

Continuous separation simulations-052621

Report date	May 26, 2021 10:53:06 AM
-------------	--------------------------

Contents

- 1. Global Definitions**
- 1.1. Parameters
- 2. Component 1**
- 2.1. Definitions
- 2.2. Geometry 1
- 2.3. Materials.....
- 2.4. Nernst-Planck Equations
- 2.5. Laminar Flow
- 2.6. Mesh 1
- 3. Study 1**
- 3.1. Stationary.....
- 3.2. Solver Configurations
- 4. Results**
- 4.1. Data Sets
- 4.2. Plot Groups

1 Global Definitions

Date	May 26, 2021 10:44:47 AM
------	--------------------------

GLOBAL SETTINGS

Name	Continuous separation simulations-052621.mph
Path	C:\Users\jrt3552\Documents\BED\final final simulations\Continuous separation simulations-052621.mph
Version	COMSOL Multiphysics 5.5 (Build: 359)
Unit system	SI

USED PRODUCTS

COMSOL Multiphysics
Chemical Reaction Engineering Module

1.1 PARAMETERS

PARAMETERS 1

Name	Expression	Value	Description
cheight	15[um]	1.5E-5 m	channel height
cwidth	200[um]	2E-4 m	channel width
cH3O0	$10^{-\text{pH0}}$ [M]	1E-4 mol/m ³	initial concentration of protons
pH0	7	7	initial solution pH
cOH0	$10^{-(14 - \text{pH0})}$ [M]	1E-4 mol/m ³	initial concentration of hydroxide
cK0	5 [mM]	5 mol/m ³	initial concentration of K
cBead0	30 [fM]	3E-11 mol/m ³	initial concentration of microbead
cCl0	cK0	5 mol/m ³	initial concentration of Cl

Name	Expression	Value	Description
dH3O	$9.103e-9[m^2/s]$	$9.103E-9 m^2/s$	diffusivity of proton
dOH	$5.286e-9[m^2/s]$	$5.286E-9 m^2/s$	diffusivity of hydroxide
dK	$1.97e-9[m^2/s]$	$1.97E-9 m^2/s$	diffusivity of microbeads
dBead	$0.785e-9[m^2/s]$	$7.85E-10 m^2/s$	diffusivity of bead
dCl	$2.033e-9[m^2/s]$	$2.033E-9 m^2/s$	diffusivity of Cl
issTOT	4.3[uA]	$4.3E-6 A$	total current passed through driving electrodes
jssTOT	$issTOT/(cwidth*cheight)$	$1433.3 A/m^2$	current density passed through channel
issBPE	2900 [nA]	$2.9E-6 A$	steady-state current through BPE
jssBPE	$issBPE/(elength*ewidth)$	$1160 A/m^2$	current density at BPE
fssBPE	$jssBPE/F$	$0.012023 mol/(m^2 \cdot s)$	flux at BPE
elength	50[um]	$5E-5 m$	length of BPE end
ewidth	50[um]	$5E-5 m$	width of BPE end
F	96485 [C/mol]	$96485 C/mol$	Faraday's constant
kfhydro	$2e-5 [1/(s)]$	$2E-5 1/s$	forward reaction rate constant for water hydrolysis
kbhydro	$1.4e11[1/(M*s)]$	$1.4E8 m^3/(s \cdot mol)$	backward reaction rate constant for

Name	Expression	Value	Description
			water hydrolysis
cH2O0	55.5 [M]	55500 mol/m ³	concentration of water
T	293 [K]	293 K	Temperature

2 Component 1

Date	Jan 3, 2020 8:56:10 AM
------	------------------------

SETTINGS

Description	Value
Unit system	Same as global system
Geometry shape order	Automatic
Avoid inverted elements by curving interior domain elements	Off

SPATIAL FRAME COORDINATES

First	Second	Third
x	y	z

MATERIAL FRAME COORDINATES

First	Second	Third
X	Y	Z

GEOMETRY FRAME COORDINATES

First	Second	Third
Xg	Yg	Zg

MESH FRAME COORDINATES

First	Second	Third
Xm	Ym	Zm

2.1 DEFINITIONS

2.1.1 Coordinate Systems

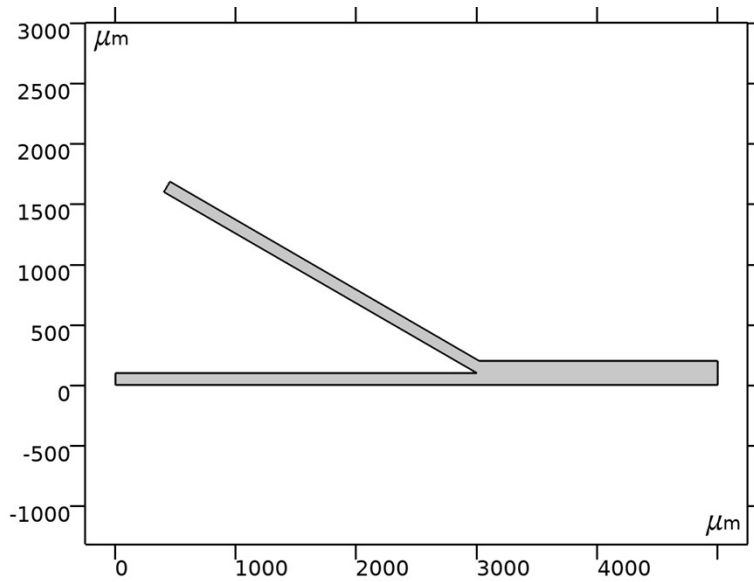
Boundary System 1

Coordinate system type	Boundary system
Tag	sys1

COORDINATE NAMES

First	Second	Third
t1	n	to

2.2 GEOMETRY 1



Geometry 1

UNITS

Length unit	μm
Angular unit	deg

GEOMETRY STATISTICS

Description	Value
Space dimension	2
Number of domains	1
Number of boundaries	12
Number of vertices	12

2.2.1 Point 3 (pt3)

POINT

Description	Value
Point coordinate	{3200, 0}

2.2.2 Point 4 (pt4)

POINT

Description	Value
Point coordinate	{3250, 0}

2.2.3 Point 1 (pt1)

POINT

Description	Value
Point coordinate	{250, 0}

2.2.4 Point 2 (pt2)

POINT

Description	Value
Point coordinate	{300, 0}

2.2.5 0 0 (pol3)

OBJECT TYPE

Description	Value
Type	Solid

COORDINATES

Description	Value
Data source	Table

COORDINATES

x (μm)	y (μm)
0	0
0	100
3000	100
401.92	1600
451.92	1686.6
3020	200
5000	200
5000	0
0	0

2.2.6 Polygon 1 (pol1)

OBJECT TYPE

Description	Value
Type	Solid

COORDINATES

Description	Value
Data source	Table

COORDINATES

x (µm)	y (µm)
0	0
0	66.7
1000	66.7
133.974	566.7
167.324	624.464
1020	133.4
2960	133.4
2960	0
0	0

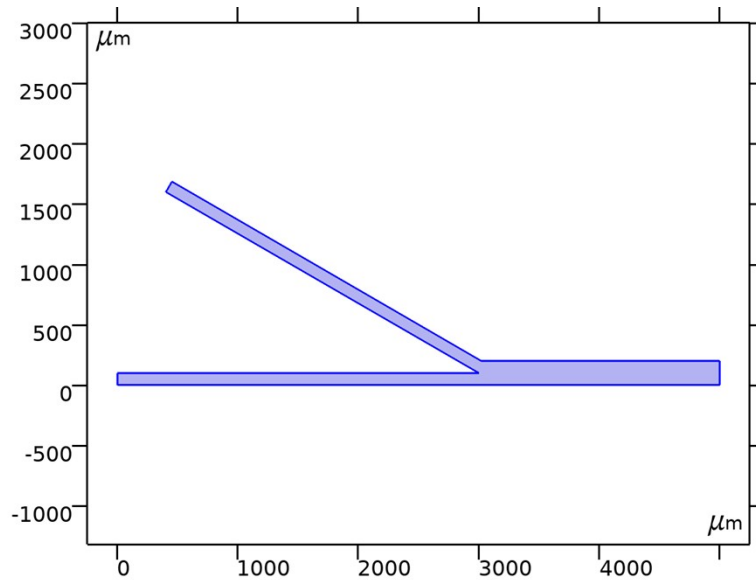
2.2.7 Form Union (fin)

SETTINGS

Description	Value
Repair tolerance	Automatic

2.3 MATERIALS

2.3.1 Water



Water

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

MATERIAL PARAMETERS

Name	Value	Unit
Dynamic viscosity	eta(T)	Pa·s
Density	rho(T)	kg/m ³

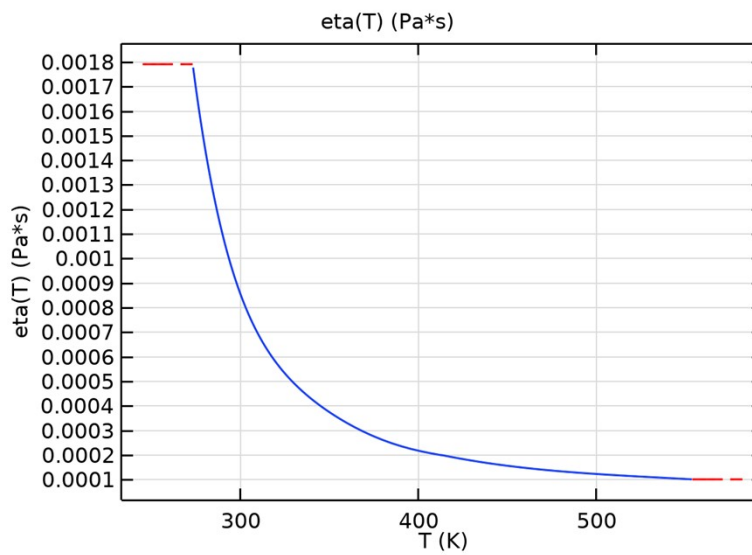
BASIC SETTINGS

Description	Value
Coefficient of thermal expansion	{{alpha_p(T), 0, 0}, {0, alpha_p(T), 0}, {0, 0, alpha_p(T)}}
Bulk viscosity	muB(T)
Dynamic viscosity	eta(T)
Ratio of specific heats	gamma_w(T)
Electrical conductivity	{{5.5e-6[S/m], 0, 0}, {0, 5.5e-6[S/m], 0}, {0, 0, 5.5e-6[S/m]}}
Heat capacity at constant pressure	Cp(T)
Density	rho(T)
Thermal conductivity	{{k(T), 0, 0}, {0, k(T), 0}, {0, 0, k(T)}}

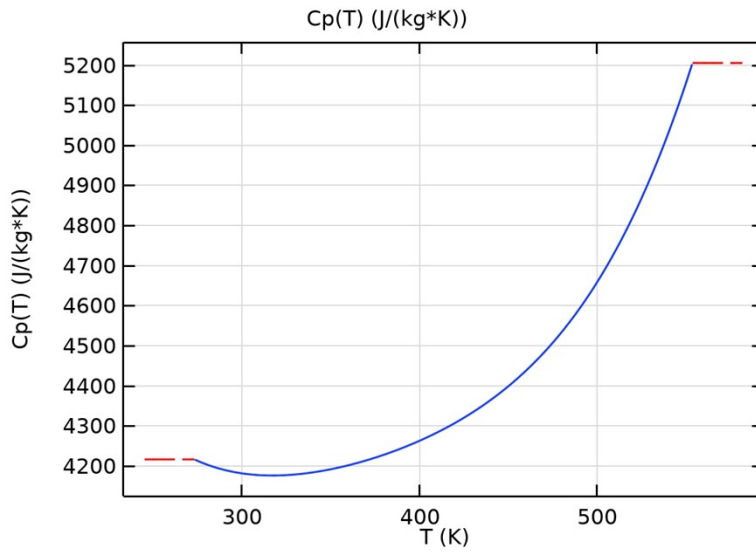
Description	Value
Speed of sound	cs(T)

FUNCTIONS

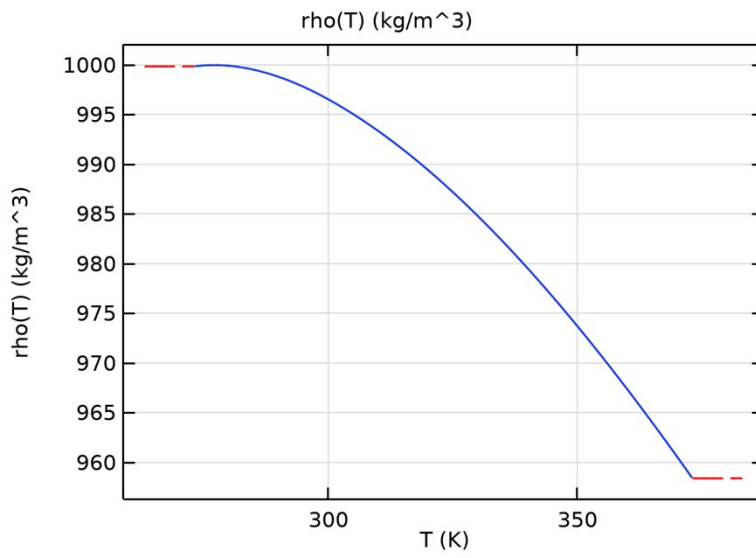
Function name	Type
eta	Piecewise
Cp	Piecewise
rho	Piecewise
k	Piecewise
cs	Interpolation
alpha_p	Analytic
gamma_w	Analytic
muB	Analytic



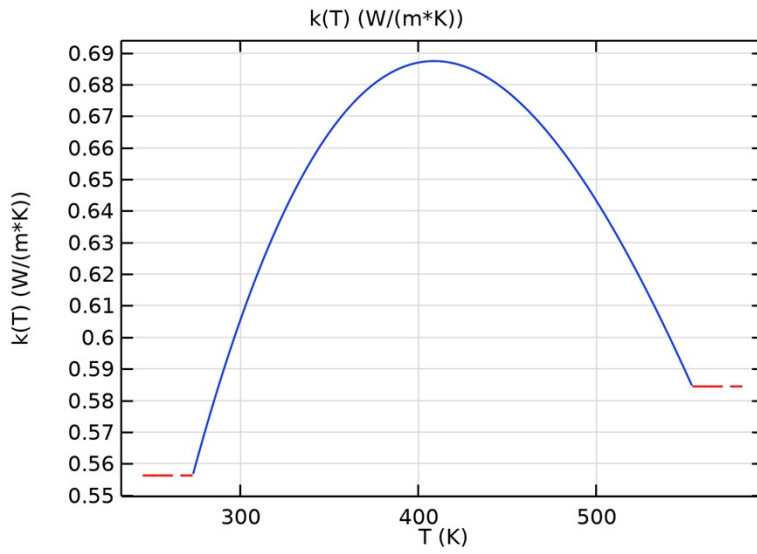
eta



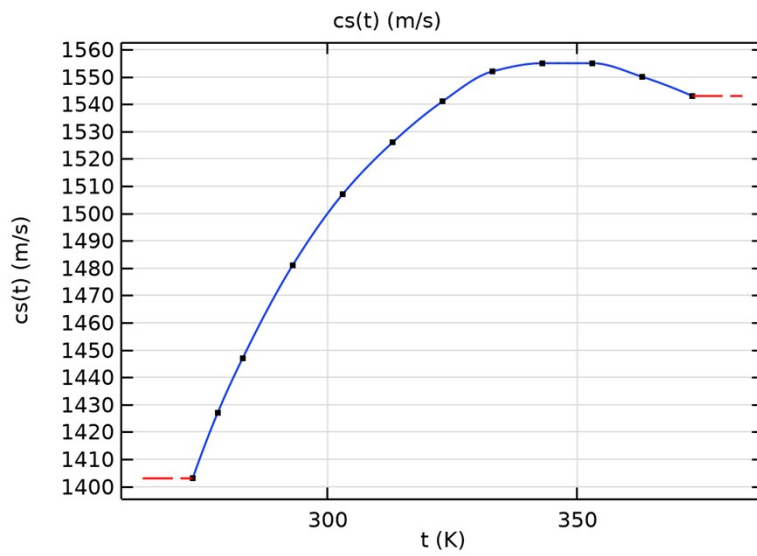
C_p



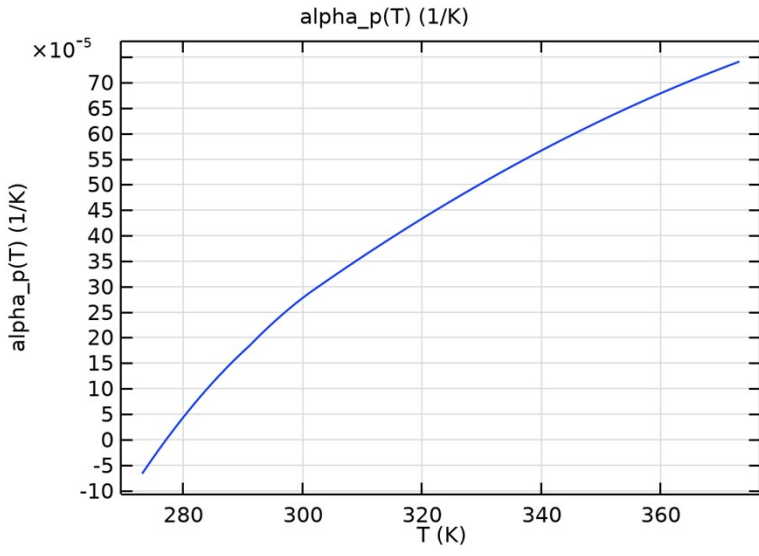
ρ



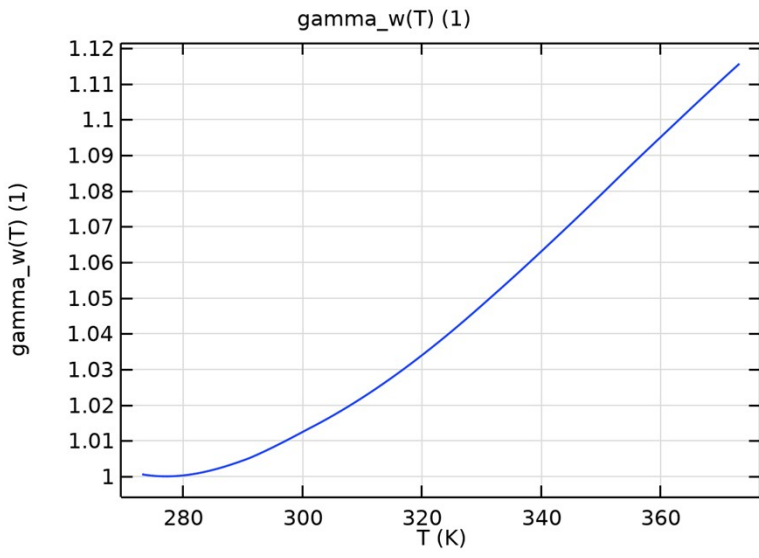
k



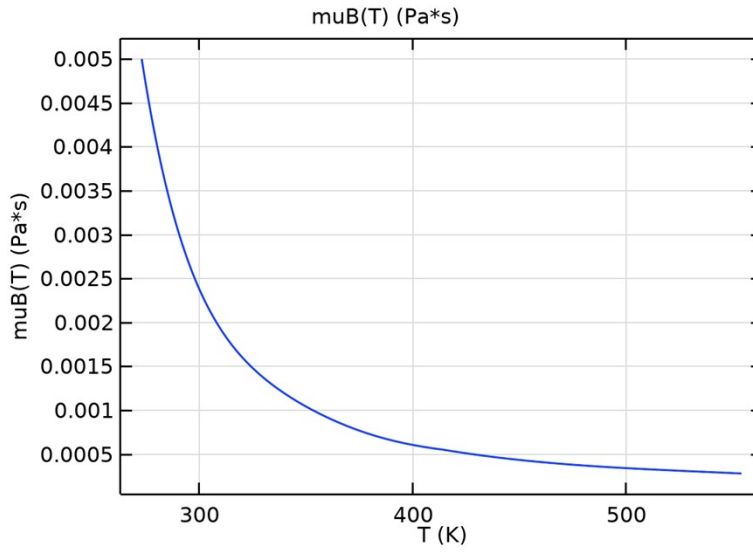
cs



alpha_p



gamma_w



μ_B

2.4 NERNST-PLANCK EQUATIONS

USED PRODUCTS

COMSOL Multiphysics
Chemical Reaction Engineering Module



Nernst-Planck Equations

SELECTION

Geometric entity level	Domain
------------------------	--------

Selection	Geometry geom1: Dimension 2: All domains
-----------	--

EQUATIONS

$$\nabla \cdot (\mathbf{J}_i + \mathbf{u}c_i) = R_i$$

$$\nabla \cdot \mathbf{i} = F \sum_i z_i R_i$$

$$\sum_i z_i c_i = 0$$

$$\mathbf{J}_i = -D_i \nabla c_i - z_i \mu_{m,i} F c_i \nabla V$$

$$\mathbf{i} = F \sum_i z_i (-D_i \nabla c_i - z_i \mu_{m,i} F c_i \nabla V)$$

2.4.1 Interface settings

Discretization

SETTINGS

Description	Value
Concentration	Quadratic
Electric potential	Quadratic

2.4.2 Variables

Name	Expression	Unit	Description	Selection	Details
domflux.cH3Ox	npe.dflux_cH3Ox+npe.cflux_cH3Ox+npe.mflux_cH3Ox	mol/(m ² .s)	Domain flux, x component	Domain 1	
domflux.cH3Oy	npe.dflux_cH3Oy+npe.cflux_cH3Oy+npe.mflux_cH3Oy	mol/(m ² .s)	Domain flux, y component	Domain 1	
domflux.cOHx	npe.dflux_cOHx+npe.cflux_cOHx+npe.mflux_cOHx	mol/(m ² .s)	Domain flux, x component	Domain 1	
domflux.cOHy	npe.dflux_cOHy+npe.cflux_cOHy+npe.mflux_cOHy	mol/(m ² .s)	Domain flux, y component	Domain 1	
domflux.cBeadx	npe.dflux_cBeadx+npe.cflux_cBeadx+npe.mflux_cBeadx	mol/(m ² .s)	Domain flux, x component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
domflux.cBeady	npe.dflux_cBeady+npe.cflux_cBeady+npe.mflux_cBeady	mol/(m ² .s)	Domain flux, y component	Domain 1	
domflux.cKx	npe.dflux_cKx+npe.cflux_cKx+npe.mflux_cKx	mol/(m ² .s)	Domain flux, x component	Domain 1	
domflux.cKy	npe.dflux_cKy+npe.cflux_cKy+npe.mflux_cKy	mol/(m ² .s)	Domain flux, y component	Domain 1	
domflux.cClx	npe.dflux_cClx+npe.cflux_cClx+npe.mflux_cClx	mol/(m ² .s)	Domain flux, x component	Domain 1	
domflux.cCly	npe.dflux_cCly+npe.cflux_cCly+npe.mflux_cCly	mol/(m ² .s)	Domain flux, y component	Domain 1	
domflux.Vx	npe.Jx	A/m ²	Domain flux, x component	Domain 1	
domflux.Vy	npe.Jy	A/m ²	Domain flux, y component	Domain 1	
npe.nx	dnx	1	Normal vector, x component	Boundaries 1–12	
npe.ny	dny	1	Normal vector, y component	Boundaries 1–12	
npe.nz	0	1	Normal vector, z component	Boundaries 1–12	
npe.nxmesh	dnxmesh	1	Normal vector (mesh), x component	Boundaries 1–12	
npe.nymesh	dnymesh	1	Normal vector (mesh), y component	Boundaries 1–12	
npe.nzmesh	0	1	Normal vector	Boundaries 1–12	

