

Applying Band Gap Engineering to Tune Linear Optical and Nonlinear Optical Properties of Noncentrosymmetric Chalcogenides $\text{La}_4\text{Ge}_3\text{Se}_x\text{S}_{12-x}$ ($x = 0, 2, 4, 6, 8, 10$)

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Table S1. Selected crystal data and unit cell parameters for $\text{La}_4\text{Ge}_3\text{Se}_x\text{S}_{12-x}$ ($x = 0, 2, 4, 6, 8, 10$)

Empirical Formula	$\text{La}_4\text{Ge}_3\text{S}_{12}$	$\text{La}_4\text{Ge}_3\text{Se}_2\text{S}_{10}$	$\text{La}_4\text{Ge}_3\text{Se}_4\text{S}_8$	$\text{La}_4\text{Ge}_3\text{Se}_6\text{S}_6$	$\text{La}_4\text{Ge}_3\text{Se}_8\text{S}_4$	$\text{La}_4\text{Ge}_3\text{Se}_{10}\text{S}_2$
Formula weight	1158.13	1247.40	1356.28	1423.43	1521.60	1616.26
Temperature	200(2) K	296(2) K	296(2) K	296(2) K	296(2) K	296(2) K
Radiation, wavelength	Mo-K α , 0.71073 Å	Mo-K α , 0.71073 Å	Mo-K α , 0.71073 Å	Mo-K α , 0.71073 Å	Mo-K α , 0.71073 Å	Mo-K α , 0.71073 Å
Crystal system	Rhombohedral					
Space group	$R\bar{3}c$ (No. 161)					
Unit cell dimensions	a=19.427(3) Å c=8.0753(13) Å	a=19.5575(6) Å c=8.1468(19) Å	a=19.6679(12) Å c=8.1950(6) Å	a=19.7766(11) Å c= 8.2448(4) Å	a=19.8976(12) Å c=8.2984(4) Å	a=20.0723(12) Å c=8.3460(5) Å
Unit cell volume	2639.2(10) Å ³	2698.64(19) Å ³	2745.3(4) Å ³	2792.6(3) Å ³	2845.3(4) Å ³	2912.1(4) Å ³
Z	6					
Density (calc)	4.372 g/cm ³	4.605 g/cm ³	4.922 g/cm ³	5.078 g/cm ³	5.328 g/cm ³	5.530 g/cm ³
Absorption coefficient	15.942 mm ⁻¹	19.221 mm ⁻¹	23.427 mm ⁻¹	25.491 mm ⁻¹	28.806 mm ⁻¹	31.713 mm ⁻¹
Final R indices [$I > 2\sigma(I)$]	$R_1 = 0.0323$, $wR_2 = 0.0634$	$R_1 = 0.0351$, $wR_2 = 0.0618$	$R_1 = 0.0323$, $wR_2 = 0.0518$	$R_1 = 0.0296$, $wR_2 = 0.0439$	$R_1 = 0.0391$, $wR_2 = 0.0709$	$R_1 = 0.0368$, $wR_2 = 0.0632$
Final R indices [all data]	$R_1 = 0.0383$, $wR_2 = 0.0763$	$R_1 = 0.0520$, $wR_2 = 0.0812$	$R_1 = 0.0415$, $wR_2 = 0.0597$	$R_1 = 0.0374$, $wR_2 = 0.0457$	$R_1 = 0.0646$, $wR_2 = 0.0840$	$R_1 = 0.0515$, $wR_2 = 0.0680$

Table S2. Refined atomic coordinates and isotropic displacement parameters for $\text{La}_4\text{Ge}_3\text{Se}_x\text{S}_{12-x}$ ($x = 0, 2, 4, 6, 8, 10$)

Atoms	Wyckoff	x	y	z	Occupancy	$U_{\text{eq}} (\text{\AA}^2)$
$\text{La}_4\text{Ge}_3\text{S}_{12}$						
La1	<i>18b</i>	0.56401(7)	0.66945(9)	0.97069(15)	1	0.0113(3)
La2	<i>6a</i>	0.3333	0.6667	0.6736(2)	1	0.0092(5)
Ge1	<i>18b</i>	0.34629(12)	0.47923(13)	1.0210(3)	1	0.0074(5)
S1	<i>18b</i>	0.2470(3)	0.4661(3)	1.1771(7)	1	0.0110(11)
S2	<i>18b</i>	0.3939(3)	0.6029(3)	0.9226(6)	1	0.0101(11)
S3	<i>18b</i>	0.6039(3)	0.6631(3)	1.3253(6)	1	0.0117(12)
S4	<i>18b</i>	0.5113(3)	0.5572(3)	0.6789(6)	1	0.0123(11)
$\text{La}_4\text{Ge}_3\text{Se}_2\text{S}_{10}$						
La1	<i>18b</i>	0.43819(6)	0.33469(7)	0.97798(14)	1	0.0188(3)
La2	<i>6a</i>	0.6667	0.3333	1.1813(2)	1	0.0138(4)
Ge1	<i>18b</i>	0.68003(10)	0.53332(11)	1.0269(2)	1	0.0125(4)
S1	<i>18b</i>	0.57954(18)	0.44671(19)	1.1871(4)	0.687(16)	0.0165(10)
Se1	<i>18b</i>	0.57954(18)	0.44671(19)	1.1871(4)	0.313(16)	0.0165(10)
S2	<i>18b</i>	0.4454(2)	0.4896(2)	1.0171(4)	0.796(15)	0.0178(12)
Se2	<i>18b</i>	0.4454(2)	0.4896(2)	1.0171(4)	0.204(15)	0.0178(12)
S3	<i>18b</i>	0.3916(2)	0.3286(2)	1.3313(5)	0.880(15)	0.0191(13)
Se3	<i>18b</i>	0.3916(2)	0.3286(2)	1.3313(5)	0.120(15)	0.0191(13)
S4	<i>18b</i>	0.5423(2)	0.2691(3)	0.9298(5)	1	0.0129(8)
$\text{La}_4\text{Ge}_3\text{Se}_4\text{S}_8$						
La1	<i>18b</i>	0.43797(5)	0.33346(6)	0.02593(9)	1	0.01817(19)
La2	<i>6a</i>	0.6667	0.3333	0.32203(13)	1	0.0112(3)
Ge1	<i>18b</i>	0.65285(8)	0.51901(8)	-0.02245(15)	1	0.0114(3)
S1	<i>18b</i>	0.75416(10)	0.53323(10)	-0.1847(2)	0.366(11)	0.0153(6)
Se1	<i>18b</i>	0.75416(10)	0.53323(10)	-0.1847(2)	0.634(11)	0.0153(6)
S2	<i>18b</i>	0.60615(18)	0.39664(18)	0.0732(3)	0.977(11)	0.0118(9)
Se2	<i>18b</i>	0.60615(18)	0.39664(18)	0.0732(3)	0.023(11)	0.0118(9)
S3	<i>18b</i>	0.39643(13)	0.33939(14)	-0.3276(3)	0.724(10)	0.0150(7)

Atoms	Wyckoff	<i>x</i>	<i>y</i>	<i>z</i>	Occupancy	U_{eq} (Å ²)
Se3	<i>18b</i>	0.39643(13)	0.33939(14)	-0.3276(3)	0.276(10)	0.0150(7)
S4	<i>18b</i>	0.49060(11)	0.44729(11)	0.3208(2)	0.522(11)	0.0156(6)
Se4	<i>18b</i>	0.49060(11)	0.44729(11)	0.3208(2)	0.478(11)	0.0156(6)
La ₄ Ge ₃ Se ₆ S ₆						
La1	<i>18b</i>	0.56130(4)	0.66595(5)	0.97383(8)	1	0.01846(18)
La2	<i>6a</i>	0.3333	0.6667	0.67836(12)	1	0.0115(2)
Ge1	<i>18b</i>	0.34722(7)	0.48143(6)	1.02229(14)	1	0.0118(3)
S1	<i>18b</i>	0.24572(7)	0.46723(8)	1.18522(16)	0.240(9)	0.0142(5)
Se1	<i>18b</i>	0.24572(7)	0.46723(8)	1.18522(16)	0.760(9)	0.0142(5)
S2	<i>18b</i>	0.39365(15)	0.60361(14)	0.9275(3)	0.965(9)	0.0119(9)
Se2	<i>18b</i>	0.39365(15)	0.60361(14)	0.9275(3)	0.035(9)	0.0119(9)
S3	<i>18b</i>	0.60387(9)	0.66028(10)	1.3274(2)	0.535(9)	0.0170(6)
Se3	<i>18b</i>	0.60387(9)	0.66028(10)	1.3274(2)	0.465(9)	0.0170(6)
S4	<i>18b</i>	0.50915(8)	0.55226(8)	0.67890(18)	0.374(8)	0.0150(5)
Se4	<i>18b</i>	0.50915(8)	0.55226(8)	0.67890(18)	0.626(8)	0.0150(5)
La ₄ Ge ₃ Se ₈ S ₄						
La1	<i>18b</i>	0.43908(7)	0.33400(8)	0.02710(15)	1	0.0200(3)
La2	<i>6a</i>	0.6667	0.3333	0.3209(2)	1	0.0155(5)
Ge1	<i>18b</i>	0.65278(12)	0.51812(12)	-0.0231(3)	1	0.0142(5)
S1	<i>18b</i>	0.75433(12)	0.53228(13)	-0.1854(3)	0.067(19)	0.0175(8)
Se1	<i>18b</i>	0.75433(12)	0.53228(13)	-0.1854(3)	0.933(19)	0.0175(8)
S2	<i>18b</i>	0.6063(2)	0.3959(2)	0.0723(5)	0.864(18)	0.0143(14)
Se2	<i>18b</i>	0.6063(2)	0.3959(2)	0.0723(5)	0.136(18)	0.0143(14)
S3	<i>18b</i>	0.39621(13)	0.34008(15)	-0.3270(3)	0.320(18)	0.0178(9)
Se3	<i>18b</i>	0.39621(13)	0.34008(15)	-0.3270(3)	0.680(18)	0.0178(9)
S4	<i>18b</i>	0.49129(13)	0.44805(12)	0.3201(3)	0.165(18)	0.0178(8)
Se4	<i>18b</i>	0.49129(13)	0.44805(12)	0.3201(3)	0.835(18)	0.0178(8)
La ₄ Ge ₃ Se ₁₀ S ₂						
La1	<i>18b</i>	0.56213(6)	0.66732(6)	0.97182(13)	1	0.0187(3)
La2	<i>6a</i>	0.3333	0.6667	0.6788(2)	1	0.0156(4)

