \frac{1}{1 - 0.63(h/w)} \frac{12\mu L}{h^3 w}$$
(S2)

116 where  $\Delta p$ , *R*, and *Q* denote the pressure drop, hydraulic resistance, and flow rate, respectively, 117 and *h*, *w*, and *L* are height, width, and length of the channel, respectively, and  $\mu$  represents 118 dynamic viscosity of a fluid. From the equation (S2), the calculated hydraulic resistances for the 119 1st and 2nd spiral channels in the MDDS device for blood separation were 13.5 and 3.01 120 ×10<sup>12</sup>Pa·s/m<sup>3</sup>, respectively, and the resistance for the CHO retention device was 1.21 121 ×10<sup>12</sup>Pa·s/m<sup>3</sup>; the average height value was applied for the channel having a trapezoidal cross-

section. Fig. S4 shows the relationship between the deformation ratio ( $\sim \Delta h / h_0$ , obtained by the 122 numerical simulation) and the dimensionless number ( $\Delta p w_0 / E h_0$ , theoretically calculated) for the 123 two PDMS spiral devices. Here, the average deformation ratio over the entire channel region was 124 used for  $\Delta h / h_0$  while the median pressure value (( $p_{inlet} + p_{outlet}) / 2$ ) was used for  $\Delta p$ . In the case 125 of the 1st spiral channel in the MDDS device,  $(\Delta p_{1stspiral} / 2 + \Delta p_{2ndspiral})$  was applied as the median 126 127 pressure value. Similar to Fig. 2l, for the MDDS device for blood separation, all the combinations of 12 flow rate conditions (from 0.25 to 3.0 mL/min with the step size of 0.25 mL/min) and 7 128 Young's modulus conditions (from 1.5 to 3.0 MPa with the step size of 0.25 MPa) were tested, 129 resulting in a total of 84 conditions, and for the CHO retention device, all the combinations of 20 130 131 flow rate conditions (from 1.0 to 20.0 mL/min with the step size of 1.0 mL/min) and 7 Young's 132 modulus conditions (from 1.5 to 3.0 MPa with the step size of 0.25 MPa) were tested, resulting in 133 a total of 140 conditions. As shown in Fig. S4, the deformation ratio does not show linear dependency on the dimensionless number  $(\Delta p w_0 / E h_0)$ , different from Fig. 21. Because the 134 135 theoretically calculated pressure drop does not reflect the channel deformation, it has a higher 136 value than its actual value under the channel deformation, and the difference between the calculated and actual values increases as the dimensionless number ( $\Delta pw_0 / Eh_0$ ) increases due to 137 138 a larger channel deformation. As a result, the deformation ratio has a nonlinear dependency on the dimensionless number  $(\Delta p w_0 / E h_0)$  with a decreasing slope. 139



**Fig. S5.** Particle trajectories in the plastic MDDS device. (a) Channel configuration of the plastic MDDS device; the red boxes represent the observation spots. (b) Particle trajectories at the Sshaped transition region and the outlet bifurcation region under various input flow rate conditions; particles having diameters of 6 (green) and 10  $\mu$ m (red) were used to mimic the movement of RBCs and WBCs, respectively (scale bar: 200  $\mu$ m). (c) Trajectories of blood cells (500× diluted blood sample) at the S-shaped transition region and the outlet bifurcation region under various

- 147 input flow rate conditions (scale bar: 200 µm). MDDS device, multi-dimensional double spiral
- 148 device; IW, inner wall; OW, outer wall; RBC, red blood cell; WBC, white blood cell.



150 Fig. S6. Trajectories of CHO cells in the original PDMS spiral device for CHO cell retention. (a)151 Channel configuration of the original PDMS spiral device; the red box represents the observation

152 spot. (b) Trajectories of CHO cells at each loop (scale bar: 500 µm). CHO cell, Chinese Hamster

<sup>153</sup> Ovary cell; IW, inner wall; OW, outer wall.



Fig. S7. Trajectories of CHO cells in the plastic spiral device for CHO cell retention. (a) Channel configuration of the plastic spiral device; the red box represents the observation spot. (b) Trajectories of CHO cells at the outlet bifurcation region under various input flow rate conditions (scale bar: 500 µm). CHO cell, Chinese Hamster Ovary cell; IW, inner wall; OW, outer wall.