Name	Expression	Unit	Description	Selection	Details
			(mesh), z component		
npe.nxc	root.nxc/npe.ncLen	1	Normal vector, x component	Boundaries 1–12	
npe.nyc	root.nyc/npe.ncLen	1	Normal vector, y component	Boundaries 1–12	
npe.nzc	0	1	Normal vector, z component	Boundaries 1–12	
npe.ncLen	$\sqrt{\text{root.nxc}^2 + \text{root.nyc}^2 + \text{eps}}$	1	Help variable	Boundaries 1–12	
npe.ndflux_cH3O	$\text{npe.dflux_cH3Ox} * \text{npe.nxc} + \text{npe.dflux_cH3Oy} * \text{npe.nyc} + \text{npe.dflux_cH3Oz} * \text{npe.nzc}$	mol/(m ² .s)	Normal diffusive flux	Boundaries 1–12	
npe.ncflux_cH3O	$\text{npe.cflux_cH3Ox} * \text{npe.nxc} + \text{npe.cflux_cH3Oy} * \text{npe.nyc} + \text{npe.cflux_cH3Oz} * \text{npe.nzc}$	mol/(m ² .s)	Normal convective flux	Boundaries 1–12	
npe.nmflux_cH3O	$\text{npe.mflux_cH3Ox} * \text{npe.nxc} + \text{npe.mflux_cH3Oy} * \text{npe.nyc} + \text{npe.mflux_cH3Oz} * \text{npe.nzc}$	mol/(m ² .s)	Normal electrophoretic flux	Boundaries 1–12	
npe.ntflux_cH3O	npe.bndFlux_cH3O	mol/(m ² .s)	Normal total flux	Boundaries 1–12	
npe.ndflux_cOH	$\text{npe.dflux_cOHx} * \text{npe.nxc} + \text{npe.dflux_cOHy} * \text{npe.nyc} + \text{npe.dflux_cOHz} * \text{npe.nzc}$	mol/(m ² .s)	Normal diffusive flux	Boundaries 1–12	
npe.ncflux_cOH	$\text{npe.cflux_cOHx} * \text{npe.nxc} + \text{npe.cflux_cOHy} * \text{npe.nyc} + \text{npe.cflux_cOHz} * \text{npe.nzc}$	mol/(m ² .s)	Normal convective flux	Boundaries 1–12	

Name	Expression	Unit	Description	Selection	Details
	$cflux_cOHz * npe.nzc$				
npe.nmflux_cOH	$npe.mflux_cOHx * npe.nxc + npe.mflux_cOHy * npe.nyc + npe.mflux_cOHz * npe.nzc$	mol/(m ² .s)	Normal electrophoretic flux	Boundaries 1–12	
npe.ntflux_cOH	$npe.bndFlux_cOH$	mol/(m ² .s)	Normal total flux	Boundaries 1–12	
npe.ndflux_cBead	$npe.dflux_cBeadx * npe.nxc + npe.dflux_cBeady * npe.nyc + npe.dflux_cBeadz * npe.nzc$	mol/(m ² .s)	Normal diffusive flux	Boundaries 1–12	
npe.ncflux_cBead	$npe.cflux_cBeadx * npe.nxc + npe.cflux_cBeady * npe.nyc + npe.cflux_cBeadz * npe.nzc$	mol/(m ² .s)	Normal convective flux	Boundaries 1–12	
npe.nmflux_cBead	$npe.mflux_cBeadx * npe.nxc + npe.mflux_cBeady * npe.nyc + npe.mflux_cBeadz * npe.nzc$	mol/(m ² .s)	Normal electrophoretic flux	Boundaries 1–12	
npe.ntflux_cBead	$npe.bndFlux_cBead$	mol/(m ² .s)	Normal total flux	Boundaries 1–12	
npe.ndflux_cK	$npe.dflux_cKx * npe.nxc + npe.dflux_cKy * npe.nyc + npe.dflux_cKz * npe.nzc$	mol/(m ² .s)	Normal diffusive flux	Boundaries 1–12	
npe.ncflux_cK	$npe.cflux_cKx * npe.nxc + npe.cflux_cKy * npe.nyc + npe.cflux_cKz * npe.nzc$	mol/(m ² .s)	Normal convective flux	Boundaries 1–12	
npe.nmflux_cK	$npe.mflux_cKx * npe.nxc + npe.mflux_cKy * npe.nyc$	mol/(m ² .s)	Normal electrophoretic flux	Boundaries 1–12	

Name	Expression	Unit	Description	Selection	Details
	e.nyc+npe.mflux_cKz*npe.nzc				
npe.ntflux_cK	npe.bndFlux_cK	mol/(m ² .s)	Normal total flux	Boundaries 1–12	
npe.ndflux_cCl	npe.dflux_cClx*npe.nxc+npe.dflux_cCly*npe.nyc+npe.dflux_cClz*npe.nzc	mol/(m ² .s)	Normal diffusive flux	Boundaries 1–12	
npe.ncflux_cCl	npe.cflux_cClx*npe.nxc+npe.cflux_cCly*npe.nyc+npe.cflux_cClz*npe.nzc	mol/(m ² .s)	Normal convective flux	Boundaries 1–12	
npe.nmflux_cCl	npe.mflux_cClx*npe.nxc+npe.mflux_cCly*npe.nyc+npe.mflux_cClz*npe.nzc	mol/(m ² .s)	Normal electrophoretic flux	Boundaries 1–12	
npe.ntflux_cCl	(npe.cflux_cClx+npe.mflux_cClx)*npe.nxc+(npe.cflux_cCly+npe.mflux_cCly)*npe.nyc+(npe.cflux_cClz+npe.mflux_cClz)*npe.nzc+npe.ndflux_cCl	mol/(m ² .s)	Normal total flux	Boundaries 1–12	
npe.bndFlux_cH3O	-dflux_spatial(cH3O)	mol/(m ² .s)	Boundary flux	Boundaries 1–12	
npe.bndFlux_cOH	-dflux_spatial(cOH)	mol/(m ² .s)	Boundary flux	Boundaries 1–12	
npe.bndFlux_cBead	-dflux_spatial(cBead)	mol/(m ² .s)	Boundary flux	Boundaries 1–12	
npe.bndFlux_	-	mol/(m	Boundary flux	Boundaries	

Name	Expression	Unit	Description	Selection	Details
cK	dflux_spatial(cK)	$^2.s$		s 1–12	
npe.bndFlux_V	-dflux_spatial(npe.V)	A/m ²	Boundary flux	Boundaries 1–12	
npe.nI	npe.bndFlux_V	A/m ²	Normal electrolyte current density	Boundaries 1–12	
npe.R_cH3O	0	mol/(m ³ .s)	Total rate expression	Domain 1	+ operation
npe.cP_cH3O	0	mol/kg	Concentration species absorbed to the solid	Domain 1	+ operation
npe.cP_cH3O	0	mol/kg	Concentration species absorbed to the solid	Boundaries 1–12	+ operation
npe.KP_cH3O	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Domain 1	+ operation
npe.KP_cH3O	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Boundaries 1–12	+ operation
npe.R_cOH	0	mol/(m ³ .s)	Total rate expression	Domain 1	+ operation
npe.cP_cOH	0	mol/kg	Concentration species absorbed to the solid	Domain 1	+ operation
npe.cP_cOH	0	mol/kg	Concentration species absorbed to the solid	Boundaries 1–12	+ operation
npe.KP_cOH	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Domain 1	+ operation
npe.KP_cOH	0	m ³ /kg	Adsorption isotherm, first	Boundaries 1–12	+ operation

Name	Expression	Unit	Description	Selection	Details
			concentration derivative		
npe.R_cBead	0	mol/(m ³ .s)	Total rate expression	Domain 1	+ operation
npe.cP_cBead	0	mol/kg	Concentration species absorbed to the solid	Domain 1	+ operation
npe.cP_cBead	0	mol/kg	Concentration species absorbed to the solid	Boundaries 1–12	+ operation
npe.KP_cBead	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Domain 1	+ operation
npe.KP_cBead	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Boundaries 1–12	+ operation
npe.R_cK	0	mol/(m ³ .s)	Total rate expression	Domain 1	+ operation
npe.cP_cK	0	mol/kg	Concentration species absorbed to the solid	Domain 1	+ operation
npe.cP_cK	0	mol/kg	Concentration species absorbed to the solid	Boundaries 1–12	+ operation
npe.KP_cK	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Domain 1	+ operation
npe.KP_cK	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Boundaries 1–12	+ operation
npe.R_cCl	0	mol/(m ³ .s)	Total rate expression	Domain 1	+ operation
npe.cP_cCl	0	mol/kg	Concentration species absorbed to	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
			the solid		
npe.cP_cCl	0	mol/kg	Concentration species absorbed to the solid	Boundaries 1–12	+ operation
npe.KP_cCl	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Domain 1	+ operation
npe.KP_cCl	0	m ³ /kg	Adsorption isotherm, first concentration derivative	Boundaries 1–12	+ operation
npe.cbf_cH3O	0	mol/(m ² .s)	Convective boundary flux	Boundaries 1–12	
npe.cbf_cOH	0	mol/(m ² .s)	Convective boundary flux	Boundaries 1–12	
npe.cbf_cBead	0	mol/(m ² .s)	Convective boundary flux	Boundaries 1–12	
npe.cbf_cK	0	mol/(m ² .s)	Convective boundary flux	Boundaries 1–12	

2.4.3 Convection, Diffusion, and Migration 1



Convection, Diffusion, and Migration 1

SELECTION

Geometric entity level	Domain
------------------------	--------

Selection	Geometry geom1: Dimension 2: All domains
-----------	--

EQUATIONS

$$\nabla \cdot (\mathbf{J}_i + \mathbf{u}c_i) = R_i$$

$$\nabla \cdot \mathbf{i} = F \sum_i z_i R_i$$

$$\sum_i z_i c_i = 0$$

$$\mathbf{J}_i = -D_i \nabla c_i - z_i \mu_{m,i} F c_i \nabla V$$

$$\mathbf{i} = F \sum_i z_i (-D_i \nabla c_i - z_i \mu_{m,i} F c_i \nabla V)$$

Convection

SETTINGS

Description	Value
Velocity field	Velocity field (spf)

Diffusion

SETTINGS

Description	Value
Material	Water (mat1)
Diffusion coefficient	User defined
Diffusion coefficient	{{dH3O, 0, 0}, {0, dH3O, 0}, {0, 0, dH3O}}
Diffusion coefficient	User defined
Diffusion coefficient	{{dOH, 0, 0}, {0, dOH, 0}, {0, 0, dOH}}
Diffusion coefficient	User defined
Diffusion coefficient	{{dBead, 0, 0}, {0, dBead, 0}, {0, 0, dBead}}
Diffusion coefficient	User defined
Diffusion coefficient	{{dK, 0, 0}, {0, dK, 0}, {0, 0, dK}}
Diffusion coefficient	User defined
Diffusion coefficient	{{dCl, 0, 0}, {0, dCl, 0}, {0, 0, dCl}}

Migration in electric field

SETTINGS

Description	Value
Mobility	Nernst - Einstein relation
Charge number	{1, -1, -1, 1, -1}

Coordinate system selection

SETTINGS

Description	Value
Coordinate system	Global coordinate system

Model input

SETTINGS

Description	Value
Temperature	User defined
Temperature	T

Variables

Name	Expression	Unit	Description	Selection	Details
cCl	$npe.ctemp/(npe.e.z_cCl+eps)$	mol/m ³	Concentration	Domain 1	
npe.D_ch3Ox _x	dH3O	m ² /s	Diffusion coefficient, xx component	Domain 1	
npe.D_ch3Oy _x	0	m ² /s	Diffusion coefficient, yx component	Domain 1	
npe.D_ch3Oz _x	0	m ² /s	Diffusion coefficient, zx component	Domain 1	
npe.D_ch3Ox _y	0	m ² /s	Diffusion coefficient, xy component	Domain 1	
npe.D_ch3Oy _y	dH3O	m ² /s	Diffusion coefficient, yy component	Domain 1	
npe.D_ch3Oz _y	0	m ² /s	Diffusion coefficient, zy component	Domain 1	
npe.D_ch3Ox _z	0	m ² /s	Diffusion coefficient, xz component	Domain 1	
npe.D_ch3Oy _z	0	m ² /s	Diffusion coefficient, yz component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
			component		
npe.D_cH3Oz z	dH3O	m ² /s	Diffusion coefficient, zz component	Domain 1	
npe.D_cOHxx	dOH	m ² /s	Diffusion coefficient, xx component	Domain 1	
npe.D_cOHyx	0	m ² /s	Diffusion coefficient, yx component	Domain 1	
npe.D_cOHxz	0	m ² /s	Diffusion coefficient, zx component	Domain 1	
npe.D_cOHxy	0	m ² /s	Diffusion coefficient, xy component	Domain 1	
npe.D_cOHyy	dOH	m ² /s	Diffusion coefficient, yy component	Domain 1	
npe.D_cOHzy	0	m ² /s	Diffusion coefficient, zy component	Domain 1	
npe.D_cOHxz	0	m ² /s	Diffusion coefficient, xz component	Domain 1	
npe.D_cOHyz	0	m ² /s	Diffusion coefficient, yz component	Domain 1	
npe.D_cOHzz	dOH	m ² /s	Diffusion coefficient, zz component	Domain 1	
npe.D_cBead xx	dBead	m ² /s	Diffusion coefficient, xx component	Domain 1	
npe.D_cBead yx	0	m ² /s	Diffusion coefficient, yx component	Domain 1	
npe.D_cBead zx	0	m ² /s	Diffusion coefficient, zx component	Domain 1	
npe.D_cBead xy	0	m ² /s	Diffusion coefficient, xy component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.D_cBead yy	dBead	m ² /s	Diffusion coefficient, yy component	Domain 1	
npe.D_cBead zy	0	m ² /s	Diffusion coefficient, zy component	Domain 1	
npe.D_cBead xz	0	m ² /s	Diffusion coefficient, xz component	Domain 1	
npe.D_cBead yz	0	m ² /s	Diffusion coefficient, yz component	Domain 1	
npe.D_cBead zz	dBead	m ² /s	Diffusion coefficient, zz component	Domain 1	
npe.D_cKxx	dK	m ² /s	Diffusion coefficient, xx component	Domain 1	
npe.D_cKyx	0	m ² /s	Diffusion coefficient, yx component	Domain 1	
npe.D_cKzx	0	m ² /s	Diffusion coefficient, zx component	Domain 1	
npe.D_cKxy	0	m ² /s	Diffusion coefficient, xy component	Domain 1	
npe.D_cKyy	dK	m ² /s	Diffusion coefficient, yy component	Domain 1	
npe.D_cKzy	0	m ² /s	Diffusion coefficient, zy component	Domain 1	
npe.D_cKxz	0	m ² /s	Diffusion coefficient, xz component	Domain 1	
npe.D_cKyz	0	m ² /s	Diffusion coefficient, yz component	Domain 1	
npe.D_cKzz	dK	m ² /s	Diffusion coefficient, zz component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.D_cClxx	dCl	m ² /s	Diffusion coefficient, xx component	Domain 1	
npe.D_cClyx	0	m ² /s	Diffusion coefficient, yx component	Domain 1	
npe.D_cClzx	0	m ² /s	Diffusion coefficient, zx component	Domain 1	
npe.D_cClxy	0	m ² /s	Diffusion coefficient, xy component	Domain 1	
npe.D_cClyy	dCl	m ² /s	Diffusion coefficient, yy component	Domain 1	
npe.D_cClzy	0	m ² /s	Diffusion coefficient, zy component	Domain 1	
npe.D_cClxz	0	m ² /s	Diffusion coefficient, xz component	Domain 1	
npe.D_cClyz	0	m ² /s	Diffusion coefficient, yz component	Domain 1	
npe.D_cClzz	dCl	m ² /s	Diffusion coefficient, zz component	Domain 1	
npe.um_ch3Oxx	npe.D_ch3Oxx / (R_const*npe.cdm1.mininput_temperature)	s·mol/kg	Mobility, xx component	Domain 1	
npe.um_ch3Oyx	npe.D_ch3Oyx / (R_const*npe.cdm1.mininput_temperature)	s·mol/kg	Mobility, yx component	Domain 1	
npe.um_ch3Ozx	npe.D_ch3Ozx / (R_const*npe.cdm1.mininput_temperature)	s·mol/kg	Mobility, zx component	Domain 1	
npe.um_ch3Oxy	npe.D_ch3Oxy / (R_const*npe.cdm1.mininput_temperature)	s·mol/kg	Mobility, xy component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.um_ch3Oyy	$npe.D_ch3Oyy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yy component	Domain 1	
npe.um_ch3Ozy	$npe.D_ch3Ozy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zy component	Domain 1	
npe.um_ch3Oxz	$npe.D_ch3Oxz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xz component	Domain 1	
npe.um_ch3Oyz	$npe.D_ch3Oyz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yz component	Domain 1	
npe.um_ch3Ozz	$npe.D_ch3Ozz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zz component	Domain 1	
npe.z_ch3O	1	1	Charge number	Domain 1	
npe.um_cOHxx	$npe.D_cOHxx / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xx component	Domain 1	
npe.um_cOHyx	$npe.D_cOHyx / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yx component	Domain 1	
npe.um_cOHzx	$npe.D_cOHzx / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zx component	Domain 1	
npe.um_cOHxy	$npe.D_cOHxy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xy component	Domain 1	
npe.um_cOHyy	$npe.D_cOHyy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yy component	Domain 1	
npe.um_cOHzy	$npe.D_cOHzy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zy component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
zy	dm1.mininput_temperature)	g	component		
npe.um_cOHxz	$npe.D_cOHxz/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xz component	Domain 1	
npe.um_cOHyz	$npe.D_cOHyz/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yz component	Domain 1	
npe.um_cOHzz	$npe.D_cOHzz/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zz component	Domain 1	
npe.z_cOH	-1	1	Charge number	Domain 1	
npe.um_cBeadxx	$npe.D_cBeadxx/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xx component	Domain 1	
npe.um_cBeadyx	$npe.D_cBeadyx/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yx component	Domain 1	
npe.um_cBeadzx	$npe.D_cBeadzx/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zx component	Domain 1	
npe.um_cBeadxy	$npe.D_cBeadxy/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xy component	Domain 1	
npe.um_cBeadyy	$npe.D_cBeadyy/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yy component	Domain 1	
npe.um_cBeadzy	$npe.D_cBeadzy/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zy component	Domain 1	
npe.um_cBeadxz	$npe.D_cBeadxz/(R_const*npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xz component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.um_cBeadyz	$npe.D_cBeadyz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yz component	Domain 1	
npe.um_cBeadzz	$npe.D_cBeadzz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zz component	Domain 1	
npe.z_cBead	-1	1	Charge number	Domain 1	
npe.um_cKxx	$npe.D_cKxx / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xx component	Domain 1	
npe.um_cKyx	$npe.D_cKyx / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yx component	Domain 1	
npe.um_cKzx	$npe.D_cKzx / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zx component	Domain 1	
npe.um_cKxy	$npe.D_cKxy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xy component	Domain 1	
npe.um_cKyy	$npe.D_cKyy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yy component	Domain 1	
npe.um_cKzy	$npe.D_cKzy / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zy component	Domain 1	
npe.um_cKxz	$npe.D_cKxz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, xz component	Domain 1	
npe.um_cKyz	$npe.D_cKyz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, yz component	Domain 1	
npe.um_cKzz	$npe.D_cKzz / (R_const * npe.cdm1.mininput_temperature)$	s·mol/kg	Mobility, zz component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	1.minput_temperature)	g	component		
npe.z_cK	1	1	Charge number	Domain 1	
npe.um_cClxx	npe.D_cClxx/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, xx component	Domain 1	
npe.um_cClyx	npe.D_cClyx/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, yx component	Domain 1	
npe.um_cClzx	npe.D_cClzx/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, zx component	Domain 1	
npe.um_cClxy	npe.D_cClxy/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, xy component	Domain 1	
npe.um_cClyy	npe.D_cClyy/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, yy component	Domain 1	
npe.um_cClzy	npe.D_cClzy/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, zy component	Domain 1	
npe.um_cClxz	npe.D_cClxz/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, xz component	Domain 1	
npe.um_cClyz	npe.D_cClyz/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, yz component	Domain 1	
npe.um_cClzz	npe.D_cClzz/(R_const*npe.cd m1.minput_temperature)	s·mol/kg	Mobility, zz component	Domain 1	
npe.z_cCl	-1	1	Charge number	Domain 1	
npe.Dav_ch3	0.5*(npe.D_ch	m ² /s	Average	Domain 1	

