**Fine Manipulation of Terahertz Wave via All-Silicon Metasurfaces**

with Independent Amplitude and Phase

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**Part1. Details about the design principles of the proposed metasurfaces.**

For the first design, the phase difference $\Delta \phi$ is set to $\Delta \phi=90^\circ$, the co- and cross-polarized components have the same energy partition. By adjusting the values of the summation of propagation phase $\sum \phi/2$ and the rotation angle $\alpha$, specific phase distributions can be set in the co- and cross- polarized components.

In the design process, the phase distribution of $\varphi_{LL}$ and $\varphi_{LR}$ as

\[
\varphi_{LL} = \frac{2\pi}{\lambda} \left( \sqrt{f^2 + r^2} - f \right) - 2\theta \tag{S1}
\]

\[
\varphi_{LR} = \frac{2\pi}{\lambda} \left( \sqrt{f^2 + r^2} - f \right) - \theta
\]

Here, the phase distribution of $\varphi_{LL}$ is controlled by changing the lengths of two rectangles $L_1$ and $L_2$, while the phase of $\varphi_{LR}$ is controlled by rotating the meta-atoms. The rotation angle $\alpha$ is described as

\[
\alpha = (\theta + \pi/2)/2 \tag{S2}
\]
When the handedness of incident wave is changed to RCP, the phase distribution of the co-polarized component would change to
\[
\varphi_{bb} = \frac{2\pi}{\lambda} \left( \sqrt{f^2 + r^2} - f \right) - 2\theta \quad \text{MERGEFORMAT (S3)}
\]
As for the cross-polarized component, the phase distribution would change to
\[
\varphi_{bl} = \frac{2\pi}{\lambda} \left( \sqrt{f^2 + r^2} - f \right) - 3\theta \quad \text{MERGEFORMAT (S4)}
\]

For the second design, the intensity ratio of the co- and cross-polarizations can be tuned by adjusting the phase difference \(\Delta\varphi\). Before the start of these designs, the transmission amplitudes and phase shifts of the meta-atoms have been simulated by parameter sweeping using CST Microwave Studio. Here, five metasurfaces are designed by adopting five groups of meta-atoms with different phase difference \(\Delta\varphi = 0^\circ, 45^\circ, 90^\circ, 135^\circ,\) and \(180^\circ\). The specific parameters of these five groups of meta-atoms are listed in the following table, and this table has been added in the Supplementary Information.

| \(\Delta\varphi\) (Degree) | L₁ | L₂ | L₁ | L₂ | L₁ | L₂ | L₁ | L₂ | L₁ | L₂ | L₁ | L₂ | L₁ | L₂ | L₁ | L₂ |
|---------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0                         | 85 | 85 | 69 | 69 | 59 | 59 | 53 | 53 | 49 | 49 | 45 | 45 | 41 | 41 | 34 | 34 |
| 45                        | 96 | 81 | 83 | 66 | 69 | 56 | 59 | 52 | 53 | 48 | 50 | 44 | 46 | 39 | 43 | 30 |
| 90                        | 100| 83 | 96 | 65 | 88 | 55 | 71 | 51 | 60 | 47 | 55 | 43 | 52 | 38 | 50 | 30 |
| 135                       | 40 | 96 | 35 | 85 | 97 | 60 | 93 | 51 | 79 | 47 | 64 | 44 | 58 | 39 | 56 | 32 |
| 180                       | 46 | 96 | 43 | 83 | 39 | 69 | 31 | 63 | 96 | 46 | 84 | 42 | 68 | 39 | 62 | 32 |

The process of designing the phase distributions of the co- and cross-polarized components is the same as that of the first design. The difference is that the adopted meta-atoms need to be selected from these five groups, which correspond to different intensity ratios of the co- and cross-polarizations.

For the third design, we select six groups of meta-atoms (48 meta-atoms), which correspond to the amplitude values 0, 0.2, 0.4, 0.6, 0.8, and 1, respectively. These meta-atoms can constitute a structure database which can achieve a free combination of transmitted amplitude from 0 to 1 and phase shifts of 0 to \(7\pi/4\). For this design, the required complex amplitude distribution of the co-polarized component is described as
\[
E = \alpha_1 e^{\frac{2\pi}{\lambda} \left( \sqrt{f^2 + (x+\varepsilon)^2 + y^2} - f \right) \cdot \theta} + \alpha_2 e^{\frac{2\pi}{\lambda} \left( \sqrt{f^2 + (x-\varepsilon)^2 + y^2} - f \right) \cdot \theta} \quad \text{MERGEFORMAT (S5)}
\]

Hence, the amplitude distribution can be described as abs\(|E|\), and the required phase is \(\text{angle}(E)\). We need consider the amplitude and phase distributions in this design simultaneously. For example, \(E(m, n)\), it is the meta-atom in the mth row and nth column. Its amplitude is abs\(|E(m, n)|\) and its phase is \(\text{angle}(E(m, n))\). According to these amplitude and phase values, we can select the meta-atom from the structure database. As for the cross-polarized component, it is arranged as linear gradient phase profile along x-axis, which can be realized by rotating the meta-atoms.
Part 2: The overall layouts of the metasurfaces corresponding to the second design.

\[ \Delta \phi = 0^\circ \quad \Delta \phi = 45^\circ \quad \Delta \phi = 90^\circ \]
\[ \Delta \phi = 135^\circ \quad \Delta \phi = 180^\circ \]

Fig. S1 The overall layouts of the metasurfaces corresponding to the second design.

Part 3. The overall layout of the metasurface corresponding to the third design.

Fig. S2 The overall layout of the metasurfaces corresponding to the third design.