Electronic Supplementary Material (ESI) for Soft Matter.

A Comparative Study of Blood Rheology across Species

Jeffrey S. Horner[a,b], Norman J. Wagner[a], and Antony N. Beris[a*]

[a] Center for Research in Soft Matter and Polymers, Department of Chemical and Biomolecular Engineering, University of Delaware, Newark, DE 19716

[b] Current address: Thermal/Fluid Component Sciences Department, Sandia National Laboratories, Albuquerque, NM 87123

* Corresponding author: Antony N. Beris, Department of Chemical and Biomolecular Engineering, University of Delaware, Newark, DE 19716, USA. Tel: +1 302 831 8018; Fax: +1 302 831 1048

Email: beris@udel.edu

Supporting Information

Data and formula used to describe hematocrit dependence

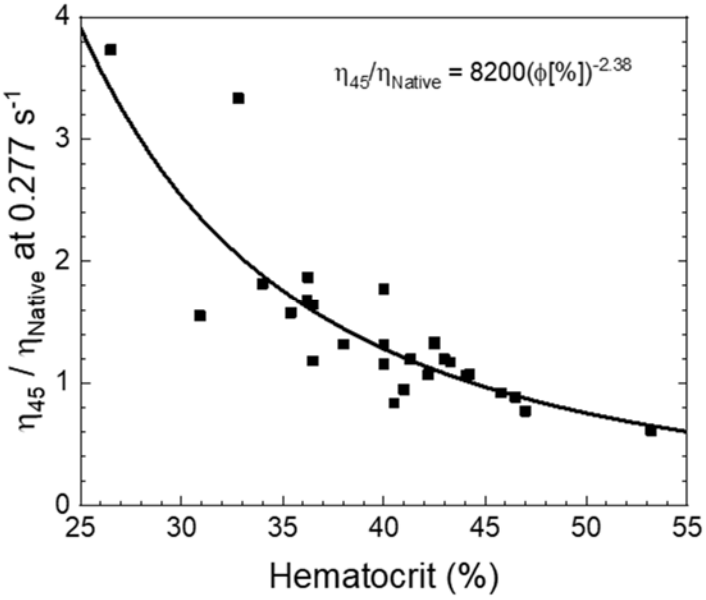

Figure S1. Ratio of 45 % hematocrit blood viscosity to native hematocrit blood viscosity at 0.277 s^{-1} for blood from various species. The power law fit used to shift original data is also shown. Data taken from Johnn et al., 1992[20]. The figure shows the relatively high impact of hematocrit to the data justifying the correction implemented in the main manuscript.

Allometric Branching Model Derivation

A fluid mechanical model for circulatory branching across species was developed by West and coworkers[29]. This model assumes vessels initially branch according to an area preservation principle, first developed by Leonardo da Vinci and therefore also known as da Vinci's rule[35], to avoid pressure pulse reflection, or equivalently, minimize hydrodynamic impedance. Once viscous effects become relevant and the pulsation is damped out, the model proposes that vessels then branch according to preserving the apparent (according to the Poiseuille profile) wall shear rate (*i.e.* $4\bar{u}_k/r_k$, where $\bar{u}_k$ and r_k are the mean blood velocity and radius of the k-th generation vessel, respectively) also known as Murray's law[29,35,43]. Mathematically, West's model can be expressed through the following equations:

$$\bar{u}_k(M) = \frac{Q_0(M)}{N_k \pi r_k(M)^2} \qquad \textbf{[S1]}$$

$$r_k(M) = r_0(M) \prod_{i=0}^{k-1} \beta_i\,(M) \approx r_0(M) \exp\left(\int_0^k ln(\beta_i)\, di\right) \qquad \textbf{[S2]}$$

$$\beta_k(M) = \begin{cases} \nu^{-\frac{1}{2}}, & k \leq \bar{k}(M) \\ \nu^{-\frac{1}{3}}, & k > \bar{k}(M) \end{cases}. \qquad \textbf{[S3]}$$

Here, k indicates the branching level (with $k = 0$ corresponding to the aorta), Q_k is the flow rate in a particular vessel, N_k is the total number of vessels at a particular branch level, β the ratio of adjacent branch level radii (*i.e.* $\beta_k = r_{k+1}/r_k$), ν is the number of "offspring" vessels for a "parent" vessel (typically $2 \leq \nu \leq 3$), and $\bar{k}$ is the critical branch level where the pulse waveform is damped out. Note that the rightmost side of **Eq. S2** is the geometric integral expression for the vessel radius at a particular branch level. This expression will be used henceforth to eliminate discontinuities associated with the branch level being a discrete variable.