Name	Expression	Unit	Description	Selection	Details
O	$3O_{xx} + npe.D_cH3O_{yy}$		diffusion coefficient		
npe.Dav_cOH	$0.5*(npe.D_cOH_{xx} + npe.D_cOH_{yy})$	m^2/s	Average diffusion coefficient	Domain 1	
npe.Dav_cBead	$0.5*(npe.D_cBead_{xx} + npe.D_cBead_{yy})$	m^2/s	Average diffusion coefficient	Domain 1	
npe.Dav_cK	$0.5*(npe.D_cK_{xx} + npe.D_cK_{yy})$	m^2/s	Average diffusion coefficient	Domain 1	
npe.Dav_cCl	$0.5*(npe.D_cCl_{xx} + npe.D_cCl_{yy})$	m^2/s	Average diffusion coefficient	Domain 1	
npe.tflux_cH3Ox	$npe.dflux_cH3Ox + npe.cflux_cH3Ox + npe.mflux_cH3Ox$	$mol/(m^2 \cdot s)$	Total flux, x component	Domain 1	+ operation
npe.tflux_cH3Oy	$npe.dflux_cH3Oy + npe.cflux_cH3Oy + npe.mflux_cH3Oy$	$mol/(m^2 \cdot s)$	Total flux, y component	Domain 1	+ operation
npe.tflux_cH3Oz	$npe.dflux_cH3Oz + npe.cflux_cH3Oz + npe.mflux_cH3Oz$	$mol/(m^2 \cdot s)$	Total flux, z component	Domain 1	+ operation
npe.dfluxMag_cH3O	$\sqrt{npe.dflux_cH3Ox^2 + npe.dflux_cH3Oy^2 + npe.dflux_cH3Oz^2}$	$mol/(m^2 \cdot s)$	Diffusive flux magnitude	Domain 1	
npe.tfluxMag_cH3O	$\sqrt{npe.tflux_cH3Ox^2 + npe.tflux_cH3Oy^2 + npe.tflux_cH3Oz^2}$	$mol/(m^2 \cdot s)$	Total flux magnitude	Domain 1	
npe.mflux_cH3Ox	$npe.z_cH3O * F_const * cH3O * (-npe.um_cH3Ox * d(npe.V, x) - npe.um_cH3Ox * y * d(npe.V, y))$	$mol/(m^2 \cdot s)$	Electrophoretic flux, x component	Domain 1	
npe.mflux_cH3Oy	$npe.z_cH3O * F_const * cH3O * (-npe.um_cH3Oy * d(npe.V, x) - npe.um_cH3Oy * x * d(npe.V, y))$	$mol/(m^2 \cdot s)$	Electrophoretic flux, y component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	$npe.um_CH3Oyx*d(npe.V,x)-npe.um_CH3Oyy*d(npe.V,y)$		component		
npe.mflux_cH3Oz	$npe.z_CH3O*F_const*cH3O*(-npe.um_CH3Ozx*d(npe.V,x)-npe.um_CH3Ozy*d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, z component	Domain 1	
npe.tflux_cOHx	$npe.dflux_cOHx+npe.cflux_cOHx+npe.mflux_cOHx$	mol/(m ² .s)	Total flux, x component	Domain 1	+ operation
npe.tflux_cOHy	$npe.dflux_cOHy+npe.cflux_cOHy+npe.mflux_cOHy$	mol/(m ² .s)	Total flux, y component	Domain 1	+ operation
npe.tflux_cOHz	$npe.dflux_cOHz+npe.cflux_cOHz+npe.mflux_cOHz$	mol/(m ² .s)	Total flux, z component	Domain 1	+ operation
npe.dfluxMag_cOH	$\sqrt{npe.dflux_cOHx^2+npe.dflux_cOHy^2+npe.dflux_cOHx^2}$	mol/(m ² .s)	Diffusive flux magnitude	Domain 1	
npe.tfluxMag_cOH	$\sqrt{npe.tflux_cOHx^2+npe.tflux_cOHy^2+npe.tflux_cOHx^2}$	mol/(m ² .s)	Total flux magnitude	Domain 1	
npe.mflux_cOHx	$npe.z_cOH*F_const*cOH*(-npe.um_cOHxx*d(npe.V,x)-npe.um_cOHxy*d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, x component	Domain 1	
npe.mflux_cOHy	$npe.z_cOH*F_const*cOH*(-npe.um_cOHyx*d(npe.V,x)-npe.um_cOHyy*d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, y component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.mflux_cOH	$npe.z_cOH * F_const * cOH * (-npe.um_cOHx * d(npe.V,x) - npe.um_cOHy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, z component	Domain 1	
npe.tflux_cBeadx	$npe.dflux_cBeadx + npe.cflux_cBeadx + npe.mflux_cBeadx$	mol/(m ² .s)	Total flux, x component	Domain 1	+ operation
npe.tflux_cBeady	$npe.dflux_cBeady + npe.cflux_cBeady + npe.mflux_cBeady$	mol/(m ² .s)	Total flux, y component	Domain 1	+ operation
npe.tflux_cBeadz	$npe.dflux_cBeadz + npe.cflux_cBeadz + npe.mflux_cBeadz$	mol/(m ² .s)	Total flux, z component	Domain 1	+ operation
npe.dfluxMag_cBead	$\sqrt{(npe.dflux_cBeadx^2 + npe.dflux_cBeady^2 + npe.dflux_cBeadz^2)}$	mol/(m ² .s)	Diffusive flux magnitude	Domain 1	
npe.tfluxMag_cBead	$\sqrt{(npe.tflux_cBeadx^2 + npe.tflux_cBeady^2 + npe.tflux_cBeadz^2)}$	mol/(m ² .s)	Total flux magnitude	Domain 1	
npe.mflux_cBeadx	$npe.z_cBead * F_const * cBead * (-npe.um_cBeadx * d(npe.V,x) - npe.um_cBeady * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, x component	Domain 1	
npe.mflux_cBeady	$npe.z_cBead * F_const * cBead * (-npe.um_cBeady * d(npe.V,x) - npe.um_cBeadx * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, y component	Domain 1	
npe.mflux_cBeadz	$npe.z_cBead * F_const * cBead * (-npe.um_cBeadz * d(npe.V,x) - npe.um_cBeadz * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, z component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	$npe.um_cBead_{zx} * d(npe.V,x) - npe.um_cBead_{zy} * d(npe.V,y)$				
npe.tflux_cKx	$npe.dflux_cKx + npe.cflux_cKx + npe.mflux_cKx$	mol/(m ² .s)	Total flux, x component	Domain 1	+ operation
npe.tflux_cKy	$npe.dflux_cKy + npe.cflux_cKy + npe.mflux_cKy$	mol/(m ² .s)	Total flux, y component	Domain 1	+ operation
npe.tflux_cKz	$npe.dflux_cKz + npe.cflux_cKz + npe.mflux_cKz$	mol/(m ² .s)	Total flux, z component	Domain 1	+ operation
npe.dfluxMag_cK	$\sqrt{npe.dflux_cKx^2 + npe.dflux_cKy^2 + npe.dflux_cKz^2}$	mol/(m ² .s)	Diffusive flux magnitude	Domain 1	
npe.tfluxMag_cK	$\sqrt{npe.tflux_cKx^2 + npe.tflux_cKy^2 + npe.tflux_cKz^2}$	mol/(m ² .s)	Total flux magnitude	Domain 1	
npe.mflux_cKx	$npe.z_cK * F_const * cK * (-npe.um_cKxx * d(npe.V,x) - npe.um_cKxy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, x component	Domain 1	
npe.mflux_cKy	$npe.z_cK * F_const * cK * (-npe.um_cKyx * d(npe.V,x) - npe.um_cKyy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, y component	Domain 1	
npe.mflux_cKz	$npe.z_cK * F_const * cK * (-npe.um_cKzx * d(npe.V,x) - npe.um_cKzy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, z component	Domain 1	
npe.tflux_cClx	$npe.dflux_cClx + npe.cflux_cClx + npe.mflux_cClx$	mol/(m ² .s)	Total flux, x component	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
npe.tflux_cCly	$npe.dflux_cCly + npe.cflux_cCly + npe.mflux_cCly$	mol/(m ² .s)	Total flux, y component	Domain 1	+ operation
npe.tflux_cClz	$npe.dflux_cClz + npe.cflux_cClz + npe.mflux_cClz$	mol/(m ² .s)	Total flux, z component	Domain 1	+ operation
npe.dfluxMag_cCl	$\sqrt{(npe.dflux_cClx^2 + npe.dflux_cCly^2 + npe.dflux_cClz^2)}$	mol/(m ² .s)	Diffusive flux magnitude	Domain 1	
npe.tfluxMag_cCl	$\sqrt{(npe.tflux_cClx^2 + npe.tflux_cCly^2 + npe.tflux_cClz^2)}$	mol/(m ² .s)	Total flux magnitude	Domain 1	
npe.mflux_cClx	$npe.z_cCl * F_const * cCl * (-npe.um_cClxx * d(npe.V,x) - npe.um_cClxy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, x component	Domain 1	
npe.mflux_cCly	$npe.z_cCl * F_const * cCl * (-npe.um_cClyx * d(npe.V,x) - npe.um_cClyy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, y component	Domain 1	
npe.mflux_cClz	$npe.z_cCl * F_const * cCl * (-npe.um_cClzx * d(npe.V,x) - npe.um_cClzy * d(npe.V,y))$	mol/(m ² .s)	Electrophoretic flux, z component	Domain 1	
npe.u	model.input.u1	m/s	Velocity field, x component	Domain 1	Meta
npe.v	model.input.u2	m/s	Velocity field, y component	Domain 1	Meta
npe.w	model.input.u3	m/s	Velocity field, z component	Domain 1	Meta
npe.cflux_cH3Ox	cH3O*npe.u	mol/(m ² .s)	Convective flux, x component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.cflux_cH3Oy	cH3O*npe.v	mol/(m ² .s)	Convective flux, y component	Domain 1	
npe.cflux_cH3Oz	cH3O*npe.w	mol/(m ² .s)	Convective flux, z component	Domain 1	
npe.cfluxMag_cH3O	sqrt(npe.cflux_cH3Ox^2+npe.cflux_cH3Oy^2+npe.cflux_cH3Oz^2)	mol/(m ² .s)	Convective flux magnitude	Domain 1	
npe.cflux_cOHx	cOH*npe.u	mol/(m ² .s)	Convective flux, x component	Domain 1	
npe.cflux_cOHy	cOH*npe.v	mol/(m ² .s)	Convective flux, y component	Domain 1	
npe.cflux_cOHz	cOH*npe.w	mol/(m ² .s)	Convective flux, z component	Domain 1	
npe.cfluxMag_cOH	sqrt(npe.cflux_cOHx^2+npe.cflux_cOHy^2+npe.cflux_cOHz^2)	mol/(m ² .s)	Convective flux magnitude	Domain 1	
npe.cflux_cBeadx	cBead*npe.u	mol/(m ² .s)	Convective flux, x component	Domain 1	
npe.cflux_cBeady	cBead*npe.v	mol/(m ² .s)	Convective flux, y component	Domain 1	
npe.cflux_cBeadz	cBead*npe.w	mol/(m ² .s)	Convective flux, z component	Domain 1	
npe.cfluxMag_cBead	sqrt(npe.cflux_cBeadx^2+npe.cflux_cBeady^2+npe.cflux_cBeadz^2)	mol/(m ² .s)	Convective flux magnitude	Domain 1	
npe.cflux_cKx	cK*npe.u	mol/(m ² .s)	Convective flux, x component	Domain 1	
npe.cflux_cKy	cK*npe.v	mol/(m	Convective	Domain 1	

Name	Expression	Unit	Description	Selection	Details
		$^2 \cdot s$)	flux, y component		
npe.cflux_cKz	$cK * npe.w$	$mol / (m^2 \cdot s)$	Convective flux, z component	Domain 1	
npe.cfluxMag_cK	$\sqrt{npe.cflux_cKx^2 + npe.cflux_cKy^2 + npe.cflux_cKz^2}$	$mol / (m^2 \cdot s)$	Convective flux magnitude	Domain 1	
npe.cflux_cClx	$cCl * npe.u$	$mol / (m^2 \cdot s)$	Convective flux, x component	Domain 1	
npe.cflux_cCl y	$cCl * npe.v$	$mol / (m^2 \cdot s)$	Convective flux, y component	Domain 1	
npe.cflux_cCl z	$cCl * npe.w$	$mol / (m^2 \cdot s)$	Convective flux, z component	Domain 1	
npe.cfluxMag_cCl	$\sqrt{npe.cflux_cClx^2 + npe.cflux_cCly^2 + npe.cflux_cClz^2}$	$mol / (m^2 \cdot s)$	Convective flux magnitude	Domain 1	
npe.dflux_ch3Ox	$-npe.D_ch3Oxx * npe.grad_ch3Ox - npe.D_ch3Oxy * npe.grad_ch3Oy - npe.D_ch3Oxz * npe.grad_ch3Oz$	$mol / (m^2 \cdot s)$	Diffusive flux, x component	Domain 1	
npe.dflux_ch3Oy	$-npe.D_ch3Oyx * npe.grad_ch3Ox - npe.D_ch3Oyy * npe.grad_ch3Oy - npe.D_ch3Oyz * npe.grad_ch3Oz$	$mol / (m^2 \cdot s)$	Diffusive flux, y component	Domain 1	
npe.dflux_ch3Oz	$-npe.D_ch3Ozx * npe.grad_ch3Ox - npe.D_ch3Ozy * npe.grad_ch3Oy - npe.D_ch3Ozz * npe.grad_ch3Oz$	$mol / (m^2 \cdot s)$	Diffusive flux, z component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	*npe.grad_cH3Ox- npe.D_cH3Ozy *npe.grad_cH3Oy- npe.D_cH3Ozz *npe.grad_cH3Oz				
npe.grad_cH3Ox	cH3Ox	mol/m ⁴	Concentration gradient, x component	Domain 1	
npe.grad_cH3Oy	cH3Oy	mol/m ⁴	Concentration gradient, y component	Domain 1	
npe.grad_cH3Oz	0	mol/m ⁴	Concentration gradient, z component	Domain 1	
npe.ctemp	- cH3O*npe.z_cH3O- cOH*npe.z_cOH- cBead*npe.z_cBead- cK*npe.z_cK	mol/m ³	Concentration	Domain 1	+ operation
npe.dflux_cOHx	- npe.D_cOHxx* npe.grad_cOHx- npe.D_cOHxy* npe.grad_cOHy- npe.D_cOHxz* npe.grad_cOHz	mol/(m ² .s)	Diffusive flux, x component	Domain 1	
npe.dflux_cOHy	- npe.D_cOHyx* npe.grad_cOHx- npe.D_cOHyy* npe.grad_cOHy- npe.D_cOHyz* npe.grad_cOHz	mol/(m ² .s)	Diffusive flux, y component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
npe.dflux_cOH _z	- npe.D_cOH _{zx} * npe.grad_cOH _x - npe.D_cOH _{zy} * npe.grad_cOH _y - npe.D_cOH _{zz} * npe.grad_cOH _z	mol/(m ² .s)	Diffusive flux, z component	Domain 1	
npe.grad_cOH _x	cOH _x	mol/m ⁴	Concentration gradient, x component	Domain 1	
npe.grad_cOH _y	cOH _y	mol/m ⁴	Concentration gradient, y component	Domain 1	
npe.grad_cOH _z	0	mol/m ⁴	Concentration gradient, z component	Domain 1	
npe.dflux_cBead _x	- npe.D_cBead _{xx} * npe.grad_cBead _x - npe.D_cBead _{xy} * npe.grad_cBead _y - npe.D_cBead _{xz} * npe.grad_cBead _z	mol/(m ² .s)	Diffusive flux, x component	Domain 1	
npe.dflux_cBead _y	- npe.D_cBead _{yx} * npe.grad_cBead _x - npe.D_cBead _{yy} * npe.grad_cBead _y - npe.D_cBead _{yz} * npe.grad_cBead _z	mol/(m ² .s)	Diffusive flux, y component	Domain 1	
npe.dflux_cBead _z	- npe.D_cBead _{zx} * npe.grad_cBead _x - npe.D_cBead _{zy} * npe.grad_cBead _y	mol/(m ² .s)	Diffusive flux, z component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	ady- npe.D_cBeadzz *npe.grad_cBe adz				
npe.grad_cBe adx	cBeadx	mol/m ⁴	Concentration gradient, x component	Domain 1	
npe.grad_cBe ady	cBeady	mol/m ⁴	Concentration gradient, y component	Domain 1	
npe.grad_cBe adz	0	mol/m ⁴	Concentration gradient, z component	Domain 1	
npe.dflux_cKx	- npe.D_cKxx*np e.grad_cKx- npe.D_cKxy*np e.grad_cKy- npe.D_cKxz*np e.grad_cKz	mol/(m ² .s)	Diffusive flux, x component	Domain 1	
npe.dflux_cKy	- npe.D_cKyx*np e.grad_cKx- npe.D_cKyy*np e.grad_cKy- npe.D_cKyz*np e.grad_cKz	mol/(m ² .s)	Diffusive flux, y component	Domain 1	
npe.dflux_cKz	- npe.D_cKzx*np e.grad_cKx- npe.D_cKzy*np e.grad_cKy- npe.D_cKzz*np e.grad_cKz	mol/(m ² .s)	Diffusive flux, z component	Domain 1	
npe.grad_cKx	cKx	mol/m ⁴	Concentration gradient, x component	Domain 1	
npe.grad_cKy	cKy	mol/m ⁴	Concentration gradient, y component	Domain 1	
npe.grad_cKz	0	mol/m ⁴	Concentration gradient, z component	Domain 1	
npe.dflux_cCl	-	mol/(m	Diffusive flux,	Domain 1	

Name	Expression	Unit	Description	Selection	Details
x	npe.D_cClxx*n pe.grad_cClx- npe.D_cClxy*n pe.grad_cCly- npe.D_cClxz*n pe.grad_cClz	² .s)	x component		
npe.dflux_cCl y	- npe.D_cClyx*n pe.grad_cClx- npe.D_cClyy*n pe.grad_cCly- npe.D_cClyz*n pe.grad_cClz	mol/(m ² .s)	Diffusive flux, y component	Domain 1	
npe.dflux_cCl z	- npe.D_cClzx*n pe.grad_cClx- npe.D_cClzy*n pe.grad_cCly- npe.D_cClzz*n pe.grad_cClz	mol/(m ² .s)	Diffusive flux, z component	Domain 1	
npe.grad_cCl x	d(cCl,x)	mol/m ⁴	Concentration gradient, x component	Domain 1	
npe.grad_cCl y	d(cCl,y)	mol/m ⁴	Concentration gradient, y component	Domain 1	
npe.grad_cCl z	0	mol/m ⁴	Concentration gradient, z component	Domain 1	
npe.kappa0x x	cH3O*npe.um_ cH3Oxx*(npe.z _cH3O*F_const)^2+cOH*npe. um_cOHxx*(np e.z_cOH*F_con st)^2+cBead*n pe.um_cBeadx x*(npe.z_cBead *F_const)^2+c K*npe.um_cKx x*(npe.z_cK*F_ const)^2+cCl* npe.um_cClxx* (npe.z_cCl*F_c onst)^2	S/m	Electrolyte conductivity, xx component	Domain 1	+ operati on

Name	Expression	Unit	Description	Selection	Details
npe.kappa0yx	$cH3O*npe.um_cH3Oyx*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHyx*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadyx*(npe.z_cBead*F_const)^2+cK*npe.um_cKyx*(npe.z_cK*F_const)^2+cCl*npe.um_cClxy*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, yx component	Domain 1	+ operation
npe.kappa0zx	$cH3O*npe.um_cH3Ozx*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHzx*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadzx*(npe.z_cBead*F_const)^2+cK*npe.um_cKzx*(npe.z_cK*F_const)^2+cCl*npe.um_cClzx*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, zx component	Domain 1	+ operation
npe.kappa0xy	$cH3O*npe.um_cH3Oxy*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHxy*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadxy*(npe.z_cBead*F_const)^2+cK*npe.um_cKxy*(npe.z_cK*F_const)^2+cCl*npe.um_cClxy*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, xy component	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
	onst)^2				
npe.kappa0y	$cH3O*npe.um_cH3Oy*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHyy*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeady*(npe.z_cBead*F_const)^2+cK*npe.um_cKy*(npe.z_cK*F_const)^2+cCl*npe.um_cCly*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, yy component	Domain 1	+ operation
npe.kappa0z	$cH3O*npe.um_cH3Ozy*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHzy*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadz*(npe.z_cBead*F_const)^2+cK*npe.um_cKzy*(npe.z_cK*F_const)^2+cCl*npe.um_cClzy*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, zy component	Domain 1	+ operation
npe.kappa0xz	$cH3O*npe.um_cH3Oxz*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHxz*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadx*(npe.z_cBead*F_const)^2+cK*npe.um_cKxz*(npe.z_cK*F_const)^2+cCl*$	S/m	Electrolyte conductivity, xz component	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
	$npe.um_cClxz^*(npe.z_cCl*F_const)^2$				
npe.kappa0yz	$cH3O*npe.um_cH3Oyz*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHyz*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadyz*(npe.z_cBead*F_const)^2+cK*npe.um_cKyz*(npe.z_cK*F_const)^2+cCl*npe.um_cClyz*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, yz component	Domain 1	+ operation
npe.kappa0zz	$cH3O*npe.um_cH3Ozz*(npe.z_cH3O*F_const)^2+cOH*npe.um_cOHzz*(npe.z_cOH*F_const)^2+cBead*npe.um_cBeadzz*(npe.z_cBead*F_const)^2+cK*npe.um_cKzz*(npe.z_cK*F_const)^2+cCl*npe.um_cClzz*(npe.z_cCl*F_const)^2$	S/m	Electrolyte conductivity, zz component	Domain 1	+ operation
npe.kappaxx	npe.kappa0xx	S/m	Electrolyte conductivity, xx component	Domain 1	
npe.kappayx	npe.kappa0yx	S/m	Electrolyte conductivity, yx component	Domain 1	
npe.kappazx	npe.kappa0zx	S/m	Electrolyte conductivity, zx component	Domain 1	