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Fig. S8. Analysis of fluidic behavior in the multiplexed device based on electric circuit analogy. 160 (a) The electric circuit model of the 25-layers-stacked device for circuit simulation. (b) The 161 pressure (or voltage) drop applied to the multiplexed device depending on the stacking number, 162 163 analyzed by circuit simulation. (c) The electric circuit model for theoretical calculation of the equivalent hydraulic resistance of the *n*-layers-stacked device ( $R_{eq}^{n}$ ), where  $R_{D}$ ,  $R_{IH}$ , and  $R_{OH}$  are 164 hydraulic resistances of the spiral device and the inlet-holes and outlet-holes connection parts 165 between layers, respectively. (d) Theoretical calculation (solid blue line) of the pressure (or 166 voltage) drop applied to the multiplexed device depending on the stacking number, compared with 167

168 the circuit simulation results (black circle). (e) The electric circuit model for theoretical calculation 169 of the flow rate (or current) applied on the first-layer device. (f) The electric circuit model for the 170 recurrence relation between flow rates into the k<sup>th</sup>-layer and (k+1)<sup>th</sup>-layer devices. IH, inlet 171 holes; OH, outlet holes; D, device.

172

We analyzed fluidic behavior in the multi-layer stacked plastic spiral device based on the electric circuit analogy using circuit simulation and theoretical calculation, where hydraulic resistance, pressure, and flow rate can be interpreted by electrical resistance, voltage, and current, respectively.<sup>4</sup> The hydraulic resistance of the spiral channel was calculated based on the equation (S2) while the hydraulic resistance of the connection parts (inlet and outlet holes) were calculated based on the following equation because they have a circular cross-section:

179 
$$R = \frac{128\mu L}{\pi d^4}$$
(S3)

where d and L are the diameter and length of the channel, respectively, and  $\mu$  represents the 180 181 dynamic viscosity of a fluid. One of the four spiral channels in the quad-version plastic CHO 182 retention device has the following channel dimension: average height of 217.5 µm, width of 1,500 183  $\mu$ m, and length of ~0.237 m. Based on the resistance equation for the channel having a rectangular cross-section, the calculated hydraulic resistance was  $\sim 2.03 \times 10^{11} \text{Pa} \cdot \text{s/m}^3$ . Because the plastic 184 spiral device consists of four spiral channels, the hydraulic resistance of the quad-version plastic 185 device  $(R_{\rm p})$  was calculated by diving the hydraulic resistance of a single device by four, resulting 186 in  $R_{\rm D} \sim 5.09 \times 10^{10} \text{Pa} \cdot \text{s/m}^3$ . The connection parts (inlet and outlet holes) have the following channel 187 dimension: diameter of 2 mm and length of 1.5 mm. Based on the resistance equation for the 188 channel having a circular cross-section, the calculated hydraulic resistance of a single hole was 189

190  $\sim 3.83 \times 10^{6} Pa \cdot s/m^{3}$ . Because the quad-version plastic device has four inlets, the hydraulic resistance of inlet holes ( $R_{\rm IH}$ ) was calculated by dividing the hydraulic resistance of a single hole 191 by four, resulting in  $R_{\rm IH} \sim 9.57 \times 10^5 \text{Pa} \cdot \text{s/m}^3$ . The quad-version plastic device has three outlets (2 192 for the inner-wall side outlets and 1 for the outer-wall side outlet), but the majority of fluid comes 193 194 from the two inner-wall side outlets by using an external flow regulator to harvest only the cellclarified portion from the outer-wall side outlet. To simply the electric circuit model, we assumed 195 196 that fluid comes out through only the two inner-wall side outlets at the same flow rate, so the hydraulic resistances of outlet holes ( $R_{\rm OH}$ ) were calculated by dividing the hydraulic resistance of 197 a single hole by two,  $R_{\rm OH} \sim 1.91 \times 10^6 \text{Pa} \cdot \text{s/m}^3$ . 198