Atoms	Wyckoff	x	y	z	Occupancy	U_{eq} (Å ²)
Ge1	<i>18b</i>	0.34685(10)	0.48149(10)	1.0251(2)	1	0.0129(4)
Se1	<i>18b</i>	0.24530(9)	0.46716(10)	1.1854(2)	1	0.0166(4)
S2	<i>18b</i>	0.39433(14)	0.60466(14)	0.9272(3)	0.577(15)	0.0131(9)
Se2	<i>18b</i>	0.39433(14)	0.60466(14)	0.9272(3)	0.423(15)	0.0131(9)
S3	<i>18b</i>	0.60401(9)	0.65991(11)	1.3267(2)	0.135(14)	0.0154(6)
Se3	<i>18b</i>	0.60401(9)	0.65991(11)	1.3267(2)	0.865(14)	0.0154(6)
S4	<i>18b</i>	0.50909(10)	0.55265(9)	0.6819(2)	0.032(14)	0.0166(6)
Se4	<i>18b</i>	0.50909(10)	0.55265(9)	0.6819(2)	0.968(14)	0.0166(6)

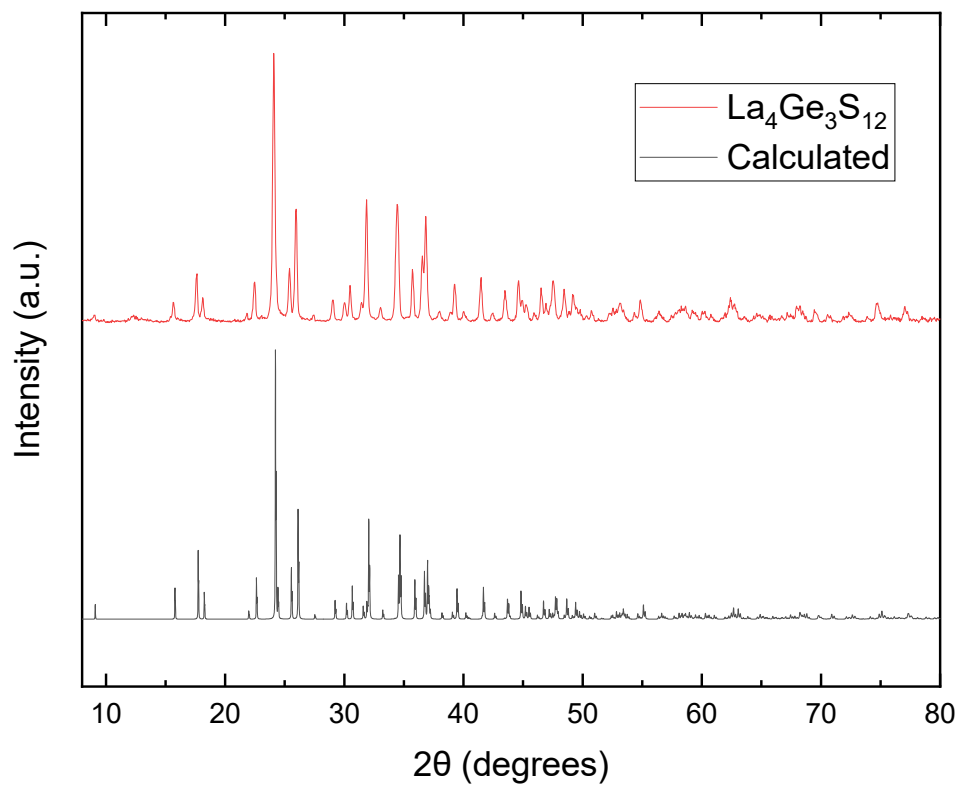
Table S3. Selected important interatomic distances (Å) in $\text{La}_4\text{Ge}_3\text{Se}_x\text{S}_{12-x}$ ($x = 0, 2, 4, 6, 8, 10$)

Atom Pairs		Distances (Å)	Atom Pairs		Distances (Å)
$\text{La}_4\text{Ge}_3\text{S}_{12}$					
La1	S3	2.868(5)	La2	S2	2.901(5)
	S2	2.911(5)		S2	2.901(5)
	S4	2.945(6)		S2	2.901(5)
	S3	2.985(5)		S2	2.913(5)
	S1	3.013(5)		S2	2.913(5)
	S4	3.021(6)		S2	2.913(5)
	S1	3.089(5)		S1	3.386(6)
Ge1	S4	2.197(6)	S1	3.386(6)	
	S3	2.200(5)	S1	3.386(6)	
	S1	2.209(6)			
	S2	2.244(6)			
$\text{La}_4\text{Ge}_3\text{Se}_2\text{S}_{10}$					
La1	Se3	2.888(4)	La2	S4	2.922(4)
	S3	2.888(4)		S4	2.922(4)
	S4	2.924(4)		S4	2.922(4)
	S2	2.979(4)		S4	2.939(4)
	S3	3.004(4)		S4	2.939(4)
	S1	3.048(3)		S4	2.939(4)
	Se2	3.073(4)		Se1	3.406(3)
	S2	3.073(4)		Se1	3.406(3)
	S1	3.118(3)			
	Ge1	Se3		2.229(4)	
S3		2.229(4)			
S4		2.237(5)			
S2		2.244(4)			
Se2		2.244(4)			
S1		2.260(4)			

Atom Pairs		Distances (Å)	Atom Pairs		Distances (Å)
La₄Ge₃Se₄S₈					
La1	Se3	2.908(2)	La2	Se2	2.934(3)
	S3	2.908(2)		S2	2.934(3)
	S2	2.920(3)		Se2	2.934(3)
	Se4	3.023(2)		S2	2.934(3)
	S4	3.023(2)		S2	2.934(3)
	S3	3.028(2)		Se2	2.947(3)
	S1	3.0745(18)		S2	2.947(3)
	Se1	3.0745(18)		Se2	2.947(3)
	S4	3.100(2)		S2	2.947(3)
	Ge1	S2		2.245(3)	Se2
Se3		2.255(3)	S2	2.947(3)	
S3		2.255(3)			
Se4		2.279(2)			
S4		2.279(2)			
S1		2.293(2)			
La₄Ge₃Se₆S₆					
La1	Se3	2.9257(17)	La2	Se2	2.947(3)
	S3	2.9257(17)		S2	2.947(3)
	S2	2.928(3)		Se2	2.947(3)
	S3	3.0514(18)		S2	2.947(3)
	Se4	3.0540(16)		S2	2.947(3)
	S4	3.0540(16)		Se2	2.957(3)
	S1	3.0928(14)		S2	2.957(3)
	Se1	3.0928(14)		Se2	2.957(3)
	S4	3.1166(16)		S2	2.957(3)
	Ge1	S2		2.253(3)	Se2
Se3		2.280(2)	S2	2.957(3)	
S3		2.280(2)			
Se4		2.2986(18)			