Name	Expression	Unit	Description	Selection	Details
			component		
npe.kappaxy	npe.kappa0xy	S/m	Electrolyte conductivity, xy component	Domain 1	
npe.kappayy	npe.kappa0yy	S/m	Electrolyte conductivity, yy component	Domain 1	
npe.kappazy	npe.kappa0zy	S/m	Electrolyte conductivity, zy component	Domain 1	
npe.kappaxz	npe.kappa0xz	S/m	Electrolyte conductivity, xz component	Domain 1	
npe.kappayz	npe.kappa0yz	S/m	Electrolyte conductivity, yz component	Domain 1	
npe.kappazz	npe.kappa0zz	S/m	Electrolyte conductivity, zz component	Domain 1	
npe.J0x	$F_const*(npe.z_cH3O*(-npe.D_cH3Oxx*npe.grad_cH3Ox-npe.D_cH3Oxy*npe.grad_cH3Oy-npe.D_cH3Oxz*npe.grad_cH3Oz+npe.z_cH3O*F_const*cH3O*(-npe.um_cH3Oxx*Vx-npe.um_cH3Oxy*Vy))+npe.z_cOH*(-npe.D_cOHxx*npe.grad_cOH$	A/m ²	Current density, x component	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
	$ \begin{aligned} & x- \\ & npe.D_cOHxy* \\ & npe.grad_cOH \\ & y- \\ & npe.D_cOHxz* \\ & npe.grad_cOH \\ & z+npe.z_cOH* \\ & F_const*cOH*(\\ & - \\ & npe.um_cOHxx \\ & *Vx- \\ & npe.um_cOHxy \\ & *Vy))+npe.z_cB \\ & ead*(- \\ & npe.D_cBeadxx \\ & *npe.grad_cBe \\ & adx- \\ & npe.D_cBeadxy \\ & *npe.grad_cBe \\ & ady- \\ & npe.D_cBeadxz \\ & *npe.grad_cBe \\ & adz+npe.z_cBe \\ & ad*F_const*cB \\ & ead*(- \\ & npe.um_cBead \\ & xx*Vx- \\ & npe.um_cBead \\ & xy*Vy))+npe.z_ \\ & cK*(- \\ & npe.D_cKxx*np \\ & e.grad_cKx- \\ & npe.D_cKxy*np \\ & e.grad_cKy- \\ & npe.D_cKxz*np \\ & e.grad_cKz+np \\ & e.z_cK*F_const \\ & *cK*(- \\ & npe.um_cKxx* \\ & Vx- \\ & npe.um_cKxy* \\ & Vy))+npe.z_cCl \\ & *(- \\ & npe.D_cClxx*n \\ & pe.grad_cClx- \\ & npe.D_cClxy*n \\ & pe.grad_cCly- \\ & npe.D_cClxz*n \end{aligned} $				

Name	Expression	Unit	Description	Selection	Details
	pe.grad_cClz+ npe.z_cCl*F_co nst*cCl*(- npe.um_cClxx* Vx- npe.um_cClxy* Vy)))				
npe.J0y	F_const*(npe.z _cH3O*(- npe.D_cH3Oyx *npe.grad_cH3 Ox- npe.D_cH3Oyy *npe.grad_cH3 Oy- npe.D_cH3Oyz *npe.grad_cH3 Oz+npe.z_cH3 O*F_const*cH3 O*(- npe.um_cH3O yx*Vx- npe.um_cH3O yy*Vy))+npe.z_ cOH*(- npe.D_cOHyx* npe.grad_cOH x- npe.D_cOHyy* npe.grad_cOH y- npe.D_cOHyz* npe.grad_cOH z+npe.z_cOH* F_const*cOH*(- npe.um_cOHyx *Vx- npe.um_cOHyy *Vy))+npe.z_cB ead*(- npe.D_cBeadyx *npe.grad_cBe adx- npe.D_cBeadyy *npe.grad_cBe ady-	A/m ²	Current density, y component	Domain 1	+ operati on

Name	Expression	Unit	Description	Selection	Details
	$\begin{aligned} & npe.D_cBeadyz \\ & *npe.grad_cBe \\ & adz+npe.z_cBe \\ & ad*F_const*cB \\ & ead*(- \\ & npe.um_cBead \\ & yx*Vx- \\ & npe.um_cBead \\ & yy*Vy))+npe.z_ \\ & cK*(- \\ & npe.D_cKyx*np \\ & e.grad_cKx- \\ & npe.D_cKyy*np \\ & e.grad_cKy- \\ & npe.D_cKyz*np \\ & e.grad_cKz+npe \\ & e.z_cK*F_const \\ & *cK*(- \\ & npe.um_cKyx* \\ & Vx- \\ & npe.um_cKyy* \\ & Vy))+npe.z_cCl \\ & *(- \\ & npe.D_cClyx*n \\ & pe.grad_cClx- \\ & npe.D_cClyy*n \\ & pe.grad_cCly- \\ & npe.D_cClyz*n \\ & pe.grad_cClz+ \\ & npe.z_cCl*F_co \\ & nst*cCl*(- \\ & npe.um_cClyx* \\ & Vx- \\ & npe.um_cClyy* \\ & Vy))) \end{aligned}$				
npe.J0z	$\begin{aligned} & F_const*(npe.z \\ & _cH3O*(- \\ & npe.D_cH3Ozx \\ & *npe.grad_cH3 \\ & Ox- \\ & npe.D_cH3Ozy \\ & *npe.grad_cH3 \\ & Oy- \\ & npe.D_cH3Ozz \\ & *npe.grad_cH3 \\ & Oz+npe.z_cH3 \\ & O*F_const*cH3 \end{aligned}$	A/m ²	Current density, z component	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
	$O*(-npe.um_cH3Oz$ $x*Vx-$ $npe.um_cH3Oz$ $y*Vy))+npe.z_c$ $OH*(-$ $npe.D_cOHzx*$ $npe.grad_cOH$ $x-$ $npe.D_cOHzy*$ $npe.grad_cOH$ $y-$ $npe.D_cOHzz*$ $npe.grad_cOH$ $z+npe.z_cOH*$ $F_const*cOH*($ $-$ $npe.um_cOHzx$ $*Vx-$ $npe.um_cOHzy$ $*Vy))+npe.z_cB$ $ead*(-$ $npe.D_cBeadzx$ $*npe.grad_cBe$ $adx-$ $npe.D_cBeadzy$ $*npe.grad_cBe$ $ady-$ $npe.D_cBeadzz$ $*npe.grad_cBe$ $adz+npe.z_cBe$ $ad*F_const*cB$ $ead*(-$ $npe.um_cBead$ $zx*Vx-$ $npe.um_cBead$ $zy*Vy))+npe.z_c$ $K*(-$ $npe.D_cKzx*np$ $e.grad_cKx-$ $npe.D_cKzy*np$ $e.grad_cKy-$ $npe.D_cKzz*np$ $e.grad_cKz+np$ $e.z_cK*F_const$ $*cK*(-$ $npe.um_cKzx*$ $Vx-$				

Name	Expression	Unit	Description	Selection	Details
	npe.um_cKzy* Vy))+npe.z_cCl *(- npe.D_cClzx*n pe.grad_cClx- npe.D_cClzy*n pe.grad_cCly- npe.D_cClzz*n pe.grad_cClz+ npe.z_cCl*F_co nst*cCl*(- npe.um_cClzx* Vx- npe.um_cClzy* Vy)))				
npe.Jx	npe.J0x	A/m ²	Current density, x component	Domain 1	
npe.Jy	npe.J0y	A/m ²	Current density, y component	Domain 1	
npe.Jz	npe.J0z	A/m ²	Current density, z component	Domain 1	
npe.V	V	V	Electric potential	Domain 1	
npe.gradVx	Vx	V/m	Potential gradient, x component	Domain 1	
npe.gradVy	Vy	V/m	Potential gradient, y component	Domain 1	
npe.gradVz	0	V/m	Potential gradient, z component	Domain 1	
npe.Res_ch3 O	- npe.D_ch3Oxx *ch3Oxx- npe.D_ch3Oxy *ch3Oxy- npe.D_ch3Oyx *ch3Oyx- npe.D_ch3Oyy *ch3Oyy+d(ch 3O*(npe.u-	mol/(m ³ .s)	Equation residual	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	$\begin{aligned} & npe.z_cH3O * npe.um_cH3O_{xx} \\ & * F_const * d(npe.V, x) - \\ & npe.z_cH3O * npe.um_cH3O_{xy} \\ & * F_const * d(npe.V, y), x) + d(cH3O * (npe.v - \\ & npe.z_cH3O * npe.um_cH3O_{yx} \\ & * F_const * d(npe.V, x) - \\ & npe.z_cH3O * npe.um_cH3O_{yy} \\ & * F_const * d(npe.V, y)), y) - \\ & npe.R_cH3O \end{aligned}$				
npe.Res_cOH	$\begin{aligned} & - \\ & npe.D_cOH_{xx} * cOH_{xx} - \\ & npe.D_cOH_{xy} * cOH_{xy} - \\ & npe.D_cOH_{yx} * cOH_{yx} - \\ & npe.D_cOH_{yy} * cOH_{yy} + d(cOH * (npe.u - \\ & npe.z_cOH * npe.um_cOH_{xx} * F_const * d(npe.V, x) - \\ & npe.z_cOH * npe.um_cOH_{xy} * F_const * d(npe.V, y)), x) + d(cOH * (\\ & npe.v - \\ & npe.z_cOH * npe.um_cOH_{yx} * F_const * d(npe.V, x) - \\ & npe.z_cOH * npe.um_cOH_{yy} * F_const * d(npe.V, y)), y) - \\ & npe.R_cOH \end{aligned}$	mol/(m ³ .s)	Equation residual	Domain 1	
npe.Res_cBea	-	mol/(m	Equation	Domain 1	

Name	Expression	Unit	Description	Selection	Details
d	$\begin{aligned} & npe.D_cBeadxx \\ & *cBeadxx- \\ & npe.D_cBeadxy \\ & *cBeadxy- \\ & npe.D_cBeadyx \\ & *cBeadyx- \\ & npe.D_cBeadyy \\ & *cBeadyy+d(cB \\ & ead*(npe.u- \\ & npe.z_cBead*n \\ & pe.um_cBeadx \\ & x*F_const*d(np \\ & e.V,x)- \\ & npe.z_cBead*n \\ & pe.um_cBeadx \\ & y*F_const*d(np \\ & e.V,y)),x)+d(cB \\ & ead*(npe.v- \\ & npe.z_cBead*n \\ & pe.um_cBeady \\ & x*F_const*d(np \\ & e.V,x)- \\ & npe.z_cBead*n \\ & pe.um_cBeady \\ & y*F_const*d(np \\ & e.V,y)),y)- \\ & npe.R_cBead \end{aligned}$	³ .s)	residual		
npe.Res_cK	$\begin{aligned} & - \\ & npe.D_cKxx*cK \\ & xx- \\ & npe.D_cKxy*cK \\ & xy- \\ & npe.D_cKyx*cK \\ & yx- \\ & npe.D_cKyy*cK \\ & yy+d(cK*(npe. \\ & u- \\ & npe.z_cK*npe.u \\ & m_cKxx*F_cons \\ & t*d(npe.V,x)- \\ & npe.z_cK*npe.u \\ & m_cKxy*F_cons \\ & t*d(npe.V,y)),x) \\ & +d(cK*(npe.v- \\ & npe.z_cK*npe.u \\ & m_cKyx*F_cons \\ & t*d(npe.V,x)- \end{aligned}$	mol/(m ³ .s)	Equation residual	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	$npe.z_cK * npe.u$ $m_cKyy * F_cons$ $t * d(npe.V, y), y)$ $- npe.R_cK$				

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
cH3O	Lagrange (Quadratic)	mol/m ³	Concentration	Spatial	Domain 1
cOH	Lagrange (Quadratic)	mol/m ³	Concentration	Spatial	Domain 1
cBead	Lagrange (Quadratic)	mol/m ³	Concentration	Spatial	Domain 1
cK	Lagrange (Quadratic)	mol/m ³	Concentration	Spatial	Domain 1
V	Lagrange (Quadratic)	V	Electric potential	Spatial	Domain 1

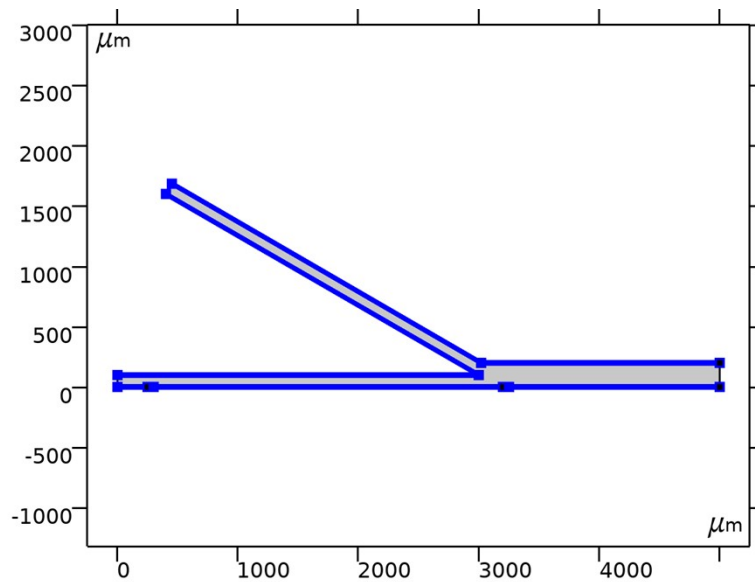
Weak expressions

Weak expression	Integration order	Integration frame	Selection
$cH3O * (npe.u * test(cH3Ox) + npe.v * test(cH3Oy)) * (isScalingSystemDomain = 0)$	4	Spatial	Domain 1
$npe.cbf_cH3O * test(cH3O)$	4	Spatial	Boundaries 1–12
$cOH * (npe.u * test(cOHx) + npe.v * test(cOHy)) * (isScalingSystemDomain = 0)$	4	Spatial	Domain 1
$npe.cbf_cOH * test(cOH)$	4	Spatial	Boundaries 1–12
$cBead * (npe.u * test(cBeadx) + npe.v * test(cBeady)) * (isScalingSystemDomain = 0)$	4	Spatial	Domain 1
$npe.cbf_cBead * test(cBead)$	4	Spatial	Boundaries 1–12
$cK * (npe.u * test(cKx) + npe.v * test(cKy)) * (isScalingSystemDomain = 0)$	4	Spatial	Domain 1

Weak expression	Integration order	Integration frame	Selection
SystemDomain==0)			
npe.cbf_cK*test(cK)	4	Spatial	Boundaries 1–12
npe.z_cH3O*F_const*cH3O*((-npe.um_cH3Oxx*d(npe.V,x)-npe.um_cH3Oxy*d(npe.V,y))*test(cH3Ox)+(-npe.um_cH3Oyx*d(npe.V,x)-npe.um_cH3Oyy*d(npe.V,y))*test(cH3Oy))	4	Spatial	Domain 1
npe.z_cOH*F_const*cOH*((-npe.um_cOHxx*d(npe.V,x)-npe.um_cOHxy*d(npe.V,y))*test(cOHx)+(-npe.um_cOHyx*d(npe.V,x)-npe.um_cOHyy*d(npe.V,y))*test(cOHy))	4	Spatial	Domain 1
npe.z_cBead*F_const*cBead*((-npe.um_cBeadxx*d(npe.V,x)-npe.um_cBeadxy*d(npe.V,y))*test(cBeadx)+(-npe.um_cBeadyx*d(npe.V,x)-npe.um_cBeadyy*d(npe.V,y))*test(cBeady))	4	Spatial	Domain 1
npe.z_cK*F_const*cK*((-npe.um_cKxx*d(npe.V,x)-npe.um_cKxy*d(npe.V,y))*test(cKx)+(-npe.um_cKyx*d(npe.V,x)-npe.um_cKyy*d(npe.V,y))*test(cKy))	4	Spatial	Domain 1
npe.dflux_cH3Ox*test(cH3Ox)+npe.dflux_cH3O	4	Spatial	Domain 1

Weak expression	Integration order	Integration frame	Selection
$y \cdot \text{test}(cH3Oy)$			
$\text{npe.dflux_cOHx} \cdot \text{test}(cOHx) + \text{npe.dflux_cOHy} \cdot \text{test}(cOHy)$	4	Spatial	Domain 1
$\text{npe.dflux_cBeadx} \cdot \text{test}(cBeadx) + \text{npe.dflux_cBeady} \cdot \text{test}(cBeady)$	4	Spatial	Domain 1
$\text{npe.dflux_cKx} \cdot \text{test}(cKx) + \text{npe.dflux_cKy} \cdot \text{test}(cKy)$	4	Spatial	Domain 1
$\text{npe.Jx} \cdot \text{test}(Vx) + \text{npe.Jy} \cdot \text{test}(Vy)$	4	Spatial	Domain 1
$\text{npe.streamline}(\text{isScalingSystemDomain} = 0)$	4	Spatial	Domain 1

2.4.4 No Flux 1



No Flux 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{J}_i = 0$$

Convection

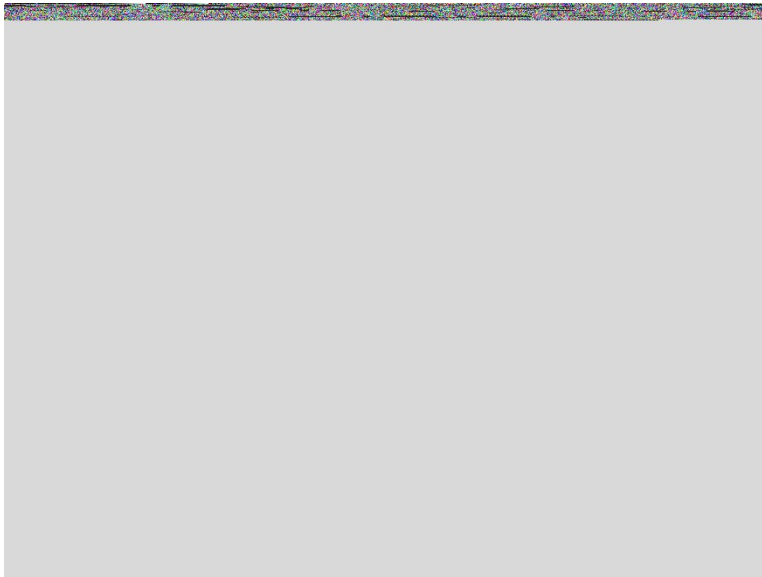
SETTINGS

Description	Value
Include	Off

Variables

Name	Expression	Unit	Description	Selection
npe.cbf_cH3O	$cH3O*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundaries 2–3, 5, 7–9, 11
npe.cbf_cOH	$cOH*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundaries 2–3, 5, 7–9, 11
npe.cbf_cBead	$cBead*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundaries 2–3, 5, 7–9, 11
npe.cbf_cK	$cK*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundaries 2–3, 5, 7–9, 11

2.4.5 Electric Insulation 1



Electric Insulation 1

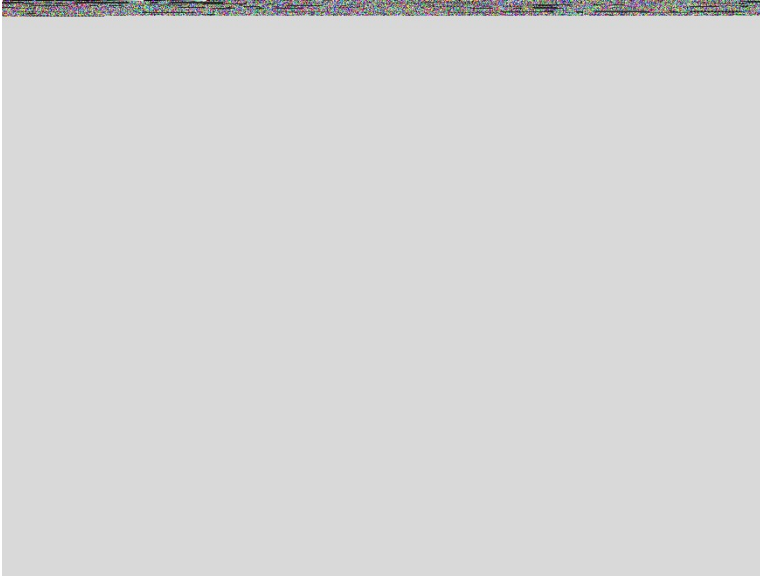
SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{i} = 0$$

2.4.6 Initial Values 1



Initial Values 1

SELECTION

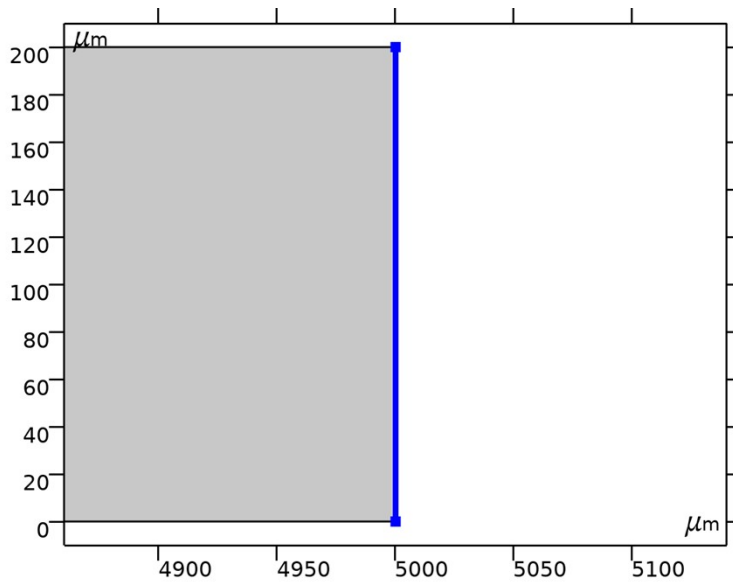
Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

Initial values

SETTINGS

Description	Value
Concentration	{cH3O0, cOH0, cBead0, cK0 + cBead0, 0}
Electric potential	0

2.4.7 Inflow 1



Inflow 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 12

EQUATIONS

$$c_i = c_{0j}$$

.....