199 An electric circuit simulation software (LTspice, Analog Devices, Inc., USA) was used for the 200 electric circuit analogy. In modeling of the electric circuit, hydraulic resistance, pressure, and flow 201 rate (40 mL/min × stacking number) were directly applied to electrical resistance, voltage, and current, respectively. For a more intuitive interpretation, a flow rate value having a 'mL/min' unit 202 203 was directly applied to a current value of the pump. Fig. S8a shows the electric circuit model for 204 the 25-layers-stacked device, where 25 device resistances (D#) are connected in parallel through 205 the inlet hole resistances (IH#) and the outlet hole resistance (OH#). Fig. 4e shows profiles of the flow rate applied to each layer of a stacked device depending on the stacking number. Fig. S8b 206 shows the pressure applied to the multiplexed device depending on the stacking number, which 207 was calculated based on the voltage applied to the pump in the circuit simulation; (10-208 209  $^{6}$ m<sup>3</sup>/mL)×(1min/60s) was multiplied to the obtained voltage value for the unit change. As expected, the result shows that the applied pressure increases as the stacking number increases because more 210 211 connection parts are engaged in the stacked device.

To obtain the closed-form formula of the hydraulic resistance and flow distribution in the multiplexed device, theoretical analysis was also performed based on the electric circuit analogy. As shown in **Fig. S8**c, from the equivalent circuit models, we can obtain the following recurrence relation between the hydraulic resistances of the *n*-layers-stacked device ( $R_{eq}^n$ ) and the (*n*+1)layers-stacked device ( $R_{eq}^{n+1}$ ) as below:

217  

$$R_{eq}^{n+1} = R_{IH} + R_{OH} + \left(\frac{1}{R_D} + \frac{1}{R_{eq}^n}\right)^{-1}$$

$$= R_C + \frac{R_{eq}^n R_D}{R_{eq}^n + R_D}$$
(S4)

218 where  $R_{\rm D}$  and  $R_{\rm C}(=R_{\rm IH}+R_{\rm OH})$  are hydraulic resistances of the spiral device and the (inlet-holes 219 and outlet-holes) connection parts between layers, respectively. Using the arithmetics of 220 determinants,<sup>5</sup> we can obtain the closed-form expression of the hydraulic resistances of the *n*-221 layers-stacked device ( $R_{\rm eq}^n$ ) as follows:

222 
$$R_{eq}^{n} = \frac{R_{c}}{2} \times \frac{(\sqrt{1+4\beta}+1)^{2n+1} + (\sqrt{1+4\beta}-1)^{2n+1}}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}}$$
(S5)

223 where  $\beta (= R_D / R_C)$  denotes the ratio between resistances of the device  $(R_D)$  and the connection-224 part  $(R_C)$ . Also, using Ohm's law, we can calculate the pressure applied to the multiplexed device 225 as follows:

226 
$$V_{\rm T}^{n} = I_{\rm T}^{n} R_{\rm eq}^{n} = n I_{0} R_{\rm eq}^{n} = \frac{n I_{0} R_{\rm c}}{2} \times \frac{\left(\sqrt{1+4\beta}+1\right)^{2n+1} + \left(\sqrt{1+4\beta}-1\right)^{2n+1}}{\left(\sqrt{1+4\beta}+1\right)^{2n} - \left(\sqrt{1+4\beta}-1\right)^{2n}}$$
(S6)

where  $I_0$  is the desired flow rate for a single device, and  $V_T^n$  and  $I_T^n (= nI_0)$  denote the pressure and the input flow rate applied to the *n*-layers-stacked device; note that  $(10^{-6}m^3/mL) \times (1min/60s)$  should be multiplied to the obtained  $V_{\rm T}^n$  value for compensation of using the unit of 'mL/min'. As shown in **Fig. S8**d, the theoretically calculated pressure was perfectly identical to the circuit simulation results at the tested conditions. To obtain the formula for the flow rate into an individual layer, we obtained the recurrence relation between flow rates into the *k* th-layer and (*k*+1) th-layer devices based on the electric circuit models in **Fig. S8**e,f, as follows:

234 
$$V_{\rm T}^n = n\mathbf{I}_0 \times R_{\rm IH} + I_1^n \times R_{\rm D} + n\mathbf{I}_0 \times R_{\rm OH}$$
(S7)

235 
$$I_1^n = \frac{(V_T^n - nI_0R_C)}{R_D}$$
(S8)

236 
$$\left(nI_{0} - \sum_{i=1}^{k} I_{i}\right) \times R_{\mathrm{IH}} + I_{k+1}^{n} \times R_{\mathrm{D}} + \left(nI_{0} - \sum_{i=1}^{k} I_{i}\right) \times R_{\mathrm{OH}} - I_{k}^{n} \times R_{\mathrm{D}} = 0$$
(S9)