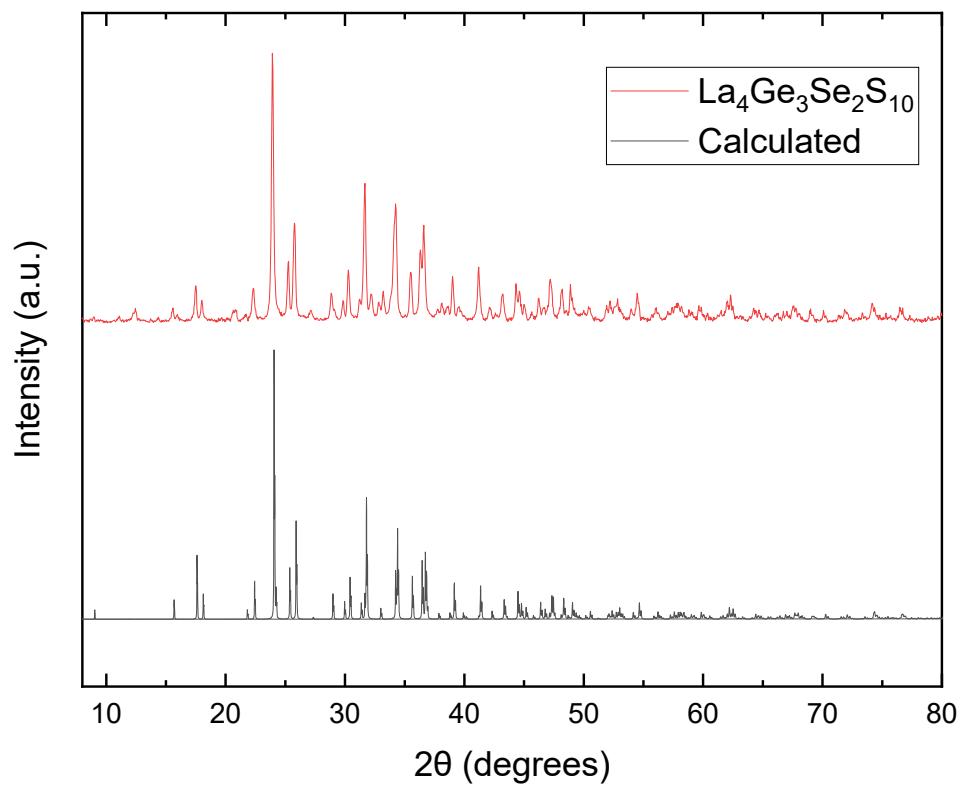
Atom Pairs		Distances (Å)	Atom Pairs		Distances (Å)
	S4	2.2986(18)			
Ge1	S1	2.3128(18)			
<hr/>					
La ₄ Ge ₃ Se ₈ S ₄					
La1	S2	2.937(4)	La2	Se2	2.957(4)
	Se3	2.954(3)		S2	2.957(4)
	S3	2.954(3)		Se2	2.957(4)
	S3	3.079(3)		S2	2.957(4)
	Se4	3.083(3)		S2	2.957(4)
	S4	3.083(3)		Se2	2.973(4)
	S1	3.109(2)		S2	2.973(4)
	Se1	3.109(2)		Se2	2.973(4)
	S4	3.128(3)		S2	2.973(4)
Ge1	S2	2.269(4)		Se2	2.973(4)
	Se3	2.309(3)		S2	2.973(4)
	S3	2.309(3)			
	Se4	2.319(3)			
	S4	2.319(3)			
	S1	2.325(3)			
<hr/>					
La ₄ Ge ₃ Se ₁₀ S ₂					
La1	S2	2.971(3)	La2	Se2	2.978(3)
	Se3	2.9745(19)		S2	2.978(3)
	S3	2.9745(19)		Se2	2.978(3)
	Se4	3.086(2)		S2	2.978(3)
	S4	3.086(2)		S2	2.978(3)
	S3	3.103(2)		Se2	2.997(3)
	Se1	3.1272(19)		S2	2.997(3)
	S4	3.136(2)		Se2	2.997(3)
	Se1	3.188(2)		S2	2.997(3)
Ge1	S2	2.309(3)		Se2	2.997(3)
	Se4	2.332(2)		S2	2.997(3)

Atom Pairs	Distances (Å)	Atom Pairs	Distances (Å)
	S4		2.332(2)
	Se1		2.333(2)
Ge1	Se3		2.339(3)
	S3		2.339(3)



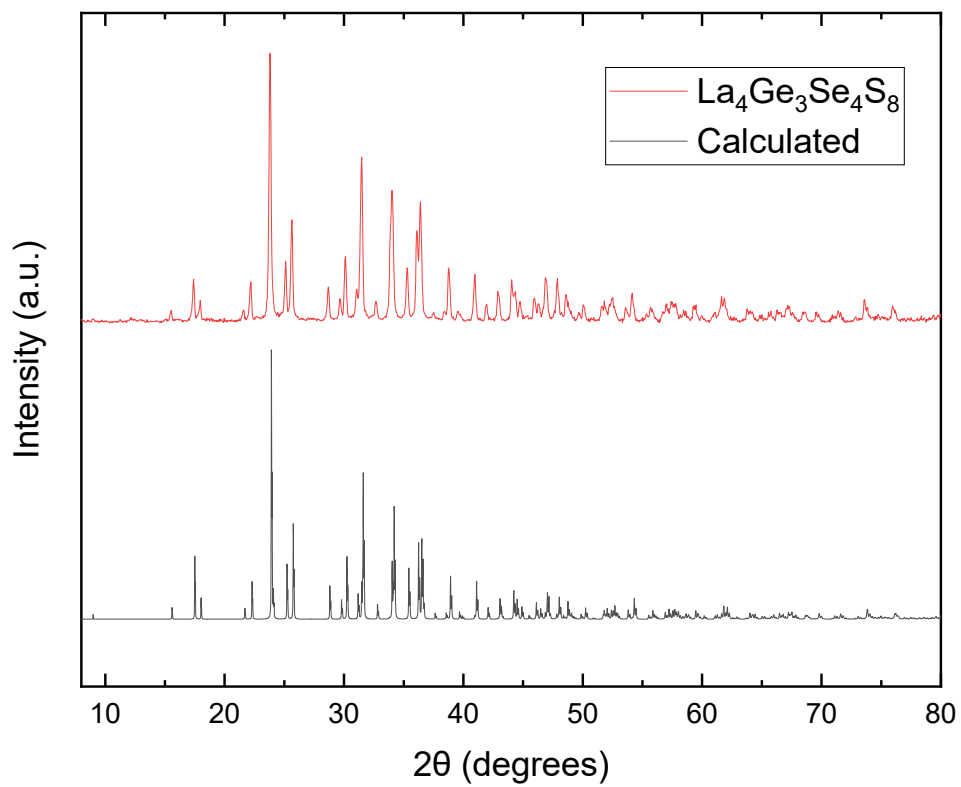
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Figure S1. The calculated and experimental X-ray diffraction patterns of $\text{La}_4\text{Ge}_3\text{S}_{12}$.



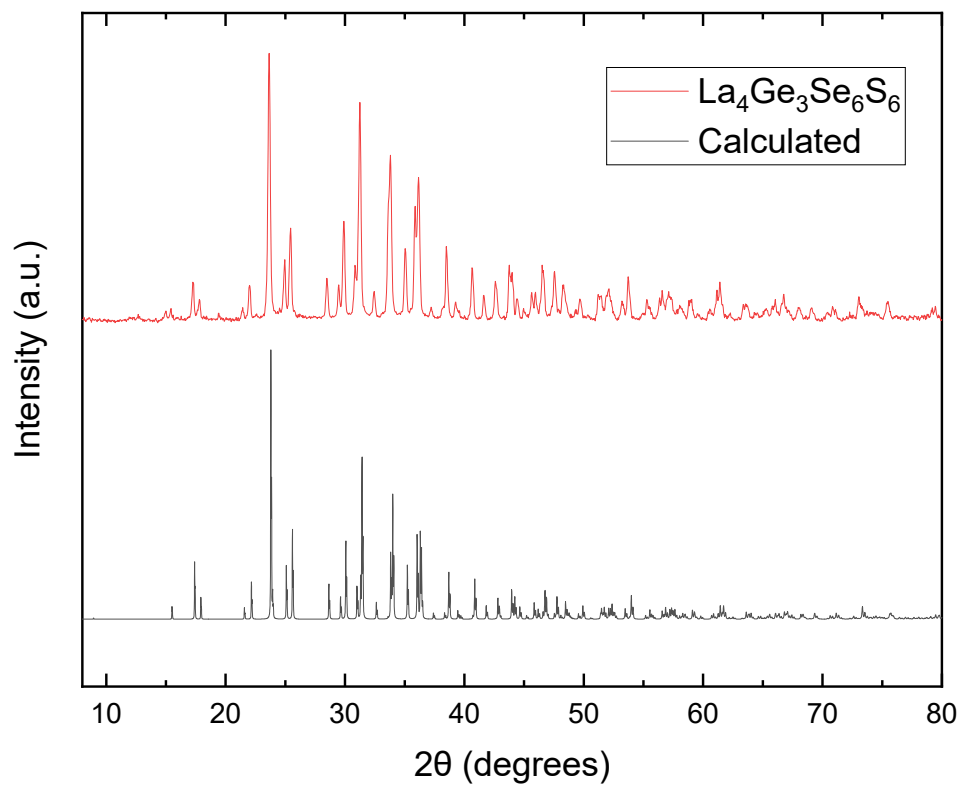
Fi

Figure S2. The calculated and experimental X-ray diffraction patterns of $\text{La}_4\text{Ge}_3\text{Se}_2\text{S}_{10}$.



