## Supplementary material to "Anomalous thermal conductivity enhancement in low dimensional resonant nanostructures due to imperfections"

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## 1. Phonon dispersions and group velocities



**Figure S1**. Phonon dispersion curves in different frequency ranges for (a) pristine GNR (0-50THz), (b) pristine GNR (5-10 THz), (c) PR-GNR (5-10 THz), (d) DR-GNR (5-10 THz), and (d) HR-GNR-2m (5-10 THz).



**Figure S2**. Comparison of phonon group velocities between Pristine GNR, PR-GNR, DR-GNR and HR-GNR-2m.

## 2. In-plane and out-of-plane decomposition of TC



**Figure S3**. The in-plane (a) and out-of-plane (b) contributions of spectrally decomposed TC for different structures.