237 
$$I_{k+1}^{n} = I_{k}^{n} - \frac{1}{\beta} \left( n I_{0} - \sum_{i=1}^{k} I_{i} \right)$$
(S10)

where  $I_k^n$  denotes the flow rate into the *k* <sup>th</sup>-layer device in the *n*-layers-stacked device. From the initial condition and the recurrence relation in the equations (S8) and (S10), we can obtain the closed-form expression of  $I_k^n$  as below:

$$I_{k}^{n} = \frac{I_{T}^{n}}{\beta^{k} 2^{2k-1}} \times \frac{(\sqrt{1+4\beta}+1)^{2n} (\sqrt{1+4\beta}-1)^{2k-1} + (\sqrt{1+4\beta}-1)^{2n} (\sqrt{1+4\beta}+1)^{2k-1}}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}}$$

$$= \frac{nI_{0}}{\beta^{k} 2^{2k-1}} \times \frac{(\sqrt{1+4\beta}+1)^{2n} (\sqrt{1+4\beta}-1)^{2k-1} + (\sqrt{1+4\beta}-1)^{2n} (\sqrt{1+4\beta}+1)^{2k-1}}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}}$$
(S11)

241

As shown in **Fig. 4**e, we found that the results from the circuit simulation and the theoretical calculation for the flow rate were perfectly identical to each other, and the results showed that the flow rate variation increased as the stacking number increased due to the more connection parts engaged in the multiplexed device. The first layer (k = 1) has the maximum variation in flow rate from the desired flow rate for a single device ( $I_0$ ). From the equation (S11), we derived a formula representing the maximum variation in flow rate as follows:

249

$$\nu = \frac{I_1^n - I_0}{I_0} = \frac{n}{2\beta} \times \left( \frac{(\sqrt{1+4\beta}+1)^{2n}(\sqrt{1+4\beta}-1) + (\sqrt{1+4\beta}-1)^{2n}(\sqrt{1+4\beta}+1)}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}} \right) - 1$$

$$= 2n \times \left( \frac{(\sqrt{1+4\beta}+1)^{2n-1} + (\sqrt{1+4\beta}-1)^{2n-1}}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}} \right) - 1$$
(S12)

250 In the equation (S12), the bracket approaches a constant value as  $n \rightarrow \infty$  as follows:

251  

$$\lim_{n \to \infty} \left( \frac{(\sqrt{1+4\beta}+1)^{2n-1} + (\sqrt{1+4\beta}-1)^{2n-1}}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}} \right) = \lim_{n \to \infty} \left( \frac{(\sqrt{1+4\beta}+1)^{-1} + (\frac{\sqrt{1+4\beta}-1}{\sqrt{1+4\beta}+1})^{2n-1}(\sqrt{1+4\beta}+1)^{-1}}{1 - (\frac{\sqrt{1+4\beta}-1}{\sqrt{1+4\beta}+1})^{2n}} \right)$$

$$= \frac{1}{\sqrt{1+4\beta}+1}$$
(S13)

252 Therefore, combining the equations (S12) and (S13), the maximum variation in flow rate,  $\nu$ , 253 diverges to infinity as  $n \to \infty$ . On the other hand, the maximum variation in flow rate,  $\nu$ , 254 converges to 0 as  $\beta \to \infty$  as follows:

$$\lim_{\beta \to \infty} v = \lim_{\beta \to \infty} \left( 2n \times \left( \frac{(\sqrt{1+4\beta}+1)^{2n-1} + (\sqrt{1+4\beta}-1)^{2n-1}}{(\sqrt{1+4\beta}+1)^{2n} - (\sqrt{1+4\beta}-1)^{2n}} \right) - 1 \right)$$
$$= \lim_{x \to \infty} \left( 2n \times \left( \frac{x^{2n-1} + (x-2)^{2n-1}}{x^{2n} - (x-2)^{2n}} \right) - 1 \right), \text{ where } x = \sqrt{1+4\beta} + 1$$
$$= \lim_{x \to \infty} \left( 2n \times \left( \frac{2x^{2n-1} + L}{-2n(-2)x^{2n-1} + L} \right) - 1 \right)$$
$$= 2n \times \frac{1}{2n} - 1 = 0$$
(S14)

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