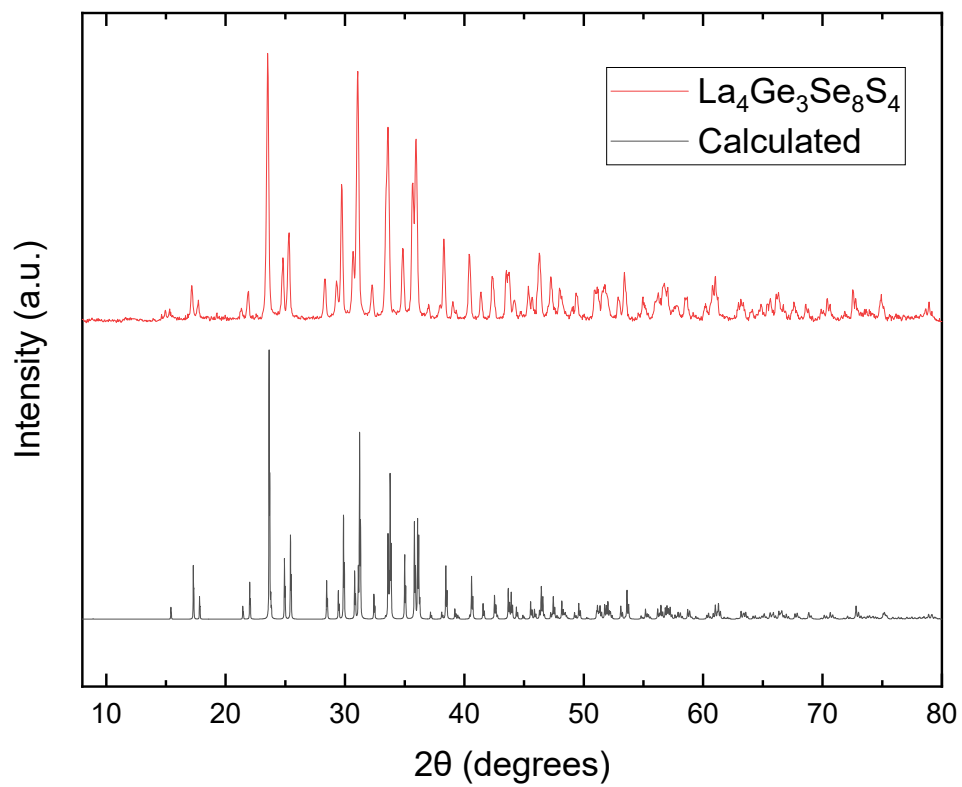
Fi

Figure S3. The calculated and experimental X-ray diffraction patterns of $\text{La}_4\text{Ge}_3\text{Se}_4\text{S}_8$.



Fi

Figure S4. The calculated and experimental X-ray diffraction patterns of $\text{La}_4\text{Ge}_3\text{Se}_6\text{S}_6$.



Fi

Figure S5. The calculated and experimental X-ray diffraction patterns of $\text{La}_4\text{Ge}_3\text{Se}_8\text{S}_4$.

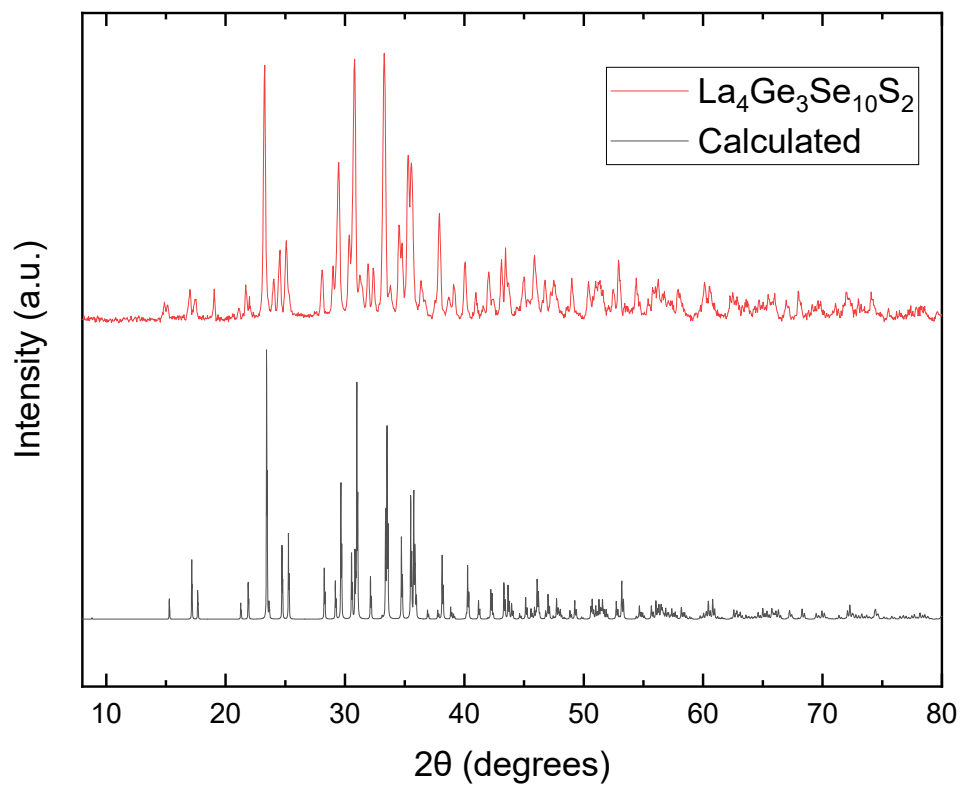


Figure S6. The calculated and experimental X-ray diffraction patterns of $\text{La}_4\text{Ge}_3\text{Se}_{10}\text{S}_2$.

