Supplementary Information: Shear Stress Dependence of Force Networks in 3D Dense Suspensions

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Figure 1: Sub Geodesics. (a) The fill path with a geodesic between the vertices $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$. The raw geodesic distance matrix contains all sub-paths that also connect a pair of vertices. (b)-(e) The set of sub-geodesics highlighted in red dotted lines. (b) $1 \rightarrow 2$, (c) $2 \rightarrow 3$, (c) $1 \rightarrow 2 \rightarrow 3$, and (e) $3 \rightarrow 4$.



Figure 2: A radial density function (RDF) plots for (A) $\mathbf{F}_A = 0.3$, (B) $\mathbf{F}_A = 0.75$, and (C) $\mathbf{F}_A = 0.9$. The particle pair interactions for the bisdisperse system have been separated by size, where small particles (S) have a radius of 1 and large particles (L) have a radius of 1.4. Two shear rates were plotted to highlight the difference between shear thinning and shear thickening. The large primary peaks identify the nearest neighbor shells, followed by much smaller secondary and tertiary peaks that indicate a limited amount of ordering in the particle configuration across distance. Beyond a radius of 6, G(d) approaches 1, indicating no long-rang order. For shear thinning, σ values of 0.055, 0.21, and 0.3 were used for $\mathbf{F}_A = 0.3$, 0.75, and 0.9 respectively. For shear thickening σ values of 5.0 were used for all three \mathbf{F}_A values. (D)-(F) Difference plots found by subtracting the shear thinning plot from the shear thickening plot of the RDF for $\mathbf{F}_A = 0.3$, $\mathbf{F}_A = 0.75$, and $\mathbf{F}_A = 0.9$ respectively.



Figure 3: RDF plot for $F_a = 0.3$ where d is the particle pair surface-to-surface distance. The particle pair interactions for the bisdisperse system have been separated by size, where small particles (S) have a radius of 1 and large particles (L) have a radius of 1.4. Two shear rates used were were $\sigma = 0.055$ for shear thinning and $\sigma = 5.0$ for shear thickening.



Figure 4: Coordination number plots for (A) $\mathbf{F}_A = 0.3$, (B) $\mathbf{F}_A = 0.75$, and (C) $\mathbf{F}_A = 0.9$ found by integrating the RDF curves. Small particles (S) have a radius of 1 and large particles (L) have a radius of 1.4.



Figure 5: The total pore volume (i.e. empty space) created by all particles in the simulation plotted as a percentage of maximum possible pore volume. Each curve corresponds to a different \mathbf{F}_A value.



Figure 6: Edge count for attractive forces where particle pair interactions have been separated by size. Small particles (S) have a radius of 1 and large particles (L) have a radius of 1.4. (A) $F_a=0.3$ under shear thinning ($\sigma=0.055$). (B) $F_a=0.3$ under shear thickening ($\sigma=5$). (C) $F_a=0.75$ under shear thinning ($\sigma=0.21$). (D) $F_a=0.75$ under shear thickening ($\sigma=0.5$). (E) $F_a=0.9$ under shear thinning ($\sigma=0.3$). (F) $F_a=0.9$ under shear thickening ($\sigma=5$).



Figure 7: (A) Edge count of different weights within the contact force network under shear thinning. σ values of 0.21 and 0.3 were used for \mathbf{F}_A =0.75, and 0.9 respectively. The contact force network for \mathbf{F}_A =0.3 is virtually non-existent and thus its edge distribution is not plotted. (B) Histogram plot of contact force edge weights under shear thickening. σ values of 5.0 were used for all three \mathbf{F}_A values.