Concentration

SETTINGS

Description	Value
Concentration	{cH3O0, cOH0, cBead0, cK0 + cBead0, 0}

Boundary condition type

SETTINGS

Description	Value
Boundary condition type	Concentration constraint

Variables

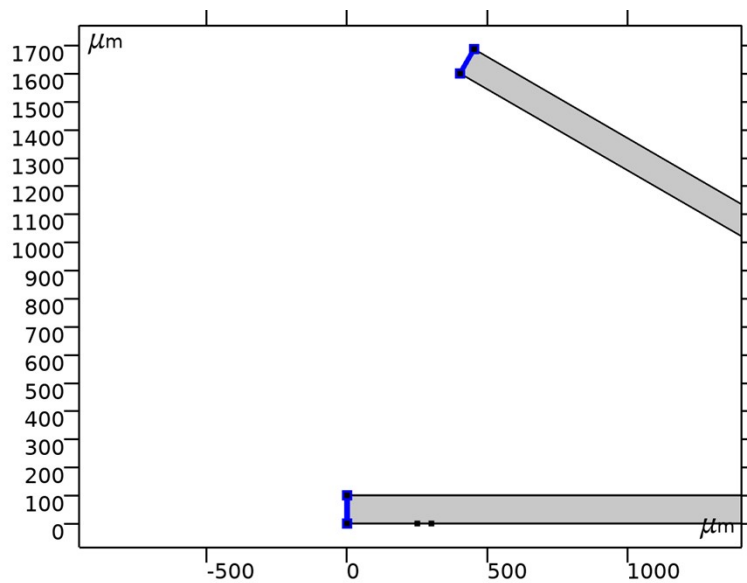
Name	Expression	Unit	Description	Selection	Details
npe.c0_	cH3O0	mol/m ³	Concentration	Boundary	+ operati

Name	Expression	Unit	Description	Selection	Details
cH3O				12	on
npe.c0_cOH	cOH0	mol/m ³	Concentration	Boundary 12	+ operation
npe.c0_cBead	cBead0	mol/m ³	Concentration	Boundary 12	+ operation
npe.c0_cK	cK0+cBead0	mol/m ³	Concentration	Boundary 12	+ operation

Constraints

Constraint	Constraint force	Shape function	Selection	Details
-cH3O+npe.c0_cH3O	-test(cH3O)	Lagrange (Quadratic)	Boundary 12	Elemental
-cOH+npe.c0_cOH	-test(cOH)	Lagrange (Quadratic)	Boundary 12	Elemental
-cBead+npe.c0_cBead	-test(cBead)	Lagrange (Quadratic)	Boundary 12	Elemental
-cK+npe.c0_cK	-test(cK)	Lagrange (Quadratic)	Boundary 12	Elemental

2.4.8 Outflow 1



Outflow 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 1, 6

EQUATIONS

$$\mathbf{n} \cdot D_i \nabla c_i = 0$$

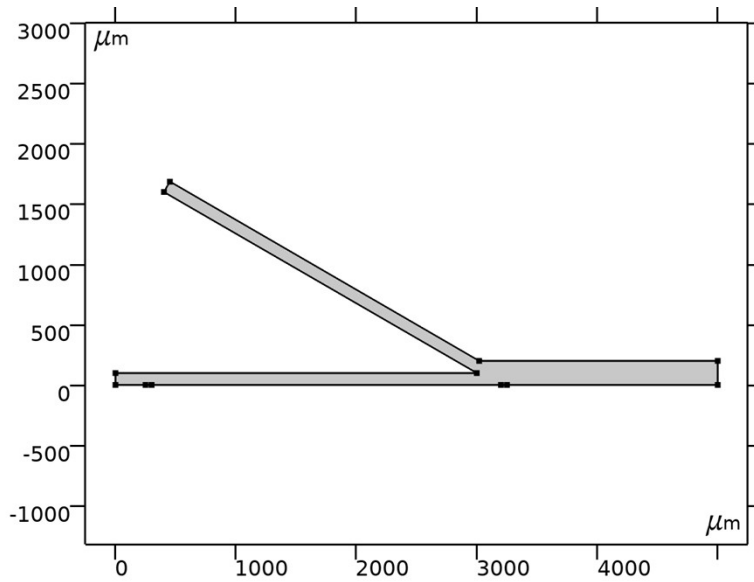
Weak expressions

Weak expression	Integration order	Integration frame	Selection
- (npe.nx*npe.u+npe.ny*npe.v+npe.nz*npe.w)*cH3O*test(cH3O)	4	Spatial	Boundaries 1, 6
- npe.z_cH3O*F_const*cH3O*(- npe.nx*(npe.um_cH3Ox*mean(d(npe.V,x))+npe.um_cH3Oxy*mean(d(npe.V,y)))+npe.um_cH3Oxz*mean(0))- npe.ny*(npe.um_cH3Ox*mean(d(npe.V,x))+npe.um_cH3Oyy*mean(d(npe.V,y)))+npe.um_cH3Oyz*mean(0))- npe.nz*(npe.um_cH3Ox*mean(d(npe.V,x))+npe.um_cH3Ozy*mean(d(npe.V,y)))+npe.um_cH3Ozz*mean(0))*test(cH3O)	4	Spatial	Boundaries 1, 6
- (npe.nx*npe.u+npe.ny*npe.v+npe.nz*npe.w)*cOH*test(cOH)	4	Spatial	Boundaries 1, 6
- npe.z_cOH*F_const*cOH*(- npe.nx*(npe.um_cOHxx*mean(d(npe.V,x))+npe.um_cOHxy*mean(d(npe.V,y)))+npe.um_cOHxz*mean(0))- npe.ny*(npe.um_cOHyx*mean(d(npe.V,x))+npe.	4	Spatial	Boundaries 1, 6

Weak expression	Integration order	Integration frame	Selection
um_cOHyy*mean(d(npe.V,y))+npe.um_cOHyz*mean(0))-npe.nz*(npe.um_cOHzx*mean(d(npe.V,x))+npe.um_cOHzy*mean(d(npe.V,y))+npe.um_cOHzz*mean(0)))*test(cOH)			
- (npe.nx*npe.u+npe.ny*npe.v+npe.nz*npe.w)*cBead*test(cBead)	4	Spatial	Boundaries 1, 6
- npe.z_cBead*F_const*cBead*(- npe.nx*(npe.um_cBeadx*mean(d(npe.V,x))+npe.um_cBeadxy*mean(d(npe.V,y))+npe.um_cBeadxz*mean(0))- npe.ny*(npe.um_cBeady*mean(d(npe.V,x))+npe.um_cBeadyy*mean(d(npe.V,y))+npe.um_cBeadyz*mean(0))- npe.nz*(npe.um_cBeadz*mean(d(npe.V,x))+npe.um_cBeadzy*mean(d(npe.V,y))+npe.um_cBeadzz*mean(0)))*test(cBead)	4	Spatial	Boundaries 1, 6
- (npe.nx*npe.u+npe.ny*npe.v+npe.nz*npe.w)*cK*test(cK)	4	Spatial	Boundaries 1, 6
-npe.z_cK*F_const*cK*(- npe.nx*(npe.um_cKxx*mean(d(npe.V,x))+npe.um_cKxy*mean(d(npe.V,y))+npe.um_cKxz*mean(0))- npe.ny*(npe.um_cKyx*mean(d(npe.V,x))+npe.um_cKyy*mean(d(npe.V,y))+npe.um_cKyz*mean(4	Spatial	Boundaries 1, 6

Weak expression	Integration order	Integration frame	Selection
$0)) - npe.nz*(npe.um_cKzx*mean(d(npe.V,x)) + npe.um_cKzy*mean(d(npe.V,y)) + npe.um_cKzz*mean(0)))*test(cK)$			

2.4.9 Electric Potential 1



Electric Potential 1

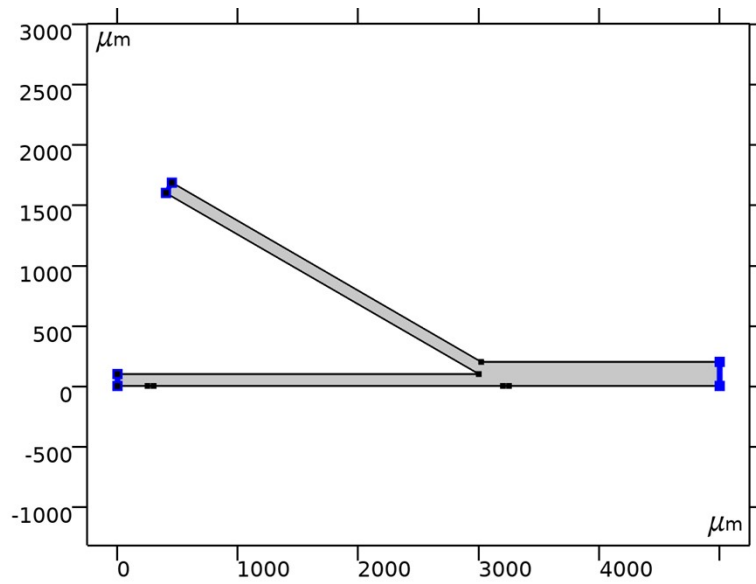
SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: No boundaries

EQUATIONS

$$V = V_0$$

2.4.10 Electric Potential 2



Electric Potential 2

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 1, 6, 12

EQUATIONS

$$V = V_0$$

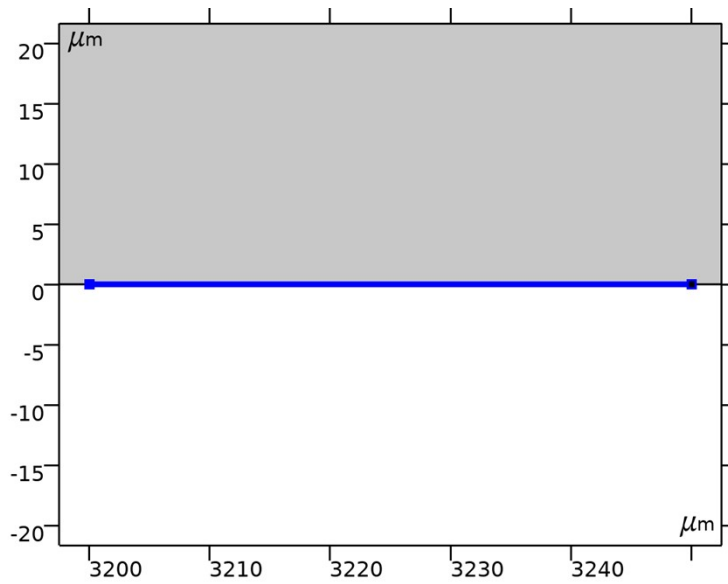
Variables

Name	Expression	Unit	Description	Selection
npe.V0	30	V	Electric potential	Boundaries 1, 6, 12

Constraints

Constraint	Constraint force	Shape function	Selection	Details
npe.V0-V	test(npe.V0-V)	Lagrange (Quadratic)	Boundaries 1, 6, 12	Elemental

2.4.11 Flux 1



Flux 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 10

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{J}_i = J_{0,i}$$

Convection

SETTINGS

Description	Value
Include	Off

Inward flux

SETTINGS

Description	Value
Species cH3O	Off
Species cOH	On
Species cBead	Off
Species cK	Off
Species cCl	Off

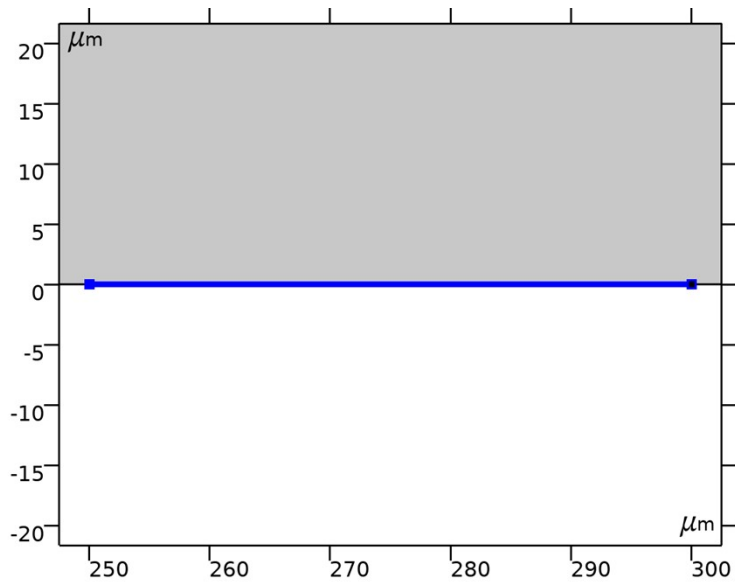
Variables

Name	Expression	Unit	Description	Selection
npe.cbf_cH3O	cH3O*(- npe.u*npe.nxmesh- npe.v*npe.nymesh- npe.w*npe.nzmesh)	mol/(m ² ·s)	Convective boundary flux	Boundary 10
npe.cbf_cOH	cOH*(- npe.u*npe.nxmesh- npe.v*npe.nymesh- npe.w*npe.nzmesh)	mol/(m ² ·s)	Convective boundary flux	Boundary 10
npe.cbf_cBead	cBead*(- npe.u*npe.nxmesh- npe.v*npe.nymesh- npe.w*npe.nzmesh)	mol/(m ² ·s)	Convective boundary flux	Boundary 10
npe.cbf_cK	cK*(- npe.u*npe.nxmesh- npe.v*npe.nymesh- npe.w*npe.nzmesh)	mol/(m ² ·s)	Convective boundary flux	Boundary 10

Weak expressions

Weak expression	Integration order	Integration frame	Selection
0	4	Spatial	Boundary 10
fssBPE*test(cOH)	4	Spatial	Boundary 10
0	4	Spatial	Boundary 10
0	4	Spatial	Boundary 10

2.4.12 Flux 2



Flux 2

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 4

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{j}_i = j_{o,i}$$

Convection

SETTINGS

Description	Value
Include	Off

Inward flux

SETTINGS

Description	Value
Species cH3O	On
Species cOH	Off
Species cBead	Off
Species cK	Off
Species cCl	Off

Description	Value
Inward flux	{fssBPE, 0, 0, 0, 0}

Variables

Name	Expression	Unit	Description	Selection
npe.cbf_cH3O	$cH3O*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundary 4
npe.cbf_cOH	$cOH*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundary 4
npe.cbf_cBead	$cBead*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundary 4
npe.cbf_cK	$cK*(-npe.u*npe.nxmesh-npe.v*npe.nymesh-npe.w*npe.nzmesh)$	mol/(m ² ·s)	Convective boundary flux	Boundary 4

Weak expressions

Weak expression	Integration order	Integration frame	Selection
fssBPE*test(cH3O)	4	Spatial	Boundary 4
0	4	Spatial	Boundary 4
0	4	Spatial	Boundary 4
0	4	Spatial	Boundary 4

2.4.13 Current Density 1



Current Density 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 10

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{i} = \dot{i}_0$$

Inward current density

SETTINGS

Description	Value
Inward current density	-jssBPE

Variables

Name	Expression	Unit	Description	Selection
npe.i0_cdens4	-jssBPE	A/m ²	Inward current density	Boundary 10

Weak expressions

Weak expression	Integration order	Integration frame	Selection
npe.i0_cdens4*test(V)	4	Spatial	Boundary 10

2.4.14 Current Density 2



Current Density 2

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 4

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{i} = i_0$$

Inward current density

SETTINGS

Description	Value
Inward current density	jssBPE

Variables

Name	Expression	Unit	Description	Selection
npe.i0_cdens5	jssBPE	A/m ²	Inward current density	Boundary 4

Weak expressions

Weak expression	Integration order	Integration frame	Selection
npe.i0_cdens5*test(V)	4	Spatial	Boundary 4

2.4.15 Reactions 1



Reactions 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: Domain 1

EQUATIONS

$$\nabla \cdot (\mathbf{J}_i + \mathbf{u}c_i) = R_i$$

$$\nabla \cdot \mathbf{i} = F \sum_i z_i R_i$$

Reaction rates

SETTINGS

Description	Value
Total rate expression	User defined
Total rate expression	kfhydro*cH2O0 - kbhydro*cH3O*cOH
Total rate expression	User defined
Total rate expression	kfhydro*cH2O0 - kbhydro*cH3O*cOH
Total rate expression	User defined
Total rate expression	0
Total rate expression	User defined
Total rate expression	0

Description	Value
Total rate expression	User defined
Total rate expression	0

Variables

Name	Expression	Unit	Description	Selection	Details
npe.R_cH3O	npe.reac1.R_cH3O	mol/(m ³ ·s)	Total rate expression	Domain 1	+ operation
npe.R_cOH	npe.reac1.R_cOH	mol/(m ³ ·s)	Total rate expression	Domain 1	+ operation
npe.R_cBead	npe.reac1.R_cBead	mol/(m ³ ·s)	Total rate expression	Domain 1	+ operation
npe.R_cK	npe.reac1.R_cK	mol/(m ³ ·s)	Total rate expression	Domain 1	+ operation
npe.R_cCl	npe.reac1.R_cCl	mol/(m ³ ·s)	Total rate expression	Domain 1	+ operation
npe.reac1.R_cH3O	model.input.R_cH3O	mol/(m ³ ·s)	Total rate expression	Domain 1	Meta
npe.reac1.R_cOH	model.input.R_cOH	mol/(m ³ ·s)	Total rate expression	Domain 1	Meta
npe.reac1.R_cBead	model.input.R_cBead	mol/(m ³ ·s)	Total rate expression	Domain 1	Meta
npe.reac1.R_cK	model.input.R_cK	mol/(m ³ ·s)	Total rate expression	Domain 1	Meta
npe.reac1.R_cCl	model.input.R_cCl	mol/(m ³ ·s)	Total rate expression	Domain 1	Meta

Weak expressions

Weak expression	Integration order	Integration frame	Selection
npe.reac1.R_cH3O*test(cH3O)	4	Spatial	Domain 1
F_const*npe.reac1.R_cH3O*npe.z_cH3O*test(V)	4	Spatial	Domain 1
npe.reac1.R_cOH*test(cOH)	4	Spatial	Domain 1
F_const*npe.reac1.R_cOH*npe.z_cOH*test(V)	4	Spatial	Domain 1
npe.reac1.R_cBead*test(cBead)	4	Spatial	Domain 1
F_const*npe.reac1.R_cBead*npe.z_cBead*test(V)	4	Spatial	Domain 1

Weak expression	Integration order	Integration frame	Selection
npe.reac1.R_cK*test(cK)	4	Spatial	Domain 1
F_const*npe.reac1.R_cK*npe.z_cK*test(V)	4	Spatial	Domain 1
F_const*npe.reac1.R_cCl*npe.z_cCl*test(V)	4	Spatial	Domain 1

2.5 LAMINAR FLOW

USED PRODUCTS

COMSOL Multiphysics
Chemical Reaction Engineering Module



Laminar Flow

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \mathbf{K}] + \mathbf{F}$$

$$\rho \nabla \cdot \mathbf{u} = 0$$

2.5.1 Interface settings

Discretization

SETTINGS

Description	Value
Discretization of fluids	P1 + P1

Physical model

SETTINGS

Description	Value
Neglect inertial term (Stokes flow)	Off
Compressibility	Incompressible flow
Enable porous media domains	Off
Reference temperature	User defined
Reference temperature	T
Reference pressure level	1[atm]

Turbulence

SETTINGS

Description	Value
Turbulence model type	None

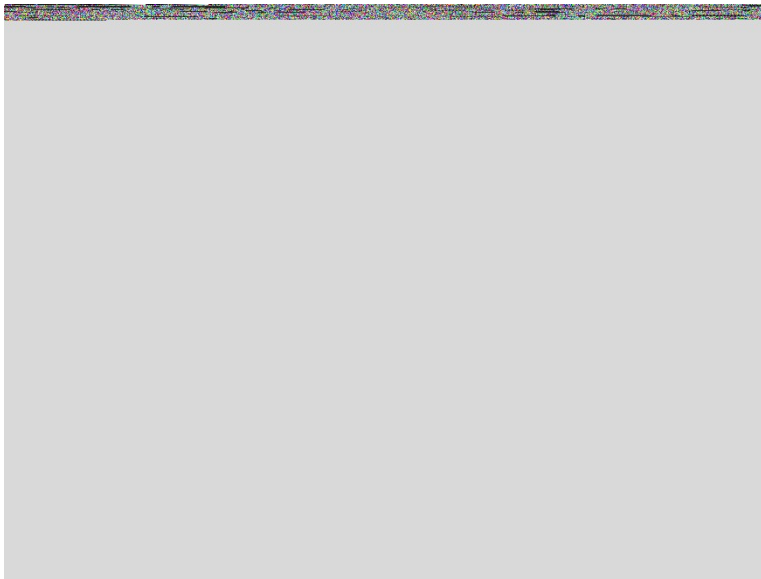
2.5.2 Variables

Name	Expression	Unit	Description	Selection	Details
spf.Tref	model.input.Tref	K	Reference temperature	Global	Meta
spf.dz	1	m	Thickness	Domain 1	
spf.pref	1[atm]	Pa	Reference pressure level	Domain 1	
spf.pA	p+spf.pref	Pa	Absolute pressure	Domain 1	
spf.hasWF	0		Help variable	Boundaries 1–12	
spf.dt_CFL	1/max(spf.maxOp(sqrt(ematic_s	s	Time step, CFL=1	Global	

Name	Expression	Unit	Description	Selection	Details
	patial(u-d(x,TIME),v-d(y,TIME))),eps)				
spf.usePseudoTimeStepping	0	1	Help variable	Global	+ operation
spf.localCFLvalue	1.3^min(niterCMP,9)+if(niterCMP>=25,9*1.3^min(-25+niterCMP,9),0)+if(niterCMP>=45,90*1.3^min(-45+niterCMP,9),0)		Local CFL number	Domain 1	
spf.locCFL	CFLCMP	1	Local CFL number	Domain 1	
spf.geometryLengthScale	4.216500000000001E-4	m	Geometry length scale	Domain 1	
spf.time_step_inv	max(sqrt(ematic_spatial(u,v)*2^gmg_level^2),spf.nu/spf.geometryLengthScale^2)	Hz	Inverse time step	Domain 1	
spf.tsti	nojac(spf.time_step_inv/spf.locCFL)	1/s	Help variable	Domain 1	
spf.nx	dnx	1	Normal vector, x component	Boundaries 1–12	
spf.ny	dny	1	Normal vector, y component	Boundaries 1–12	
spf.nz	0	1	Normal vector, z component	Boundaries 1–12	
spf.nxmesh	dnxmesh	1	Normal vector, x component	Boundaries 1–12	
spf.nymesh	dnymesh	1	Normal vector, y	Boundaries 1–12	

Name	Expression	Unit	Description	Selection	Details
			component		
spf.nzmesh	0	1	Normal vector, z component	Boundaries 1–12	

2.5.3 Fluid Properties 1



Fluid Properties 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \mathbf{K}] + \mathbf{F}$$

$$\rho \nabla \cdot \mathbf{u} = 0$$

$$\mathbf{K} = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

Fluid properties

SETTINGS

Description	Value
Density	From material
	Newtonian
Dynamic viscosity	From material

Model input

SETTINGS

Description	Value
Temperature	Common model input

USED PRODUCTS

COMSOL Multiphysics

PROPERTIES FROM MATERIAL

Property	Material	Property group
Density	Water	Basic
Dynamic viscosity	Water	Basic

Variables

Name	Expression	Unit	Description	Selection	Details
spf.mu	material.mu	Pa·s	Dynamic viscosity	Domain 1	Meta
spf.rho	subst(material.rho,spf.fp1.mininput_temperature,spf.Trho,spf.fp1.mininput_pressure,spf.prho)	kg/m ³	Density	Domain 1	Meta
spf.Trho	spf.Tref	K	Temperature for density evaluation	Domain 1	
spf.prho	spf.pref	Pa	Pressure for the evaluation of density	Domain 1	
spf.rhoref	subst(material.rho,spf.fp1.mininput_temperature,spf.Tref,spf.fp1.mininput_pressure,spf.pref)	kg/m ³	Reference density	Domain 1	Meta