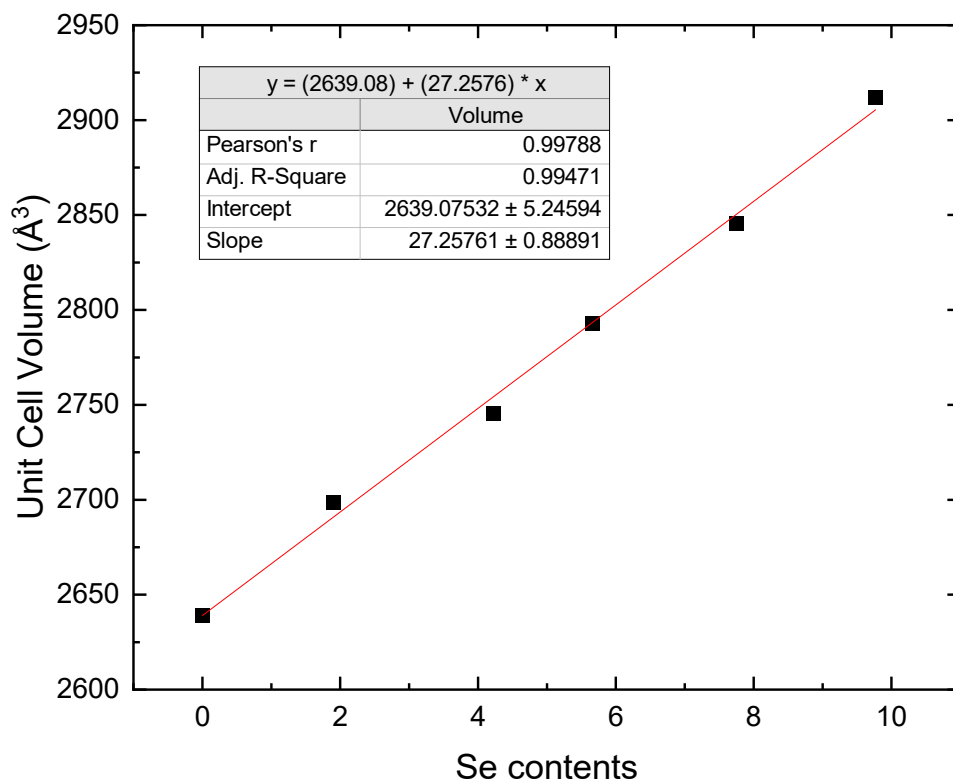
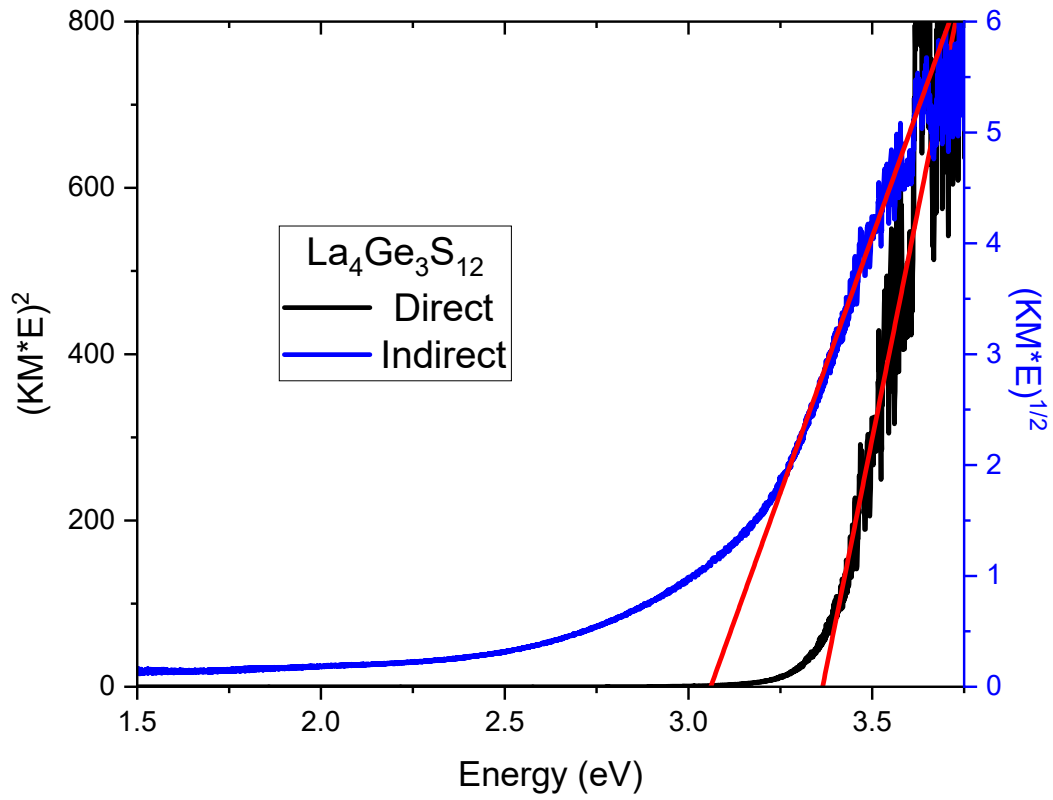
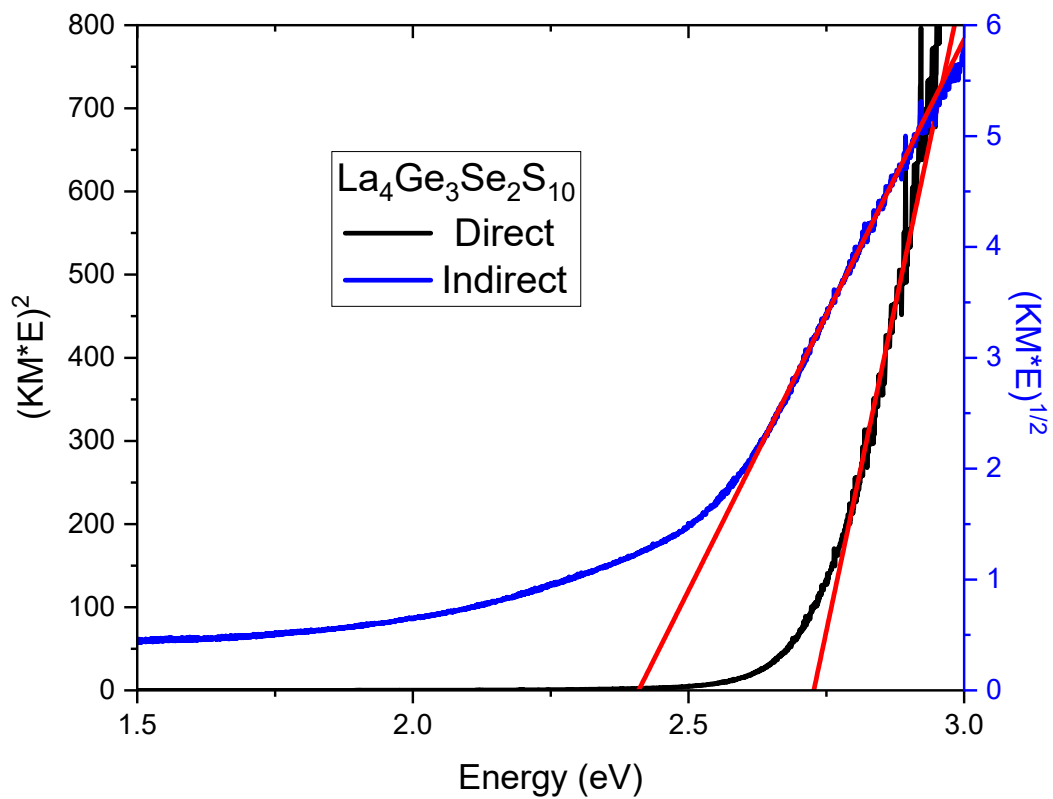


Figure S7. A comparison of Se contents and unit cell volume in $\text{La}_4\text{Ge}_3\text{Se}_x\text{S}_{12-x}$ ($x = 0, 2, 4, 6, 8, 10$), emphasizing the linear increase of volume as Se is incorporated into the crystal structure.



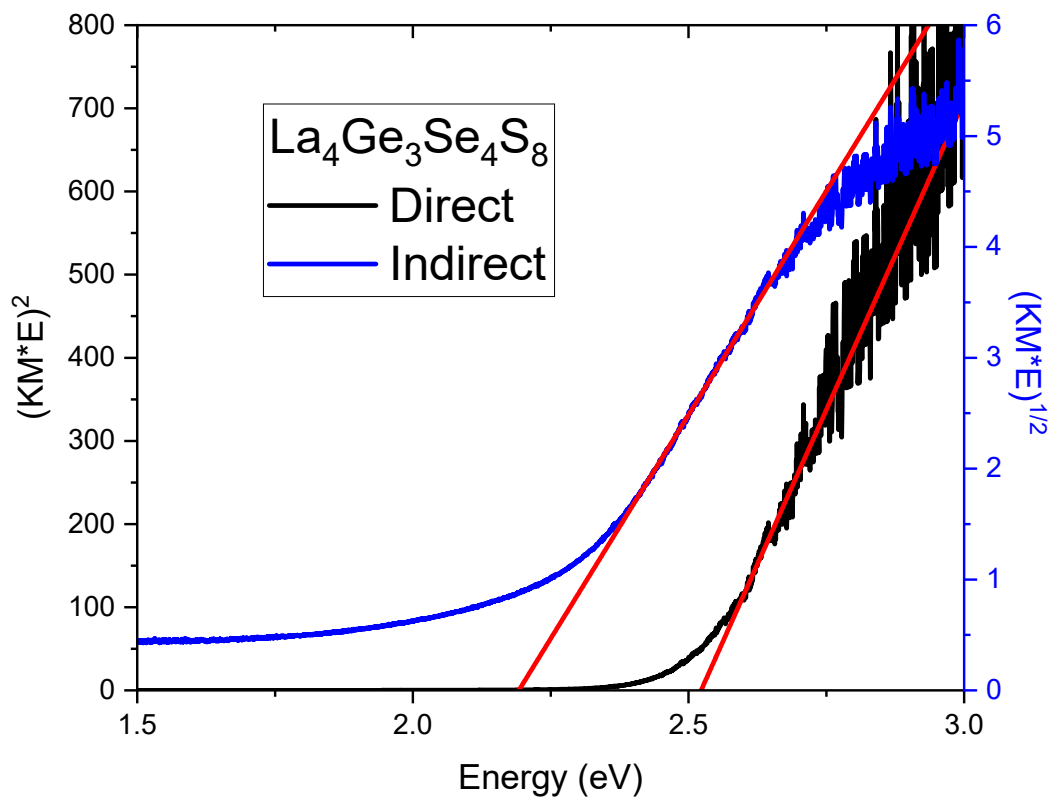
Fi

gure S8. Tauc plots for allowed direct and indirect transitions of $\text{La}_4\text{Ge}_3\text{S}_{12}$.



Fi

Figure S9. Tauc plots for allowed direct and indirect transitions of $\text{La}_4\text{Ge}_3\text{Se}_2\text{S}_{10}$.



Fi

Figure S10. Tauc plots for allowed direct and indirect transitions of $\text{La}_4\text{Ge}_3\text{Se}_4\text{S}_8$.