In the original model, West and coworkers approximated $\bar{k}$ to be the point where the hydrodynamic wave impedance is equal to the viscous resistance[29]. For a total number of C generation of vessels, this results to $C - \bar{k}$ Murray law-following generations, which, to a good approximation, can be considered as a constant. This has been evaluated from West *et al.*[27] as $C - \bar{k} =$ 15 or 22 depending whether one uses $\nu =$ 3 or 2. Under those conditions, the apparent shear rate for $k \geq \bar{k}$ can therefore be considered as constant and equal to $4\bar{u}_c/r_c$, where $\bar{u}_c$ and r_c are the mean blood velocity and radius of the capillary vessels, respectively. Furthermore, if we use the additional assumptions proposed by West *et al.*[29] that those values are universal constants among all species, we can estimate those values from human data as[27] $\bar{u}_c = \bar{u}_{0,h}/250$, and $r_c = r_{0,h}/10^4$, where $\bar{u}_{0,h}$ and $r_{0,h}$ are the mean blood velocity ($\bar{u}_0 \approx 11.44\ cm/s$) and radius of the human aorta ($\bar{r}_0 = 1.64\ cm$), respectively[41]. These values result to $4\bar{u}_c/r_c = 1120\ s^{-1}$, which corresponds to a nominal wall shear stress (using a Newtonian viscous model and a viscosity about three times the plasma viscosity, 0.0016 Pa.s) of 5.36 Pa[41]. This value agrees reasonably to recent asymptotic lower vessel radius data reported by Seymour and coworkers (see Fig. 2[35] also evaluated based on a Newtonian viscosity multiplying the apparent wall shear rate as obtained from the average velocity and radius).

To estimate the apparent shear rate of the larger arteries, $k = 0,1, \ldots, \bar{k} - 1$, one can start with the aorta, $k = 0$, and use allometric expressions offered by Schmidt-Nielsen and Holt and coworkers for the aorta radius and mean velocity as $r_0 \propto M^{0.36}$ and $\bar{u}_0 \propto M^{0.07}$, respectively[5,42]. Note that there can be considerable variation across studies in the aortic allometric scaling exponents, with other authors proposing values of 0.393 and –0.048 for the radius and mean velocity, respectively[35]. Nevertheless, the apparent wall shear rate in the aorta is universally agreed to exhibit a negative scaling with respect to body mass. This negative scaling is valid for arteries up to the point that this shear rate will become equal to the universal value for the smaller vessels quoted above (*i.e.,* $k = 0,1,\dots,\bar{k} - 1$. The behavior of the apparent wall shear rate as a function of body mass and for different branch levels can be better seen in Fig. S2 below. Note, that the apparent shear rate at the critical branch level and beyond remains constant.

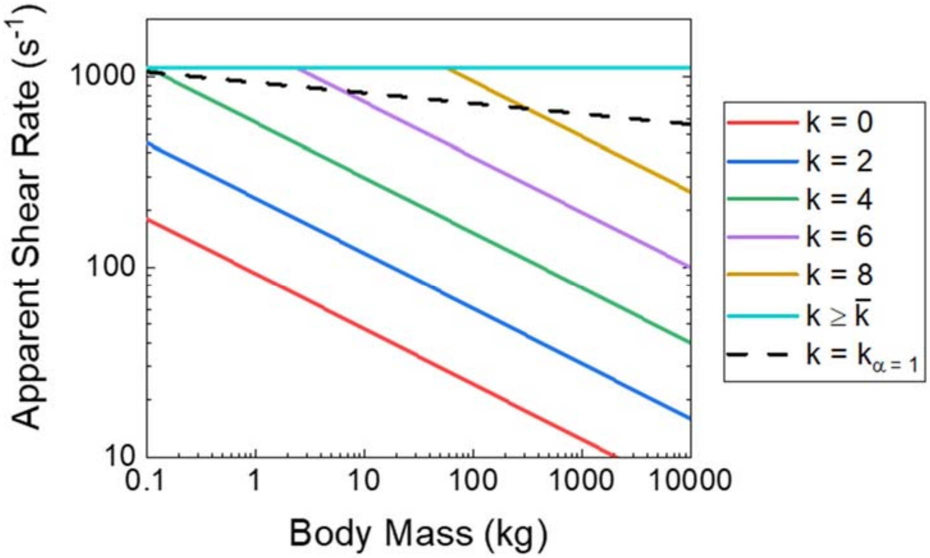

Figure S2. Apparent shear rate in blood vessels at different branching levels as a function of body mass.

Note that in Fig. S2, before $\bar{k}$, all vessels obey a –0.29 scaling with body mass. However, at and after $\bar{k}$, all vessels transition to zero scaling with body mass. According to West and coworkers' original proposition, after $\bar{k}$, the shear rate is maintained, and vessels branch according to an area increasing branching[29]. For reference, the shear rate at $k_{\alpha=1}$ has also been plotted as a dashed line. Note that the scaling at this reference point does not follow the –0.29 scaling because the value of $k_{\alpha=1}$ is also changing with respect to body mass. To evaluate this point, values of $\nu = 2.5$, $\omega_h = 80\,bpm$, $\rho = 1060\,kg/m^3$, and –1/4 allometric scaling for the pulsation frequency were used.