Name	Expression	Unit	Description	Selection	Details
spf.mumat	material.mu	Pa·s	Dynamic viscosity	Domain 1	Meta
spf.srijxx	ux	1/s	Strain rate tensor, xx component	Domain 1	
spf.srijyx	0.5*(vx+uy)	1/s	Strain rate tensor, yx component	Domain 1	
spf.srijzx	0	1/s	Strain rate tensor, zx component	Domain 1	
spf.srijxy	0.5*(uy+vx)	1/s	Strain rate tensor, xy component	Domain 1	
spf.srijyy	vy	1/s	Strain rate tensor, yy component	Domain 1	
spf.srijzy	0	1/s	Strain rate tensor, zy component	Domain 1	
spf.srijxz	0	1/s	Strain rate tensor, xz component	Domain 1	
spf.srijyz	0	1/s	Strain rate tensor, yz component	Domain 1	
spf.srijzz	0	1/s	Strain rate tensor, zz component	Domain 1	
spf.rrijxx	0	1/s	Rotation rate tensor, xx component	Domain 1	
spf.rrijyx	0.5*(vx-uy)	1/s	Rotation rate tensor, yx component	Domain 1	
spf.rrijzx	0	1/s	Rotation rate tensor, zx component	Domain 1	
spf.rrijxy	0.5*(uy-vx)	1/s	Rotation rate tensor,	Domain 1	

Name	Expression	Unit	Description	Selection	Details
			xy component		
spf.rrijyy	0	1/s	Rotation rate tensor, yy component	Domain 1	
spf.rrijzy	0	1/s	Rotation rate tensor, zy component	Domain 1	
spf.rrijxz	0	1/s	Rotation rate tensor, xz component	Domain 1	
spf.rrijyz	0	1/s	Rotation rate tensor, yz component	Domain 1	
spf.rrijzz	0	1/s	Rotation rate tensor, zz component	Domain 1	
spf.sr	$\sqrt{2 \cdot \text{spf.srijxx}^2 + 2 \cdot \text{spf.srijxy}^2 + 2 \cdot \text{spf.srijxz}^2 + 2 \cdot \text{spf.srijyx}^2 + 2 \cdot \text{spf.srijyy}^2 + 2 \cdot \text{spf.srijyz}^2 + 2 \cdot \text{spf.srijzx}^2 + 2 \cdot \text{spf.srijzy}^2 + 2 \cdot \text{spf.srijzz}^2 + \epsilon}$	1/s	Shear rate	Domain 1	
spf.rr	$\sqrt{2 \cdot \text{spf.rrijxx}^2 + 2 \cdot \text{spf.rrijxy}^2 + 2 \cdot \text{spf.rrijxz}^2 + 2 \cdot \text{spf.rrijyx}^2 + 2 \cdot \text{spf.rrijyy}^2 + 2 \cdot \text{spf.rrijyz}^2 + 2 \cdot \text{spf.rrijzx}^2 + 2 \cdot \text{spf.rrijzy}^2 + 2 \cdot \text{spf.rrijzz}^2 + \epsilon}$	1/s	Rotation rate	Domain 1	

Name	Expression	Unit	Description	Selection	Details
	ps)				
spf.divu	$u_x + v_y$	1/s	Divergence of velocity field	Domain 1	
spf.Fx	0	N/m ³	Volume force, x component	Domain 1	+ operation
spf.Fy	0	N/m ³	Volume force, y component	Domain 1	+ operation
spf.Fz	0	N/m ³	Volume force, z component	Domain 1	+ operation
spf.U	$\sqrt{u^2 + v^2}$	m/s	Velocity magnitude	Domain 1	
spf.vorticityx	0	1/s	Vorticity field, x component	Domain 1	
spf.vorticityy	0	1/s	Vorticity field, y component	Domain 1	
spf.vorticityz	$v_x - u_y$	1/s	Vorticity field, z component	Domain 1	
spf.vort_magn	$\sqrt{\text{spf.vorticityx}^2 + \text{spf.vorticityy}^2 + \text{spf.vorticityz}^2}$	1/s	Vorticity magnitude	Domain 1	
spf.cellRe	$0.25 * \text{spf.rho} * \sqrt{\text{emetric_spatial}(u - d(x, \text{TIME}), v - d(y, \text{TIME})) / \text{emetric2_spatial}} / \text{spf.mu}$	1	Cell Reynolds number	Domain 1	
spf.nu	$\text{spf.mu} / \text{spf.rho}$	m ² /s	Kinematic viscosity	Domain 1	
spf.betaT	0	1/Pa	Isothermal compressibility coefficient	Domain 1	
spf.Qm	0	kg/(m ³ .s)	Source term	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
					on
spf.Fgtotx	0	N/m ³	Gravity force, x component	Domain 1	+ operation
spf.Fgoty	0	N/m ³	Gravity force, y component	Domain 1	+ operation
spf.Fgotz	0	N/m ³	Gravity force, z component	Domain 1	+ operation
spf.mu_eff	spf.mu+spf.muT	Pa·s	Dynamic viscosity	Domain 1	
spf.muT	0	Pa·s	Turbulent dynamic viscosity	Domain 1	
spf.T_stressx	spf.K_stressx - p*spf.nxmesh	N/m ²	Total stress, x component	Boundaries 1–12	+ operation
spf.T_stressy	spf.K_stressy - p*spf.nymesh	N/m ²	Total stress, y component	Boundaries 1–12	+ operation
spf.T_stressz	spf.K_stressz - p*spf.nzmesh	N/m ²	Total stress, z component	Boundaries 1–12	+ operation
spf.K_stressx	spf.mu_eff*(2*ux*spf.nxmesh+(uy+vx)*spf.nymesh)	N/m ²	Viscous stress, x component	Boundaries 1–12	+ operation
spf.K_stressy	spf.mu_eff*((vx+uy)*spf.nxmesh+2*vy*spf.nymesh)	N/m ²	Viscous stress, y component	Boundaries 1–12	+ operation
spf.K_stressz	0	N/m ²	Viscous stress, z component	Boundaries 1–12	+ operation
spf.K_stress_tensorxx	2*spf.mu_eff*ux	N/m ²	Viscous stress tensor, xx component	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
spf.K_stress_tensoryx	spf.mu_eff*(v x+uy)	N/m ²	Viscous stress tensor, yx component	Domain 1	+ operati on
spf.K_stress_tensorzx	0	N/m ²	Viscous stress tensor, zx component	Domain 1	+ operati on
spf.K_stress_tensoryy	spf.mu_eff*(uy+vx)	N/m ²	Viscous stress tensor, xy component	Domain 1	+ operati on
spf.K_stress_tensoryy	2*spf.mu_eff *vy	N/m ²	Viscous stress tensor, yy component	Domain 1	+ operati on
spf.K_stress_tensoryz	0	N/m ²	Viscous stress tensor, zy component	Domain 1	+ operati on
spf.K_stress_tensorzx	0	N/m ²	Viscous stress tensor, xz component	Domain 1	+ operati on
spf.K_stress_tensoryz	0	N/m ²	Viscous stress tensor, yz component	Domain 1	+ operati on
spf.K_stress_tensorz	0	N/m ²	Viscous stress tensor, zz component	Domain 1	+ operati on
spf.K_stress_tensor_testx x	2*spf.mu_eff *test(ux)	N/m ²	Viscous stress tensor test, xx component	Domain 1	+ operati on
spf.K_stress_tensor_testy x	spf.mu_eff*(t est(vx)+test(uy))	N/m ²	Viscous stress tensor test, yx component	Domain 1	+ operati on
spf.K_stress_tensor_testz	0	N/m ²	Viscous stress	Domain 1	+ operati on

Name	Expression	Unit	Description	Selection	Details
x			tensor test, zx component		
spf.K_stress_tensor_testxy	spf.mu_eff*(test(uy)+test(vx))	N/m ²	Viscous stress tensor test, xy component	Domain 1	+ operation
spf.K_stress_tensor_testyy	2*spf.mu_eff*test(vy)	N/m ²	Viscous stress tensor test, yy component	Domain 1	+ operation
spf.K_stress_tensor_testzy	0	N/m ²	Viscous stress tensor test, zy component	Domain 1	+ operation
spf.K_stress_tensor_testxz	0	N/m ²	Viscous stress tensor test, xz component	Domain 1	+ operation
spf.K_stress_tensor_testyz	0	N/m ²	Viscous stress tensor test, yz component	Domain 1	+ operation
spf.K_stress_tensor_testzz	0	N/m ²	Viscous stress tensor test, zz component	Domain 1	+ operation
spf.upwind_helpx	u-d(x,TIME)	m/s	Upwind term, x component	Domain 1	+ operation
spf.upwind_helpy	v-d(y,TIME)	m/s	Upwind term, y component	Domain 1	+ operation
spf.upwind_helpz	0	m/s	Upwind term, z component	Domain 1	+ operation
spf.tau_vdxx	2*spf.mu*spf.srijxx	Pa	Viscous stress	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
			tensor, xx component		
spf.tau_vdyx	$2*spf.mu*spf.srijyx$	Pa	Viscous stress tensor, yx component	Domain 1	+ operation
spf.tau_vdzx	$2*spf.mu*spf.srijzx$	Pa	Viscous stress tensor, zx component	Domain 1	+ operation
spf.tau_vdxy	$2*spf.mu*spf.srijxy$	Pa	Viscous stress tensor, xy component	Domain 1	+ operation
spf.tau_vdyy	$2*spf.mu*spf.srijyy$	Pa	Viscous stress tensor, yy component	Domain 1	+ operation
spf.tau_vdzy	$2*spf.mu*spf.srijzy$	Pa	Viscous stress tensor, zy component	Domain 1	+ operation
spf.tau_vdxz	$2*spf.mu*spf.srijxz$	Pa	Viscous stress tensor, xz component	Domain 1	+ operation
spf.tau_vdyz	$2*spf.mu*spf.srijyz$	Pa	Viscous stress tensor, yz component	Domain 1	+ operation
spf.tau_vdzz	$2*spf.mu*spf.srijzz$	Pa	Viscous stress tensor, zz component	Domain 1	+ operation
spf.Qvd	$spf.tau_vdxx*ux+spf.tau_vdxy*uy+spf.tau_vdyy*vy+spf.tau_vdzy*vz$	W/m ³	Viscous dissipation	Domain 1	+ operation
spf.epsilon_p	1	1	Porosity	Domain 1	
spf.Fst_tensorrxx	0	N/m ²	Surface tension	Domain 1	+ operation

Name	Expression	Unit	Description	Selection	Details
			force, xx component		
spf.Fst_tensoryx	0	N/m ²	Surface tension force, yx component	Domain 1	+ operation
spf.Fst_tensorzx	0	N/m ²	Surface tension force, zx component	Domain 1	+ operation
spf.Fst_tensoryy	0	N/m ²	Surface tension force, yy component	Domain 1	+ operation
spf.Fst_tensoryz	0	N/m ²	Surface tension force, zy component	Domain 1	+ operation
spf.Fst_tensorz	0	N/m ²	Surface tension force, xz component	Domain 1	+ operation
spf.Fst_tensoryz	0	N/m ²	Surface tension force, yz component	Domain 1	+ operation
spf.Fst_tensorz	0	N/m ²	Surface tension force, zz component	Domain 1	+ operation
spf.continuityEquation	spf.rho*spf.divu	kg/(m ³ .s)	Continuity equation	Domain 1	
spf.contCoeff	spf.rho	kg/m ³	Help variable	Domain 1	
spf.res_u	px+spf.rho*u*ux+spf.rho*v*uy-(d(2*ux,x)+d(uy+vx,y))*spf.mu-spf.Fx	N/m ³	Equation residual	Domain 1	

Name	Expression	Unit	Description	Selection	Details
spf.res_v	spf.rho*u*vx +py+spf.rho *v*vy- (d(vx+uy,x)+ d(2*vy,y))*sp f.mu-spf.Fy	N/m ³	Equation residual	Domain 1	
spf.res_p	spf.rho*spf.d ivu	kg/(m ³ .s)	Pressure equation residual	Domain 1	

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
u	Lagrange (Linear)	m/s	Velocity field, x component	Spatial	Domain 1
v	Lagrange (Linear)	m/s	Velocity field, y component	Spatial	Domain 1
p	Lagrange (Linear)	Pa	Pressure	Spatial	Domain 1

Weak expressions

Weak expression	Integration order	Integration frame	Selection
(p- spf.K_stress_tensorxx)*test(ux) - spf.K_stress_tensorxy*test(uy) - spf.K_stress_tensoryx*test(vx) +(p- spf.K_stress_tensoryy)*test(vy)	2	Spatial	Domain 1
spf.Fx*test(u)+spf.Fy*test(v)	2	Spatial	Domain 1
spf.rho*(- (d(u,x)*u+d(u,y)*v)*test(u)- (d(v,x)*u+d(v,y)*v)*test(v))	2	Spatial	Domain 1
- spf.continuityEquation*test(p)	2	Spatial	Domain 1
spf.streamlinens	2	Spatial	Domain 1
(spf.usePseudoTimeStepping >0)*spf.rho*spf.tsti*(-(u- nojac(u))*test(u)-(v- nojac(v))*test(v))	2	Spatial	Domain 1

2.5.4 Initial Values 1



Initial Values 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

Initial values

SETTINGS

Description	Value
Velocity field, x component	0
Velocity field, y component	0
Velocity field, z component	0
Pressure	0

Coordinate system selection

SETTINGS

Description	Value
Coordinate system	Global coordinate system

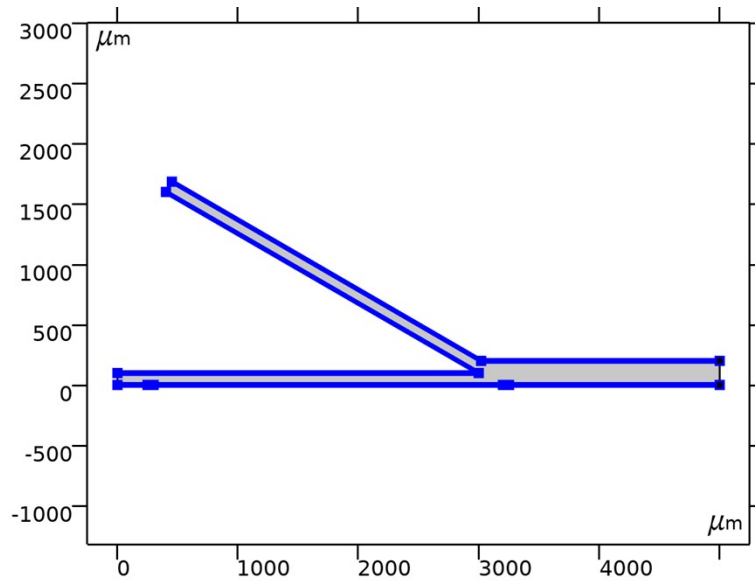
USED PRODUCTS

COMSOL Multiphysics

Variables

Name	Expression	Unit	Description	Selection
spf.u_initx	0	m/s	Velocity field, x component	Domain 1
spf.u_inity	0	m/s	Velocity field, y component	Domain 1
spf.u_initz	0	m/s	Velocity field, z component	Domain 1
spf.p_init	0	Pa	Pressure	Domain 1

2.5.5 Wall 1



Wall 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

EQUATIONS

$$\mathbf{u} = \mu_{eo} \mathbf{E}_t$$

$$\mu_{eo} = -\frac{\epsilon_r \epsilon_0 \zeta}{\mu}, \quad \mathbf{E}_t = \mathbf{E} - (\mathbf{E} \cdot \mathbf{n})\mathbf{n}$$

Boundary condition

SETTINGS

Description	Value
Wall condition	Electroosmotic velocity
Electric field	User defined
Electric field	{-npe.gradVx, -npe.gradVy, 0}

Description	Value
Electroosmotic mobility	Built - in expression
Zeta potential	-(0.080)[V]
Relative permittivity	80

Wall movement

SETTINGS

Description	Value
Translational velocity	Automatic from frame
Sliding wall	Off

Variables

Name	Expression	Unit	Description	Selection	Details
spf.KStressn_avx	spf.K_stress_tens orxx*spf.nxmesh +spf.K_stress_tens orxy*spf.nymesh +spf.K_stress_tens orz*spf.nzmesh	N/m ²	Average viscous stress, x component	Boundaries 2-5, 7-11	
spf.KStressn_avy	spf.K_stress_tens oryx*spf.nxmesh +spf.K_stress_tens oryy*spf.nymesh +spf.K_stress_tens oryz*spf.nzmesh	N/m ²	Average viscous stress, y component	Boundaries 2-5, 7-11	
spf.KStressn_avz	spf.K_stress_tens orz*spf.nxmesh +spf.K_stress_tens orzy*spf.nymesh +spf.K_stress_tens orz*spf.nzmesh	N/m ²	Average viscous stress, z component	Boundaries 2-5, 7-11	
spf.KStressTestn_avx	spf.K_stress_tens or_testxx*spf.nx mesh+spf.K_stress_tens or_testxy*spf.nymesh+spf.K_stress_tens or_testxz*spf.nz mesh	N/m ²	Average viscous stress, x component	Boundaries 2-5, 7-11	
spf.KStressTestn_ave	spf.K_stress_tens	N/m ²	Average	Boundaries	

Name	Expression	Unit	Description	Selection	Details
stn_avy	or_testyx*spf.nx mesh+spf.K_stre ss_tensor_testy *spf.nymesh+sp f.K_stress_tensor _testyz*spf.nzm esh		viscous stress, y component	s 2-5, 7- 11	
spf.KStressTe stn_avz	spf.K_stress_tens or_testzx*spf.nx mesh+spf.K_stre ss_tensor_testzy *spf.nymesh+sp f.K_stress_tensor _testzz*spf.nzm esh	N/m ²	Average viscous stress, z component	Boundarie s 2-5, 7- 11	
spf.ujumpx	spf.u_herex- spf.u_therex	m/s	Velocity jump, x component	Boundarie s 2-5, 7- 11	
spf.ujumpy	spf.u_herey- spf.u_therey	m/s	Velocity jump, y component	Boundarie s 2-5, 7- 11	
spf.ujumpz	spf.u_herez- spf.u_therez	m/s	Velocity jump, z component	Boundarie s 2-5, 7- 11	
spf.meshVol	meshvol_spatial	m		Boundarie s 2-5, 7- 11	
spf.meshVoll nt	down(meshvol_s patial)	m ²	Volume of interior mesh element	Boundarie s 2-5, 7- 11	
spf.c_here	24*nojac(down(spf.mu))*spf.me shVol/spf.mesh Vollnt	1	Intermediate variable	Boundarie s 2-5, 7- 11	
spf.sigma_dg _ns	4*spf.c_here	Pa·s/m		Boundarie s 2-5, 7- 11	
spf.rhoFace	down(spff.rho)	kg/m ³	Density face value	Boundarie s 2-5, 7- 11	
spf.umxTnFa ce	(spf.upwind_hel px*spf.nxmesh+ spf.upwind_help	m/s	Relative velocity on face	Boundarie s 2-5, 7- 11	

Name	Expression	Unit	Description	Selection	Details
	y*spf.nymesh+spf.upwind_helpz*spf.nzmesh<0)*(spf.upwind_helpx*spf.nymesh+spf.upwind_helpy*spf.nymesh+spf.upwind_helpz*spf.nzmesh)				
spf.upwind_ns	spf.rhoFace*spf.umxTnFace*(spf.ujumpx*test(spfu_herex)+spf.ujumpy*test(spfu_herey)+spf.ujumpz*test(spfu_herez))	Pa	Upwind term	Boundaries 2-5, 7-11	
spf.zeta	(-0.08)[V]	V	Zeta potential	Boundaries 2-5, 7-11	
spf.epsilonr	80	1	Relative permittivity	Boundaries 2-5, 7-11	
spf.ubndx	spf.ueox+spf.utr _x +spf.usx	m/s	Velocity at boundary, x component	Boundaries 2-5, 7-11	
spf.ubndy	spf.ueoy+spf.utr _y +spf.usy	m/s	Velocity at boundary, y component	Boundaries 2-5, 7-11	
spf.ubndz	spf.ueoz+spf.utr _z +spf.usz	m/s	Velocity at boundary, z component	Boundaries 2-5, 7-11	
spf.usx	0	m/s	Velocity of sliding wall, x component	Boundaries 2-5, 7-11	
spf.usy	0	m/s	Velocity of sliding wall, y component	Boundaries 2-5, 7-11	
spf.usz	0	m/s	Velocity of sliding wall, z component	Boundaries 2-5, 7-11	

Name	Expression	Unit	Description	Selection	Details
spf.utrx	0	m/s	Velocity of moving wall, x component	Boundaries 2-5, 7-11	
spf.utry	0	m/s	Velocity of moving wall, y component	Boundaries 2-5, 7-11	
spf.utrz	0	m/s	Velocity of moving wall, z component	Boundaries 2-5, 7-11	
spf.uLeakage _x	0	m/s	Leakage velocity, x component	Boundaries 2-5, 7-11	+ operation
spf.uLeakage _y	0	m/s	Leakage velocity, y component	Boundaries 2-5, 7-11	+ operation
spf.uLeakage _z	0	m/s	Leakage velocity, z component	Boundaries 2-5, 7-11	+ operation
spf.Ex	model.input.E1	V/m	Electric field, x component	Boundaries 2-5, 7-11	Meta
spf.Ey	model.input.E2	V/m	Electric field, y component	Boundaries 2-5, 7-11	Meta
spf.Ez	model.input.E3	V/m	Electric field, z component	Boundaries 2-5, 7-11	Meta
spf.mueo	- spf.zeta*epsilon ₀ _const*spf.epsilonr/spf.mu	m ² /(V·s)	Electroosmotic mobility	Boundaries 2-5, 7-11	
spf.ueox	(spf.Ex-spfnxmesh*(spf.nxmesh*spf.Ex+spf.nymesh*spf.Ey+spf.nzmesh*spf.Ez))*spf.mueo	m/s	Electroosmotic velocity, x component	Boundaries 2-5, 7-11	
spf.ueoy	(spf.Ey-spfnymesh*(spf.nxmesh*spf.Ex+	m/s	Electroosmotic velocity, y	Boundaries 2-5, 7-	