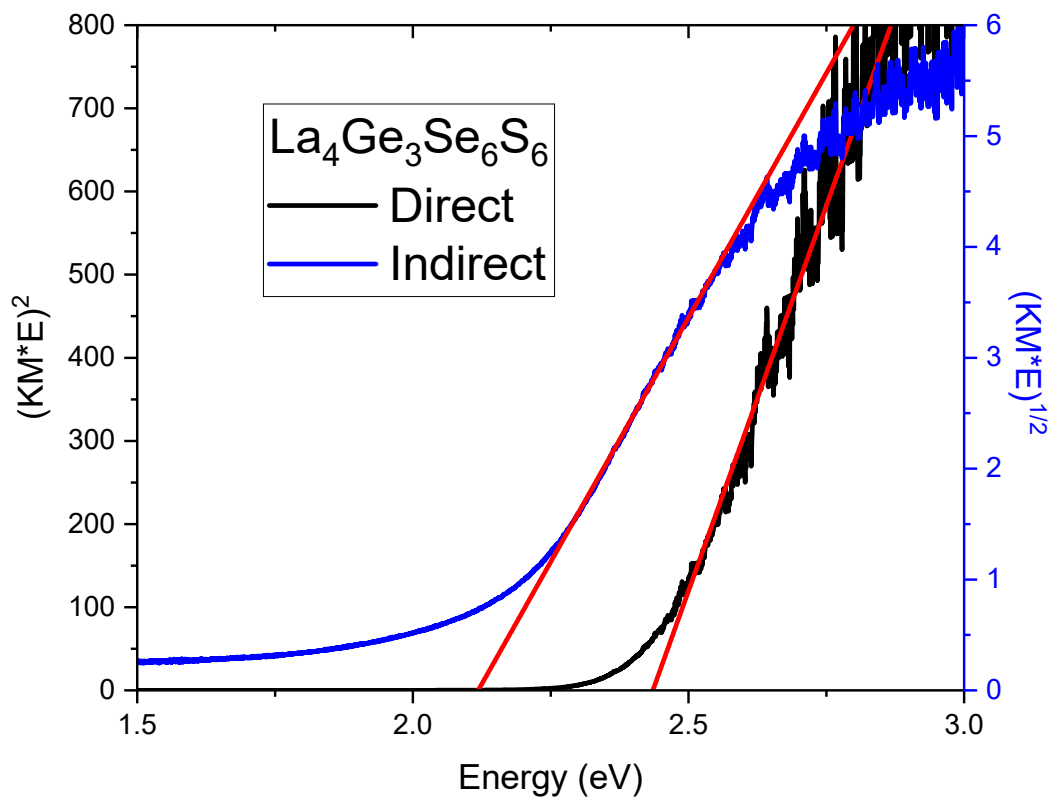
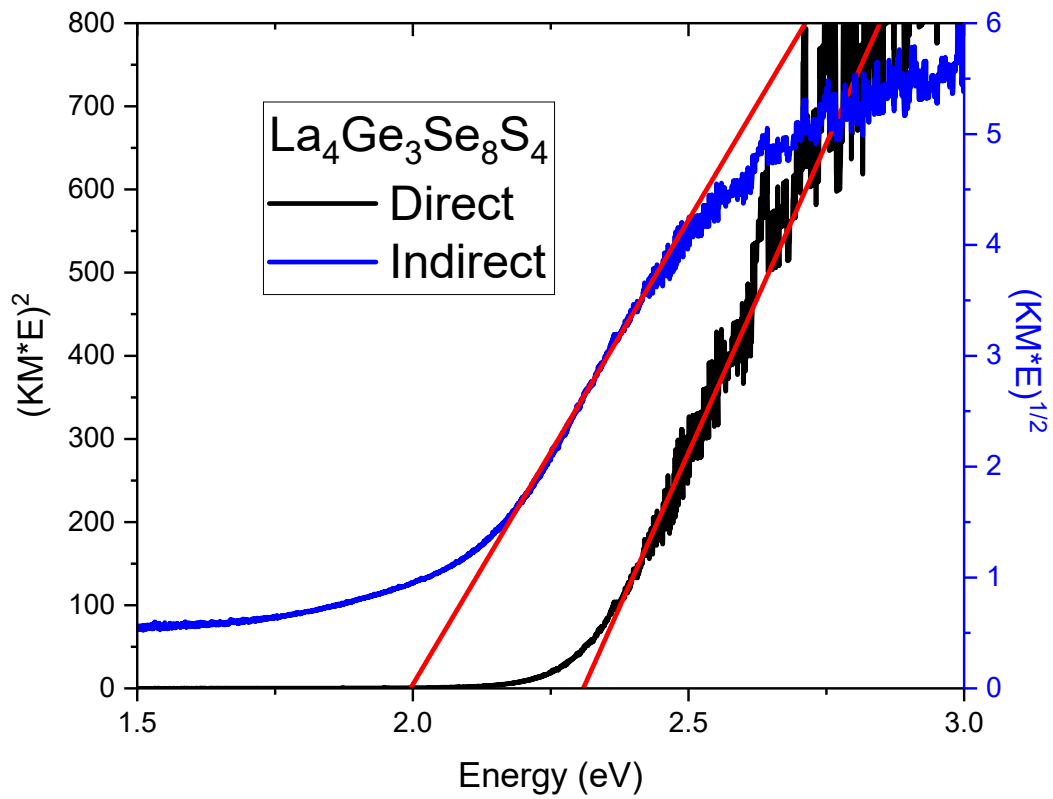
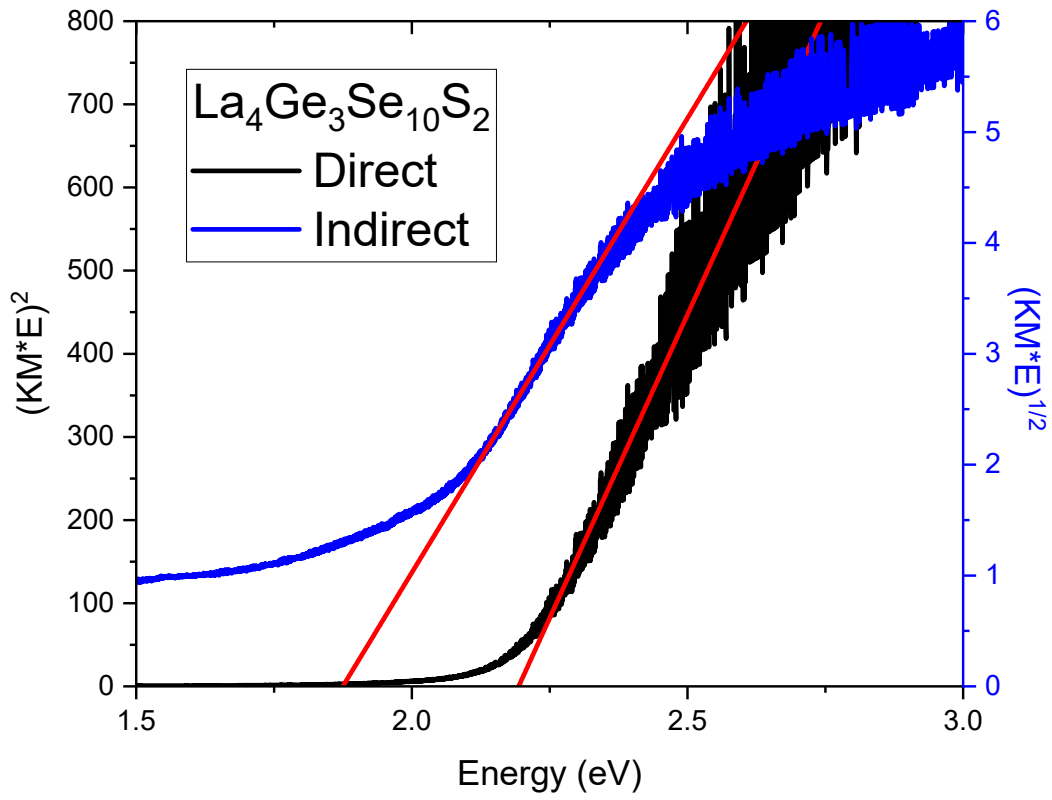


Figure S11. Tauc plots for allowed direct and indirect transitions of $\text{La}_4\text{Ge}_3\text{Se}_6\text{S}_6$.



Fi

Figure S12. Tauc plots for allowed direct and indirect transitions of $\text{La}_4\text{Ge}_3\text{Se}_8\text{S}_4$.



Fi

Figure S13. Tauc plots for allowed direct and indirect transitions of $\text{La}_4\text{Ge}_3\text{Se}_{10}\text{S}_2$.

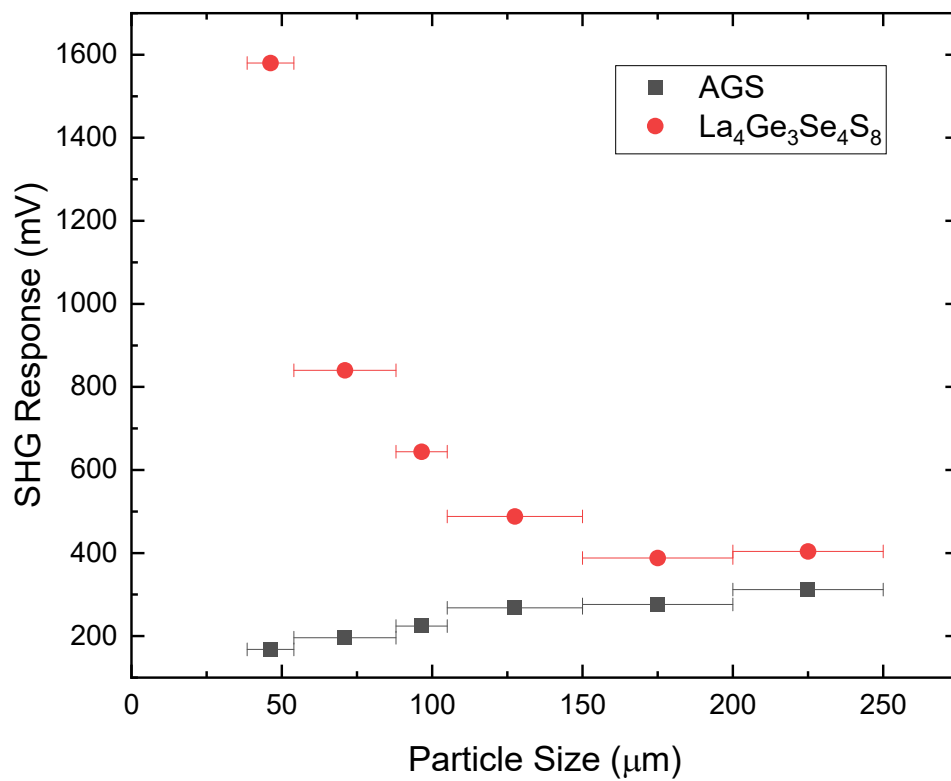


Figure S14. SHG intensities of La₄Ge₃Se₄S₈ and AgGaS₂ @2.09 μm measured at variable particle size samples.

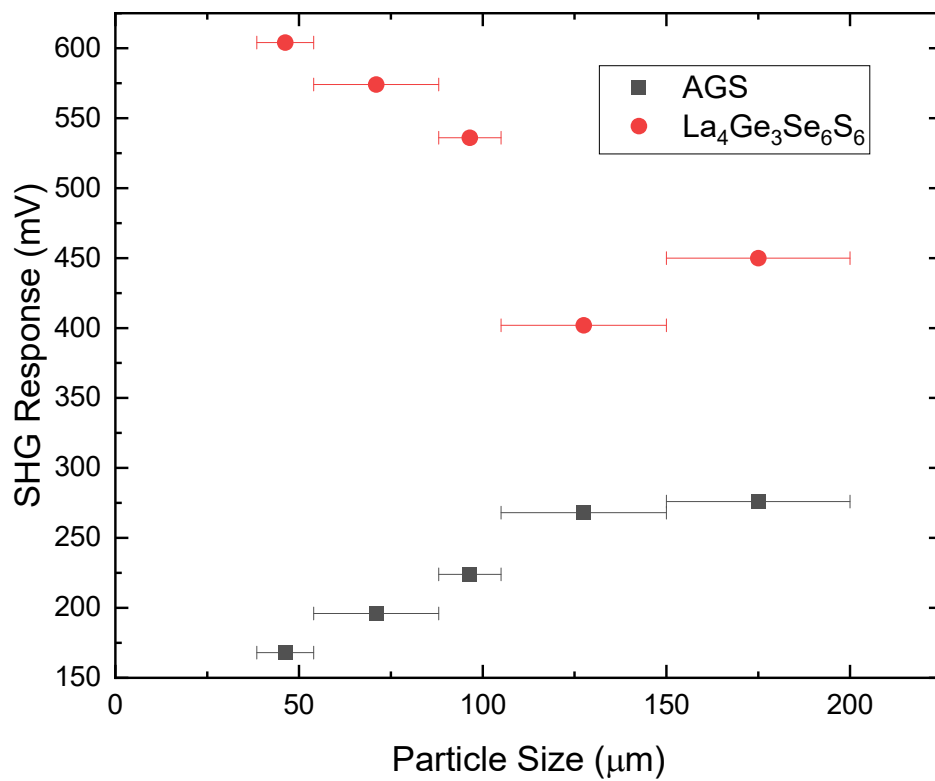


Figure S15. SHG intensities of La₄Ge₃Se₆S₆ and AgGaS₂ @2.09 μm measured at variable particle size samples.

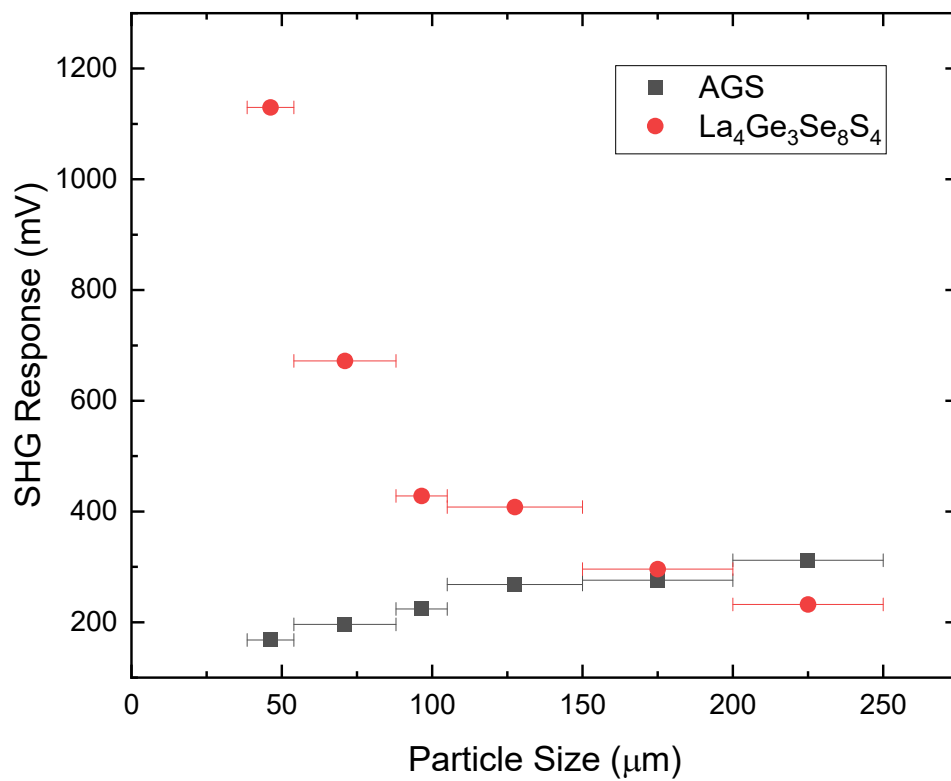


Figure S16. SHG intensities of La₄Ge₃Se₈S₄ and AgGaS₂ @2.09 μm measured at variable particle size samples.

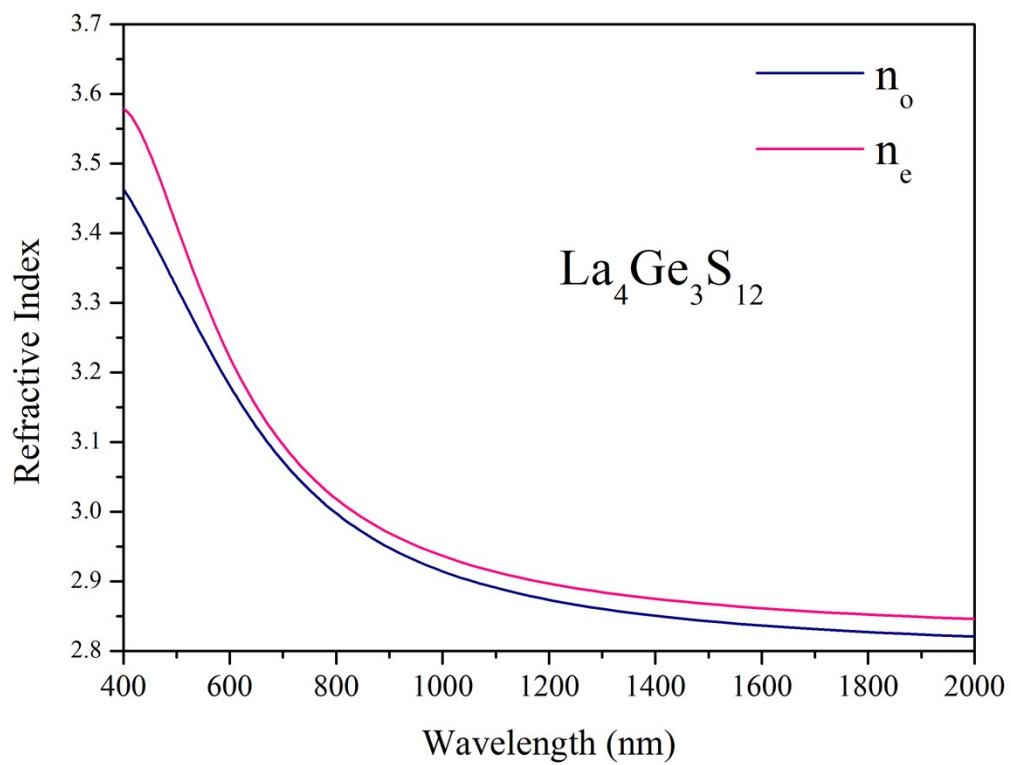


Figure S17. Calculated birefringence of $\text{La}_4\text{Ge}_3\text{S}_{12}$ versus wavelength of the fundamental light.