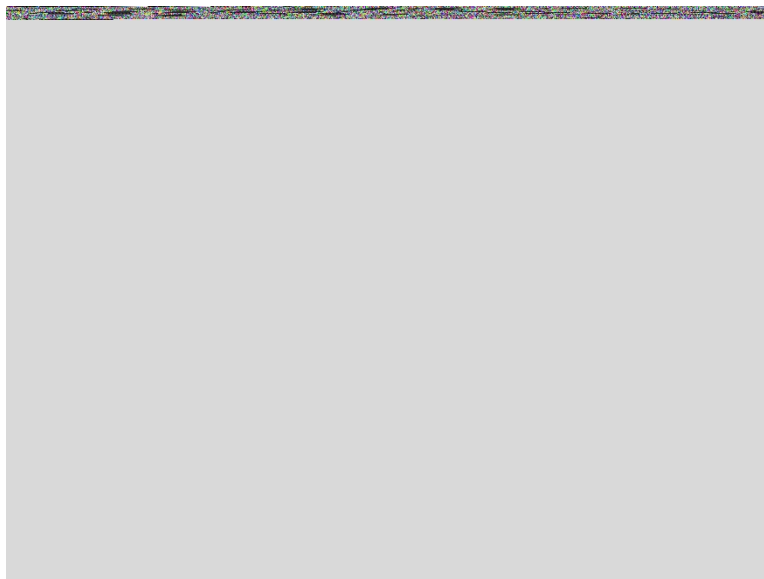
Name	Expression	Unit	Description	Selection	Details
	spf.nymesh*spf.Ey+spf.nzmesh*spf.Ez))*spf.mu_eo		component	11	
spf.ueoz	(spf.Ez-spfnzmesh*(spf.nxmesh*spf.Ex+spf.nymesh*spf.Ey+spf.nzmesh*spf.Ez))*spf.mu_eo	m/s	Electroosmotic velocity, z component	Boundaries 2-5, 7-11	
spf.u_herex	u	m/s	Intermediate variable, x component	Boundaries 2-5, 7-11	
spf.u_herey	v	m/s	Intermediate variable, y component	Boundaries 2-5, 7-11	
spf.u_herez	0	m/s	Intermediate variable, z component	Boundaries 2-5, 7-11	
spf.u_therex	spf.ubndx+spf.uLeakagex	m/s	Intermediate variable, x component	Boundaries 2-5, 7-11	
spf.u_therey	spf.ubndy+spf.uLeakagey	m/s	Intermediate variable, y component	Boundaries 2-5, 7-11	
spf.u_therez	spf.ubndz+spf.uLeakagez	m/s	Intermediate variable, z component	Boundaries 2-5, 7-11	
spf.contCoeffFace	down(spfc.contCoeff)	kg/m ³	Help variable	Boundaries 2-5, 7-11	
spf.upwindCont	spf.contCoeffFace*(spf.ujumpx*spf.nxmesh+spf.ujumpy*spf.nymesh+spf.ujumpz*spf.nzmesh)*test(p)	kg ² /(m ³ .s ³)	Upwind term for continuity equation	Boundaries 2-5, 7-11	
spf.pFace	p	Pa	Pressure face value	Boundaries 2-5, 7-11	
spf.consFlux	spf.pFace*(-test(spfc.u_herex))	W/m ²	Conservative flux	Boundaries 2-5, 7-11	+ operation

Name	Expression	Unit	Description	Selection	Details
	*spf.nxmesh-test(spf.u_herey) *spf.nymesh-test(spf.u_herez) *spf.nzmesh			11	

Weak expressions

Weak expression	Integration order	Integration frame	Selection
spf.KStressn_avx*test(spf.u_herex)+spf.KStressn_avy*test(spf.u_herey)+spf.KStressn_avz*test(spf.u_herez)+spf.KStressTestn_avx*spf.ujumpx+spf.KStressTestn_avy*spf.ujumpy+spf.KStressTestn_avz*spf.ujumpz- spf.sigma_dg_ns*spf.ujumpx*test(spf.u_herex)- spf.sigma_dg_ns*spf.ujumpy*test(spf.u_herey)- spf.sigma_dg_ns*spf.ujumpz*test(spf.u_herez)+spf.upwind_ns+spf.upwindCont+spf.con sFlux	2	Spatial	Boundaries 2-5, 7-11

2.5.6 Inlet 1



Inlet 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 12

EQUATIONS

$$\mathbf{n}^T[-p\mathbf{I} + \mathbf{K}]\mathbf{n} = -\hat{p}_0$$

$$\hat{p}_0 \geq p_0, \quad \mathbf{u} \cdot \mathbf{t} = 0$$

Boundary condition

SETTINGS

Description	Value
Boundary condition	Pressure

Pressure conditions

SETTINGS

Description	Value
Pressure	0
Suppress backflow	On
Flow direction	Normal flow

USED PRODUCTS

COMSOL Multiphysics

Variables

Name	Expression	Unit	Description	Selection
spf.KStressn_avx	spf.K_stress_tenso rxx*spf.nxmesh+s pf.K_stress_tensor xy*spf.nymesh+sp f.K_stress_tensorxz *spf.nzmesh	N/m ²	Average viscous stress, x component	Boundary 12
spf.KStressn_avy	spf.K_stress_tenso ryx*spf.nxmesh+s pf.K_stress_tensor yy*spf.nymesh+sp f.K_stress_tensoryz *spf.nzmesh	N/m ²	Average viscous stress, y component	Boundary 12
spf.KStressn_avz	spf.K_stress_tenso rzx*spf.nxmesh+s	N/m ²	Average viscous	Boundary 12

Name	Expression	Unit	Description	Selection
	pf.K_stress_tensorzy*spf.nymesh+spf.K_stress_tensorzz*spf.nzmesh		stress, z component	
spf.KStressTestn_avx	spf.K_stress_tensor_testxx*spf.nxmesh+spf.K_stress_tensor_testxy*spf.nymesh+spf.K_stress_tensor_testxz*spf.nzmesh	N/m ²	Average viscous stress, x component	Boundary 12
spf.KStressTestn_avy	spf.K_stress_tensor_testyx*spf.nxmesh+spf.K_stress_tensor_testyy*spf.nymesh+spf.K_stress_tensor_testyz*spf.nzmesh	N/m ²	Average viscous stress, y component	Boundary 12
spf.KStressTestn_avz	spf.K_stress_tensor_testzx*spf.nxmesh+spf.K_stress_tensor_testzy*spf.nymesh+spf.K_stress_tensor_testzz*spf.nzmesh	N/m ²	Average viscous stress, z component	Boundary 12
spf.ujumpx	spf.ut_herex-spf.ut_therex	m/s	Velocity jump, x component	Boundary 12
spf.ujumpy	spf.ut_herey-spf.ut_therey	m/s	Velocity jump, y component	Boundary 12
spf.ujumpz	spf.ut_herez-spf.ut_therez	m/s	Velocity jump, z component	Boundary 12
spf.meshVol	meshvol_spatial	m		Boundary 12
spf.meshVolInt	down(meshvol_spatial)	m ²	Volume of interior mesh element	Boundary 12
spf.c_here	96/spf.epsilon_p	Pa-s/m	Intermediate variable	Boundary 12
spf.sigma_dg_ns	4*spf.ct_here	Pa-s/m		Boundary 12

Name	Expression	Unit	Description	Selection
spf.rhoFace	down(spf.rho)	kg/m ³	Density face value	Boundary 12
spf.umxTnFace	spf.upwind_helpx* spf.nxmesh+spf.u pwind_helpy*spf.n ymesh+spf.upwin d_helpz*spf.nzmes h	m/s	Relative velocity on face	Boundary 12
spf.upwind_ns	spf.backflowPenaltyConv*spf.uNormal	W/m ²	Upwind term	Boundary 12
spf.p0	0	Pa	Pressure	Boundary 12
spf.f0	spf.p0+spf.uNormal*(spf.backflowPenaltyDiff+spf.backflowPenaltyConv)*(spf.uNormal>0)	N/m ²	Normal stress	Boundary 12
spf.un_here	u*nojac(spf.nxmesh)+v*nojac(spf.ny mesh)	m/s	Intermediate variable	Boundary 12
spf.ut_herex	u- spf.un_here*nojac(spf.nxmesh)	m/s	Intermediate variable, x component	Boundary 12
spf.ut_herey	v- spf.un_here*nojac(spf.nymesh)	m/s	Intermediate variable, y component	Boundary 12
spf.ut_herez	- spf.un_here*nojac(spf.nzmesh)	m/s	Intermediate variable, z component	Boundary 12
spf.un_there	0	m/s	Intermediate variable	Boundary 12
spf.ut_therex	- spf.un_there*nojac(spf.nxmesh)	m/s	Intermediate variable, x component	Boundary 12
spf.ut_therey	- spf.un_there*nojac(spf.nymesh)	m/s	Intermediate variable, y component	Boundary 12
spf.ut_therez	- spf.un_there*nojac(spf.nzmesh)	m/s	Intermediate variable, z component	Boundary 12
spf.ct_here	24*nojac(down((spf.mu+spf.muT)/s	Pa-s/m	Intermediate variable	Boundary 12

Name	Expression	Unit	Description	Selection
	$pf.\epsilon_p) * spf.meshVol / spf.meshVollnt$			
spf.inl1.Te0xx	0		Viscoelastic stress tensor, xx component	Boundary 12
spf.inl1.Te0xy	0		Viscoelastic stress tensor, xy component	Boundary 12
spf.inl1.Te0xz	0		Viscoelastic stress tensor, xz component	Boundary 12
spf.inl1.Te0yy	0		Viscoelastic stress tensor, yy component	Boundary 12
spf.inl1.Te0yz	0		Viscoelastic stress tensor, yz component	Boundary 12
spf.inl1.Te0zz	0		Viscoelastic stress tensor, zz component	Boundary 12
spf.uNormal	$u * nojac(spfxmesh) + v * nojac(spfy mesh)$	m/s	Normal velocity	Boundary 12
spf.backflowPenaltyDiff	$spf.c_here * \min(\text{down}(spf.\mu) + spf.\mu T) * spf.meshVol / spf.meshVollnt, \text{down}(spf.\rho) * \text{abs}(spf.uNormal) / \text{down}(spf.\epsilon_p))$	$kg^2 / (m^4 \cdot s^2)$	Backflow penalty parameter, diffusive contribution	Boundary 12
spf.backflowPenaltyConv	$spf.\rho_{Face} * spf.u_{mxTnFace} / spf.\epsilon_p^2$	$kg / (m^2 \cdot s)$	Backflow penalty parameter, convective contribution	Boundary 12

Weak expressions

Weak expression	Integration order	Integration frame	Selection
- spf.f0*(test(u)*spf.nxmesh+test(v)*spf.nymesh)	2	Spatial	Boundary 12
spf.KStressn_avx*test(spf.ut_herex)+spf.KStressn_avy*test(spf.ut_herey)+spf.KStressn_avz*test(spf.ut_herez)+spf.KStressTestn_avx*spf.ujumpx+spf.KStressTestn_avy*spf.ujumpy+spf.KStressTestn_avz*spf.ujumpz- spf.sigma_dg_ns*spf.ujumpx*test(spf.ut_herex)- spf.sigma_dg_ns*spf.ujumpy*test(spf.ut_herey)- spf.sigma_dg_ns*spf.ujumpz*test(spf.ut_herez)	2	Spatial	Boundary 12

2.5.7 Outlet 1



Outlet 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 1, 6

EQUATIONS

$$\mathbf{n}^T[-p\mathbf{I} + \mathbf{K}]\mathbf{n} = -p_0$$

$$\mathbf{u} \cdot \mathbf{t} = 0$$

Boundary condition

SETTINGS

Description	Value
Boundary condition	Pressure

Pressure conditions

SETTINGS

Description	Value
Pressure	0
Normal flow	On
Suppress backflow	Off

USED PRODUCTS

COMSOL Multiphysics

Variables

Name	Expression	Unit	Description	Selection
spf.KStressn_avx	spf.K_stress_tensorxx* spf.nxmesh+spf.K_stre ss_tensorxy*spf.nymes h+spf.K_stress_tensorex z*spf.nzmesh	N/m ²	Average viscous stress, x component	Boundarie s 1, 6
spf.KStressn_avy	spf.K_stress_tensoryx* spf.nxmesh+spf.K_stre ss_tensoryy*spf.nymes h+spf.K_stress_tensory z*spf.nzmesh	N/m ²	Average viscous stress, y component	Boundarie s 1, 6
spf.KStressn_avz	spf.K_stress_tensorzx* spf.nxmesh+spf.K_stre ss_tensorzy*spf.nymes h+spf.K_stress_tensorz z*spf.nzmesh	N/m ²	Average viscous stress, z component	Boundarie s 1, 6
spf.KStressTestn_avx	spf.K_stress_tensor_tes txx*spf.nxmesh+spf.K_ stress_tensor_testxy*s pf.nymesh+spf.K_stres s_tensor_testxz*spf.nz	N/m ²	Average viscous stress, x component	Boundarie s 1, 6

Name	Expression	Unit	Description	Selection
	mesh			
spf.KStressTestn_a vy	spf.K_stress_tensor_tes tyx*spf.nxmesh+spf.K_ stress_tensor_testyy*s pf.nymesh+spf.K_stres s_tensor_testyz*spf.nz mesh	N/m ²	Average viscous stress, y component	Boundarie s 1, 6
spf.KStressTestn_a vz	spf.K_stress_tensor_tes tzx*spf.nxmesh+spf.K_ stress_tensor_testzy*s pf.nymesh+spf.K_stres s_tensor_testzz*spf.nz mesh	N/m ²	Average viscous stress, z component	Boundarie s 1, 6
spf.ujumpx	spf.ut_herex- spf.ut_therex	m/s	Velocity jump, x component	Boundarie s 1, 6
spf.ujumpy	spf.ut_herey- spf.ut_therey	m/s	Velocity jump, y component	Boundarie s 1, 6
spf.ujumpz	spf.ut_herez- spf.ut_therez	m/s	Velocity jump, z component	Boundarie s 1, 6
spf.meshVol	meshvol_spatial	m		Boundarie s 1, 6
spf.meshVolInt	down(meshvol_spatial)	m ²	Volume of interior mesh element	Boundarie s 1, 6
spf.sigma_dg_ns	4*spf.ct_here	Pa·s/m		Boundarie s 1, 6
spf.p0	0	Pa	Pressure	Boundarie s 1, 6
spf.f0	spf.p0	N/m ²	Normal stress	Boundarie s 1, 6
spf.un_here	u*nojac(spf.nxmesh)+ v*nojac(spf.nymesh)	m/s	Intermediat e variable	Boundarie s 1, 6
spf.ut_herex	u- spf.un_here*nojac(spf. nxmesh)	m/s	Intermediat e variable, x component	Boundarie s 1, 6
spf.ut_herey	v- spf.un_here*nojac(spf. nymesh)	m/s	Intermediat e variable, y component	Boundarie s 1, 6
spf.ut_herez	-	m/s	Intermediat	Boundarie

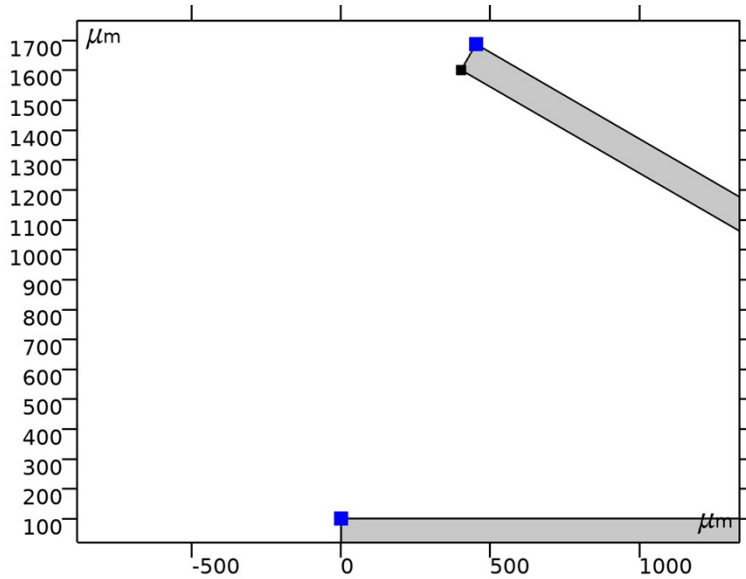
Name	Expression	Unit	Description	Selection
	spf.un_here*nojac(spfnzmesh)		e variable, z component	s 1, 6
spf.un_there	0	m/s	Intermediate variable	Boundaries 1, 6
spf.ut_there_x	- spf.un_there*nojac(spfnxmesh)	m/s	Intermediate variable, x component	Boundaries 1, 6
spf.ut_there_y	- spf.un_there*nojac(spfnymesh)	m/s	Intermediate variable, y component	Boundaries 1, 6
spf.ut_there_z	- spf.un_there*nojac(spfnzmesh)	m/s	Intermediate variable, z component	Boundaries 1, 6
spf.ct_here	24*nojac(down((spf.mu+spf.muT)/spf.epsilon_p))*spf.meshVol/spf.meshVollnt	Pa·s/m	Intermediate variable	Boundaries 1, 6
spf.out1.Uav	0	m/s	Average velocity	Global
spf.out1.Uavfdf	0	m/s	Average velocity	Global
spf.out1.Mflow	spf.out1.intFlow(spfrho*(spf.nx*u+spf.ny*v))	kg/s	Mass flow	Global

Weak expressions

Weak expression	Integration order	Integration frame	Selection
- spf.f0*(test(u)*spf.nxmesh+test(v)*spf.nymesh)	2	Spatial	Boundaries 1, 6
spf.KStressn_avx*test(spf.ut_herex)+spf.KStressn_avy*test(spf.ut_herey)+spf.KStressn_avz*test(spf.ut_herez)+spf.KStressTestn_avx*spf.ujumpx+spf.KStressTestn_avy*spf.ujumpy+spf.KStressTestn_avz*spf.ujumpz-spfi.sigma_dg_ns*spf.ujumpx*test(spf.ut_herex)-spfi.sigma_dg_ns*spf.ujumpy*test(spf.ut_herey)-spfi.sigma_dg_ns*spf.ujumpz	2	Spatial	Boundaries 1, 6

Weak expression	Integration order	Integration frame	Selection
mpz*test(spf.ut_herez)			

2.5.8 Pressure Point Constraint 1



Pressure Point Constraint 1

SELECTION

Geometric entity level	Point
Selection	Geometry geom1: Dimension 0: Points 2, 6

EQUATIONS

$$p = p_0$$

Pressure constraint

SETTINGS

Description	Value
Pressure	0

USED PRODUCTS

COMSOL Multiphysics

Variables

Name	Expression	Unit	Description	Selection
------	------------	------	-------------	-----------

Name	Expression	Unit	Description	Selection
spf.p0	0	Pa	Pressure	Points 2, 6

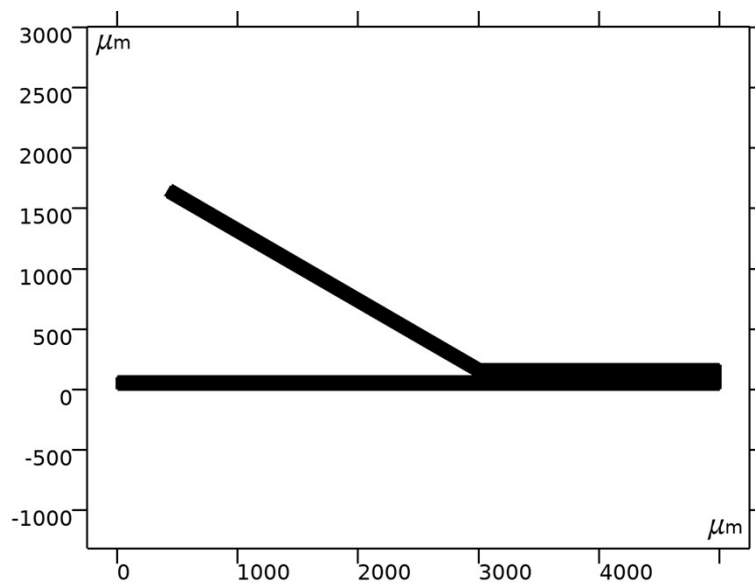
Constraints

Constraint	Constraint force	Shape function	Selection	Details
-p+spf.p0	test(-p)	Lagrange (Linear)	Points 2, 6	Elemental

2.6 MESH 1

MESH STATISTICS

Description	Value
Minimum element quality	0.6544
Average element quality	0.97
Triangle	34139
Quad	620
Edge element	1777
Vertex element	12



Mesh 1

2.6.1 Size (size)

SETTINGS

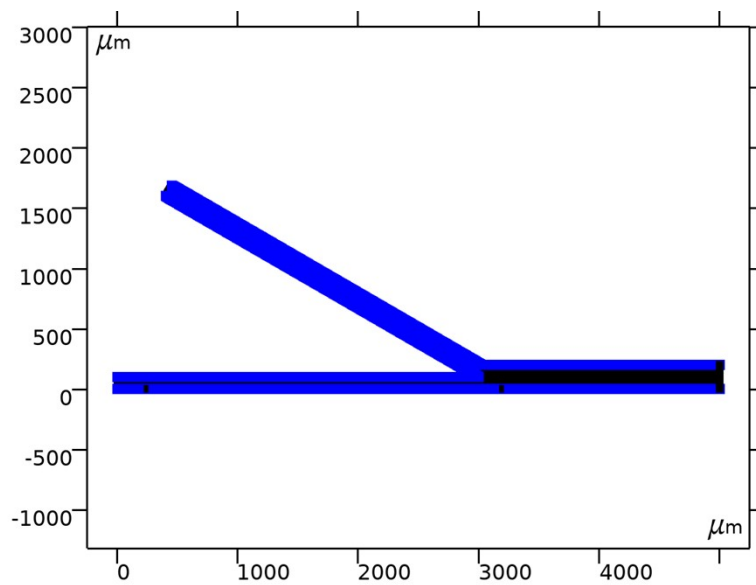
Description	Value
-------------	-------

Description	Value
Calibrate for	Fluid dynamics
Maximum element size	11.3
Minimum element size	0.0337
Curvature factor	0.2
Maximum element growth rate	1.05
Predefined size	Extremely fine