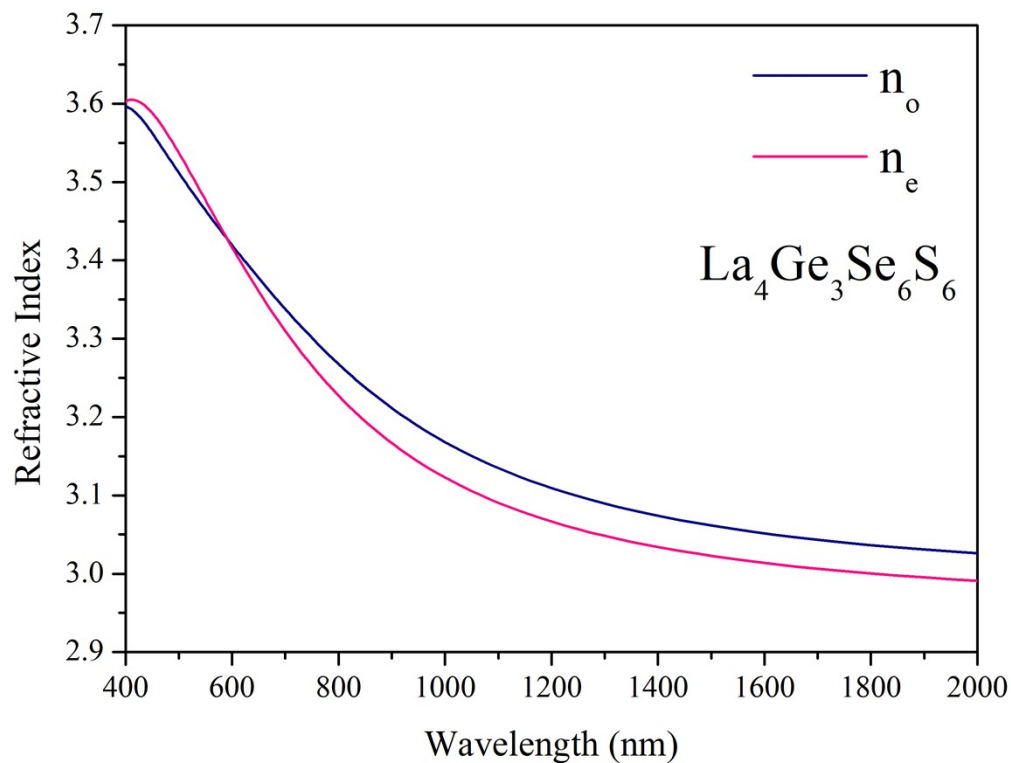
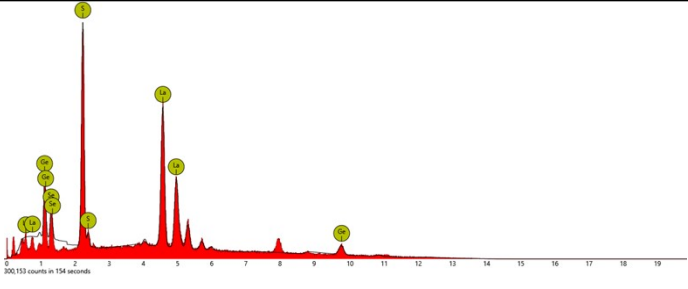
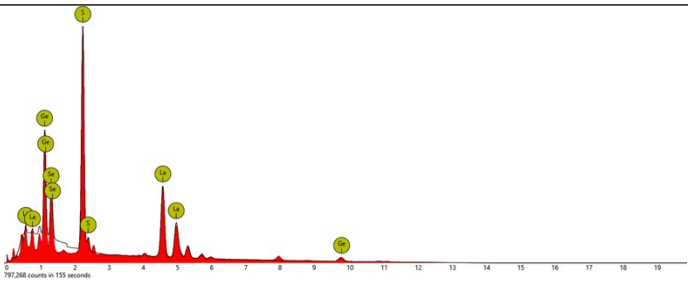
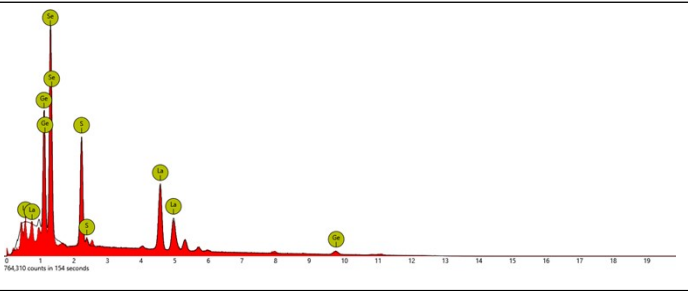
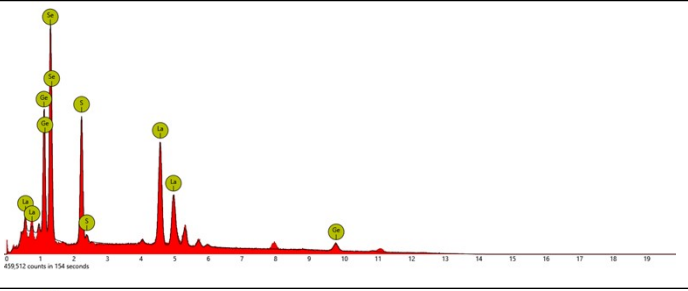
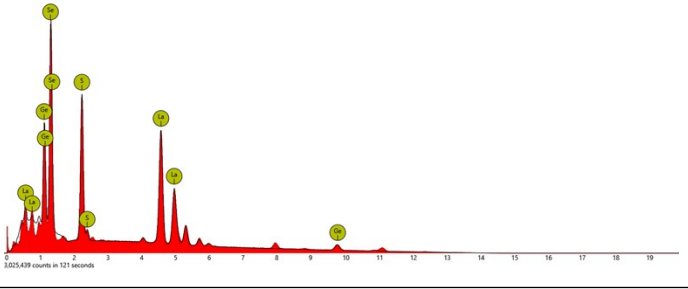
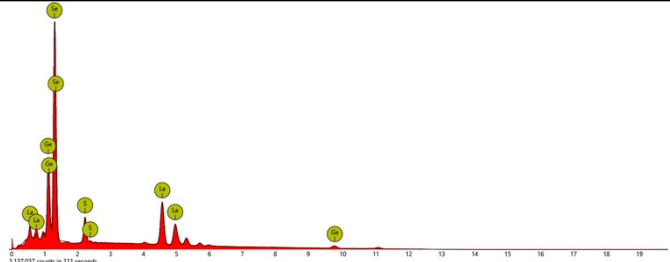
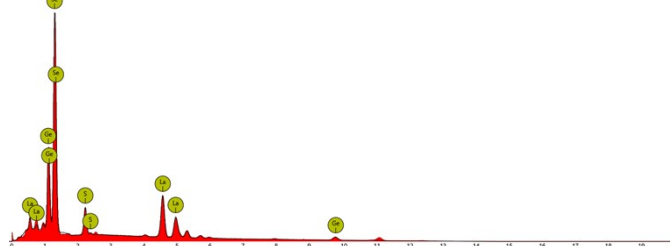


Figure S18. Calculated birefringence of $\text{La}_4\text{Ge}_3\text{Se}_6\text{S}_6$ versus wavelength of the fundamental light.

Table S4. Results of EDX analyses of selected crystals of $\text{La}_4\text{Ge}_3\text{Se}_2\text{S}_{10}$, $\text{La}_4\text{Ge}_3\text{Se}_6\text{S}_6$, and $\text{La}_4\text{Ge}_3\text{Se}_{10}\text{S}_2$.

Spectrum	Normalized Composition
	$\text{La}_{4.1}\text{Ge}_3\text{Se}_{2.2}\text{S}_{9.6}$

 <p>200,133 counts in 154 seconds</p>	$\text{La}_{4.0}\text{Ge}_3\text{Se}_{2.0}\text{S}_{9.1}$
 <p>197,268 counts in 155 seconds</p>	$\text{La}_{3.9}\text{Ge}_3\text{Se}_{2.1}\text{S}_{8.7}$
Average:	$\text{La}_{4.0(1)}\text{Ge}_3\text{Se}_{2.1(1)}\text{S}_{9.1(1)}$
 <p>194,310 counts in 154 seconds</p>	$\text{La}_{3.6}\text{Ge}_3\text{Se}_{6.2}\text{S}_{5.7}$
 <p>405,512 counts in 154 seconds</p>	$\text{La}_{3.8}\text{Ge}_3\text{Se}_{5.6}\text{S}_{4.9}$
 <p>400,439 counts in 121 seconds</p>	$\text{La}_{4.3}\text{Ge}_3\text{Se}_{6.1}\text{S}_{5.8}$
Average:	$\text{La}_{3.9(1)}\text{Ge}_3\text{Se}_{5.9(1)}\text{S}_{5.5(1)}$

 <p>EDS spectrum for $\text{La}_{4.6}\text{Ge}_3\text{Se}_{10.4}\text{S}_{1.6}$. The x-axis represents energy in keV from 0 to 19. The y-axis represents counts. Peaks are labeled with their corresponding elements: La (~0.5 keV), Ge (~1.1 keV), Se (~2.0 keV), S (~2.3 keV), Ge (~4.5 keV), La (~4.8 keV), and S (~10.0 keV).</p>	$\text{La}_{4.6}\text{Ge}_3\text{Se}_{10.4}\text{S}_{1.6}$
 <p>EDS spectrum for $\text{La}_{4.2}\text{Ge}_3\text{Se}_{10.1}\text{S}_{1.6}$. The x-axis represents energy in keV from 0 to 19. The y-axis represents counts. Peaks are labeled with their corresponding elements: La (~0.5 keV), Ge (~1.1 keV), Se (~2.0 keV), S (~2.3 keV), Ge (~4.5 keV), La (~4.8 keV), and S (~10.0 keV).</p>	$\text{La}_{4.2}\text{Ge}_3\text{Se}_{10.1}\text{S}_{1.6}$