2.6.2 Size 1 (size1)

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 2-3, 5, 7-9, 11



Size 1

SETTINGS

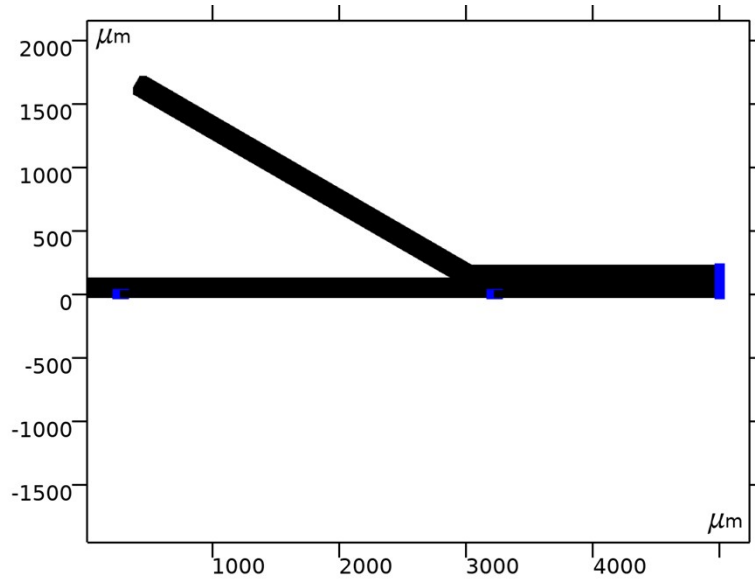
Description	Value
Calibrate for	Fluid dynamics
Maximum element size	11.3
Minimum element size	0.0337
Curvature factor	0.2
Maximum element growth rate	1.05

Description	Value
Predefined size	Extremely fine

2.6.3 Size 2 (size2)

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 4, 10, 12



Size 2

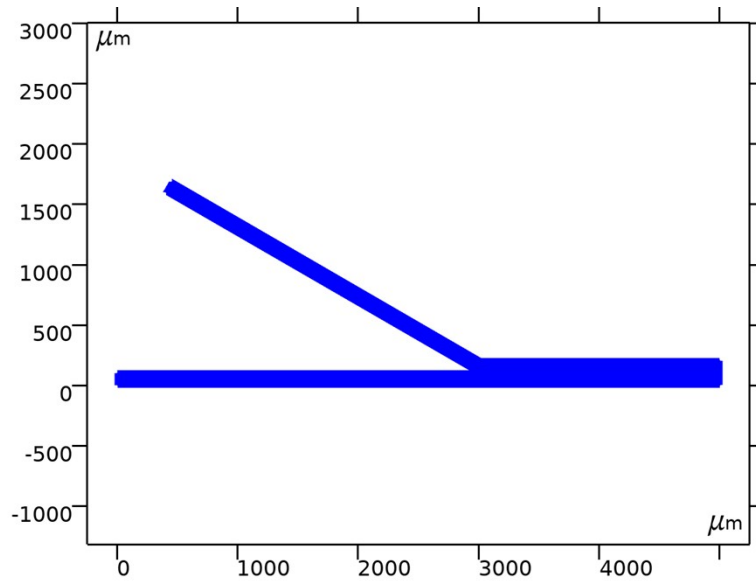
SETTINGS

Description	Value
Calibrate for	Fluid dynamics
Maximum element size	1.5
Minimum element size	0.0207
Curvature factor	0.2
Curvature factor	Off
Resolution of narrow regions	Off
Maximum element growth rate	1.05
Maximum element growth rate	Off
Predefined size	Extremely fine
Custom element size	Custom

2.6.4 Corner Refinement 1 (cr1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: Domain 1



Corner Refinement 1

2.6.5 Free Triangular 1 (ftri1)

SELECTION

Geometric entity level	Domain
Selection	Remainin g



Free Triangular 1

2.6.6 Boundary Layers 1 (bl1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: Domain 1



Boundary Layers 1

SETTINGS

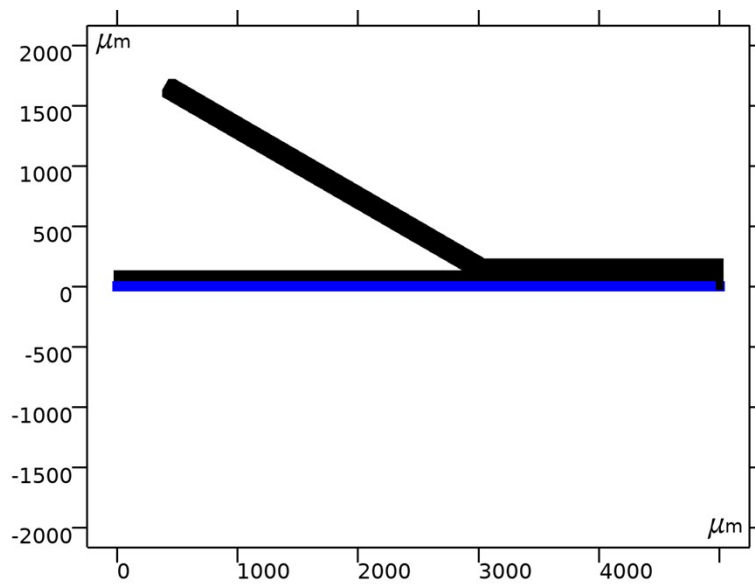
Description	Value
-------------	-------

Description	Value
Handling of sharp corners	Trimming

Boundary Layer Properties 1 (blp1)

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 2, 4-5, 10-11



Boundary Layer Properties 1

SETTINGS

Description	Value
Number of boundary layers	1
Thickness adjustment factor	10

3 Study 1

COMPUTATION INFORMATION

Computation time	1 min 2 s
CPU	Intel64 Family 6 Model 44 Stepping 2, 12 cores
Operating system	Windows 7

3.1 STATIONARY

STUDY SETTINGS

Description	Value
Include geometric nonlinearity	Off

PHYSICS AND VARIABLES SELECTION

Physics interface	Discretization
Nernst-Planck Equations (npe)	physics
Laminar Flow (spf)	physics

MESH SELECTION

Geometry	Mesh
Geometry 1 (geom1)	mesh1

3.2 SOLVER CONFIGURATIONS

3.2.1 Solution 1

Compile Equations: Stationary (st1)

STUDY AND STEP

Description	Value
Use study	
Use study step	

LOG

```

<---- Compile Equations: Stationary in Study 1/Solution 1 (sol1) -----
--
Started at May 26, 2021 10:46:57 AM.
Geometry shape order: Linear
Running on Intel64 Family 6 Model 44 Stepping 2, GenuineIntel.
Using 1 socket with 12 cores in total on CNS-R-WEL870984.
Available memory: 110.59 GB.
Time: 3 s.
Physical memory: 1.96 GB
Virtual memory: 2.12 GB
Ended at May 26, 2021 10:47:01 AM.
----- Compile Equations: Stationary in Study 1/Solution 1 (sol1) -----
->

```

Dependent Variables 1 (v1)

INITIAL VALUES OF VARIABLES SOLVED FOR

Description	Value
Method	Solution
Solution	

LOG

```

<---- Dependent Variables 1 in Study 1/Solution 1 (sol1) -----
--
Started at May 26, 2021 10:47:02 AM.
Initial values of variables solved for: Solution 1 (sol1).
Solution time: 0 s.
Physical memory: 1.93 GB
Virtual memory: 2.09 GB
Ended at May 26, 2021 10:47:02 AM.
----- Dependent Variables 1 in Study 1/Solution 1 (sol1) -----
->

```

Concentration (comp1.cBead) (comp1_cBead)

GENERAL

Description	Value
Field components	comp1.cBead
Internal variables	{comp1.uflux.cBead, comp1.dflux.cBead}

Concentration (comp1.cH3O) (comp1_cH3O)

GENERAL

Description	Value
Field components	comp1.cH3O
Internal variables	{comp1.uflux.cH3O, comp1.dflux.cH3O}

Concentration (comp1.cK) (comp1_cK)

GENERAL

Description	Value
Field components	comp1.cK
Internal variables	{comp1.uflux.cK, comp1.dflux.cK}

Concentration (comp1.cOH) (comp1_cOH)

GENERAL

Description	Value
Field components	comp1.cOH
Internal variables	{comp1.uflux.cOH, comp1.dflux.cOH}

Pressure (comp1.p) (comp1_p)

GENERAL

Description	Value
Field components	comp1. p

Velocity field (comp1.u) (comp1_u)

GENERAL

Description	Value
Field components	{comp1.u, comp1.v}
Internal variables	comp1.spf.isFluidHasBeenSolved

Electric potential (comp1.V) (comp1_V)

GENERAL

Description	Value
Field components	comp1.V
Internal variables	{comp1.uflux.V, comp1.dflux.V}

Stationary Solver 1 (s1)

GENERAL

Description	Value
Defined by study step	

LOG

```
<----- Stationary Solver 1 in Study 1/Solution 1 (sol1) -----  
--  
Started at May 26, 2021 10:47:02 AM.  
Nonlinear solver  
Number of degrees of freedom solved for: 418417 (plus 17831 internal DOFs).  
Nonsymmetric matrix found.  
Scales for dependent variables:  
Concentration (compl.cBead): 8.6e+06  
Concentration (compl.cH3O): 95  
Concentration (compl.cK): 3.1e+03  
Concentration (compl.cOH): 1.5e+02  
Pressure (compl.p): 1e+02  
Velocity field (compl.u): 2.7e+02  
Electric potential (compl.V): 30  
Orthonormal null-space function used.  
Iter      SolEst      ResEst      Damping      Stepsize #Res #Jac #Sol   LinErr  
LinRes  
   1      0.00011      5.1e+03      0.0100000      0.00012    2   1    2   3.8e-  
13 5.1e-16  
   2      0.0001      4.6e+03      0.1000000      0.00011    3   2    4   1.3e-  
12 5.1e-16  
   3      4.4e-05      5.4e+02      1.0000000      0.00048    5   3    6   1.6e-  
11 4.4e-16  
Solution time: 57 s.  
Physical memory: 5.31 GB  
Virtual memory: 6.03 GB  
Ended at May 26, 2021 10:47:59 AM.  
----- Stationary Solver 1 in Study 1/Solution 1 (sol1) -----  
->
```

Advanced (aDef)

ASSEMBLY SETTINGS

Description	Value
Reuse sparsity pattern	On

Fully Coupled 1 (fc1)

GENERAL

Description	Value
Linear solver	

METHOD AND TERMINATION

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6
Maximum number of iterations	100

Direct 1 (d1)

GENERAL

Description	Value
Solver	PARDIS O
Pivoting perturbation	1.0E-13

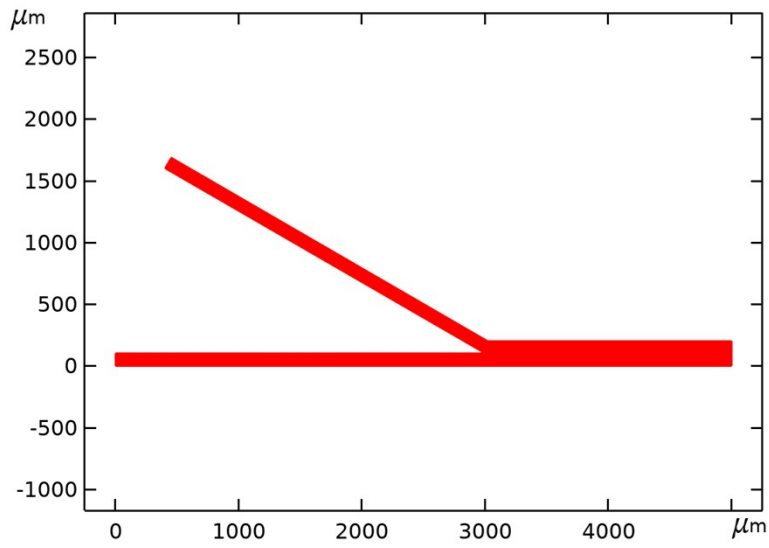
4 Results

4.1 DATA SETS

4.1.1 Study 1/Solution 1

SOLUTION

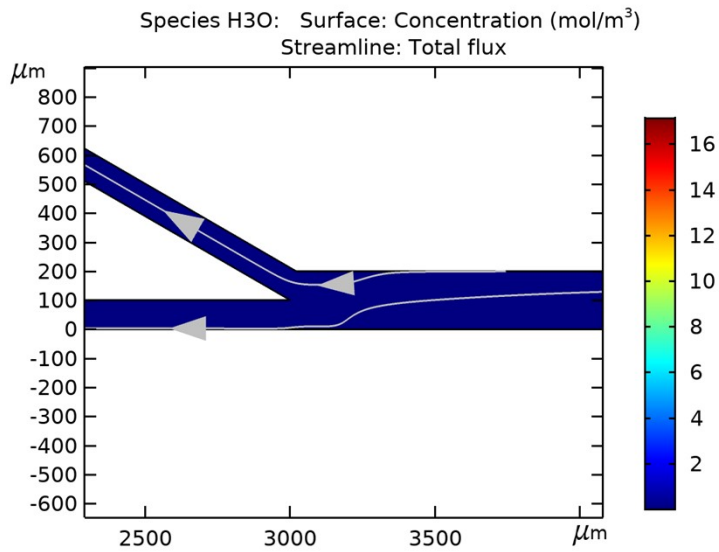
Description	Value
Solution	
Component	Save Point Geometry 1



Dataset: Study 1/Solution 1

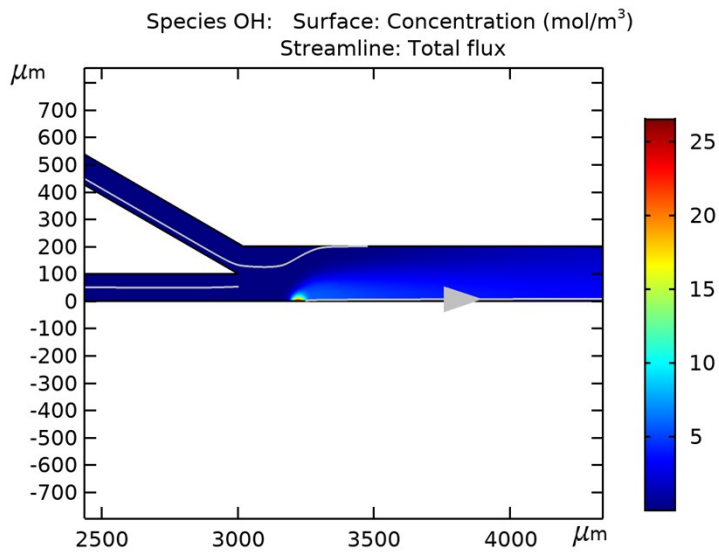
4.2 PLOT GROUPS

4.2.1 Concentration, H3O (npe)



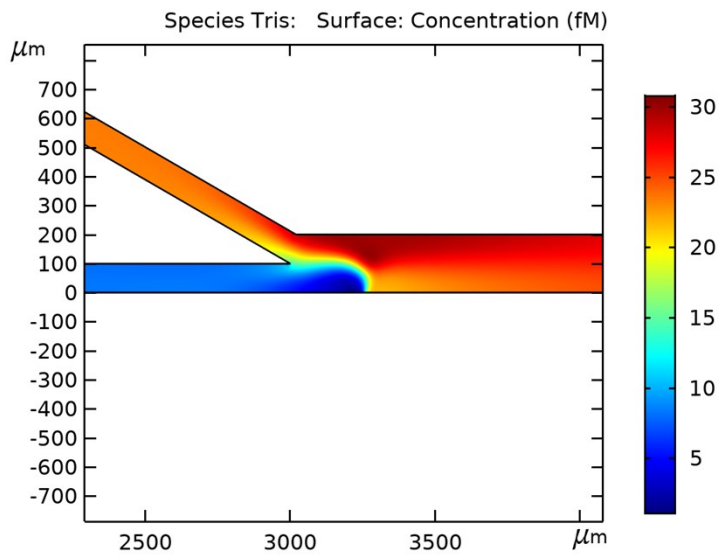
Species H3O: Surface: Concentration (mol/m³) Streamline: Total flux

4.2.2 Concentration, OH (npe)



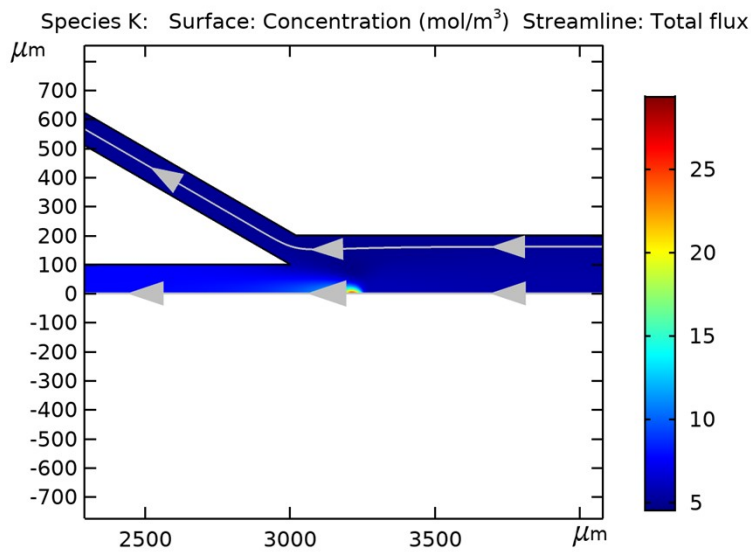
Species OH: Surface: Concentration (mol/m³) Streamline: Total flux

4.2.3 Concentration, bead (npe)



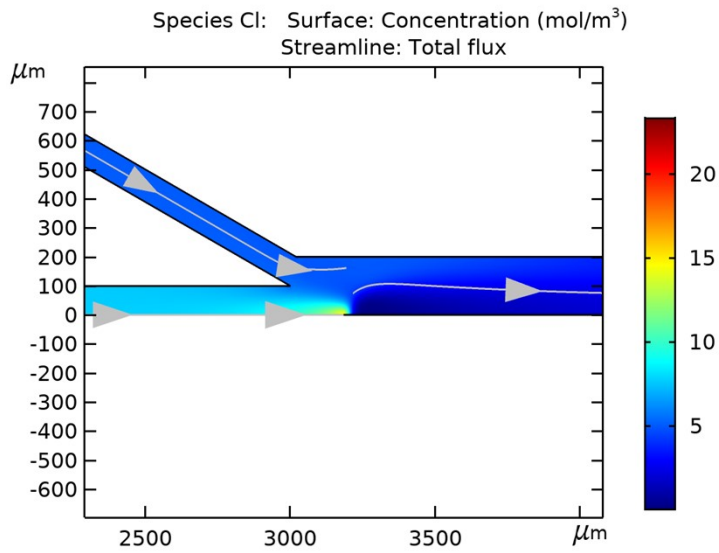
Species Tris: Surface: Concentration (fM)

4.2.4 Concentration, K (npe)



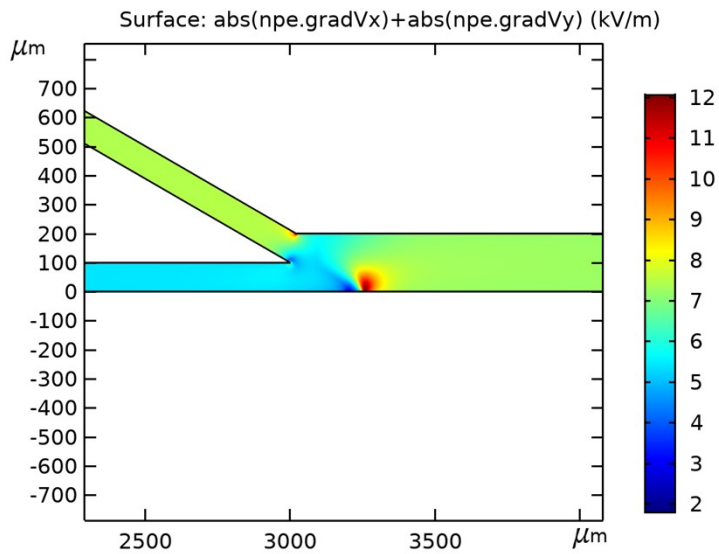
Species K: Surface: Concentration (mol/m³) Streamline: Total flux

4.2.5 Concentration, Cl (npe)



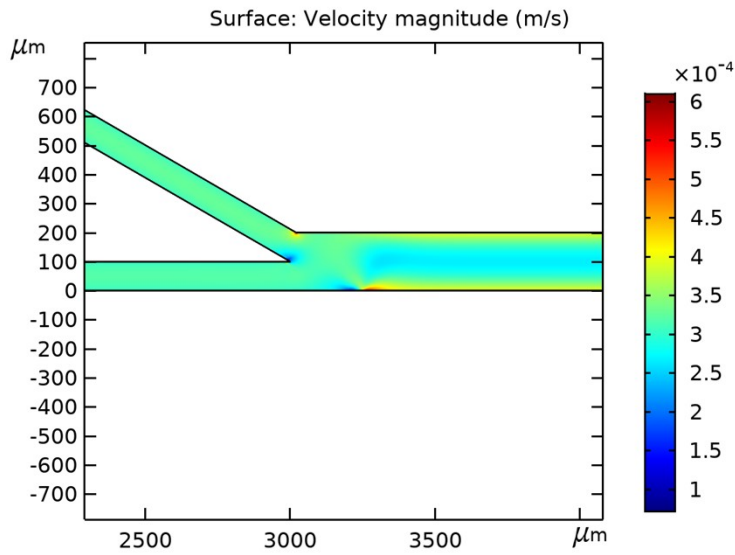
Species Cl: Surface: Concentration (mol/m³) Streamline: Total flux

4.2.6 Electric field (npe)



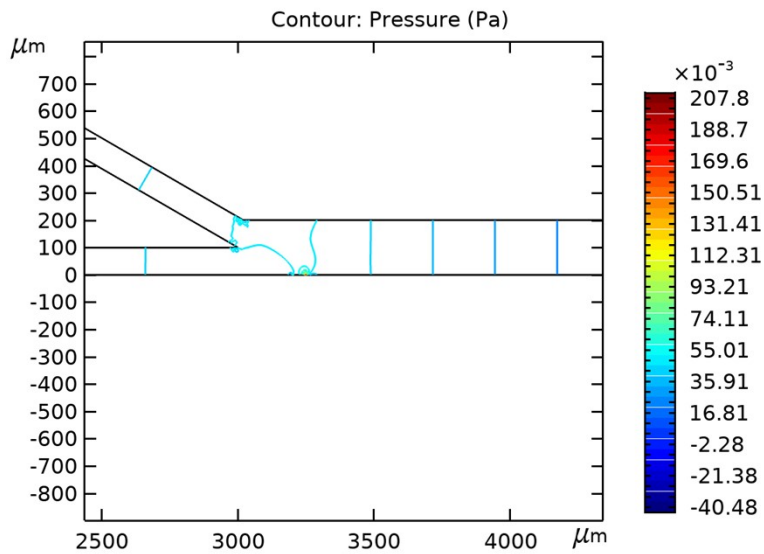
Surface: $\text{abs}(\text{npe.gradVx}) + \text{abs}(\text{npe.gradVy})$ (kV/m)

4.2.7 Velocity (spf)



Surface: Velocity magnitude (m/s)

4.2.8 Pressure (spf)



Contour: Pressure (Pa)