Electronic Supplementary Material (ESI) for Soft Matter.

Supplementary information for

Persistence length regulates emergent dynamics in active roller ensembles

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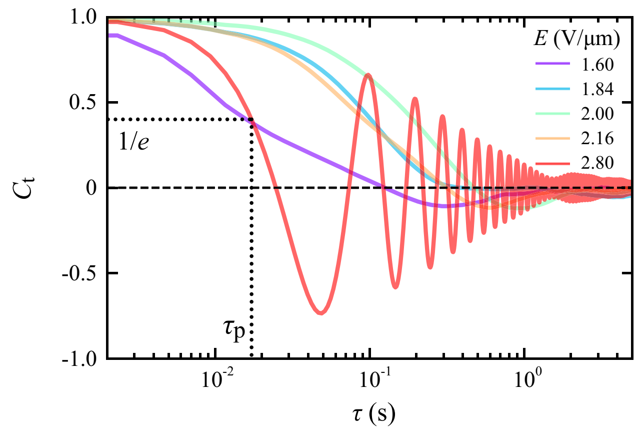
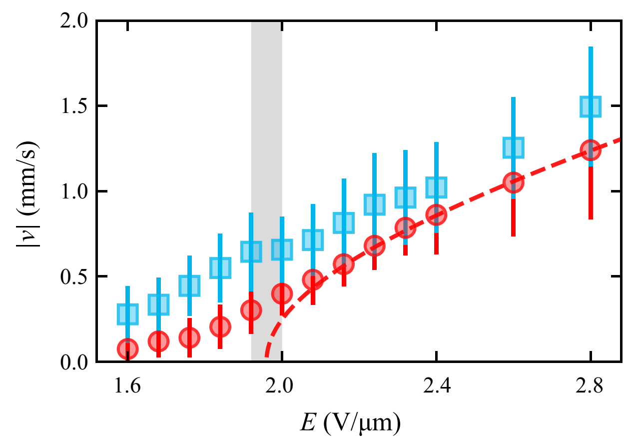
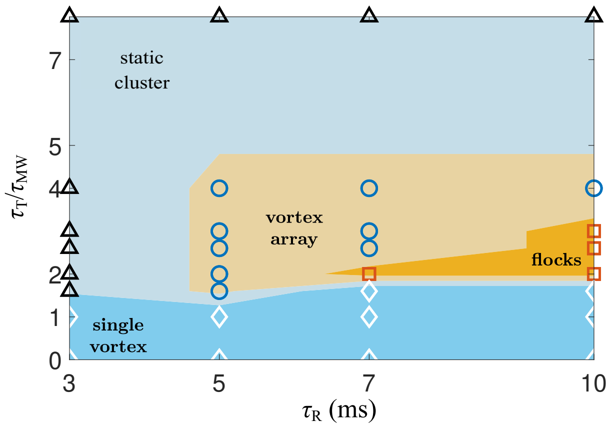


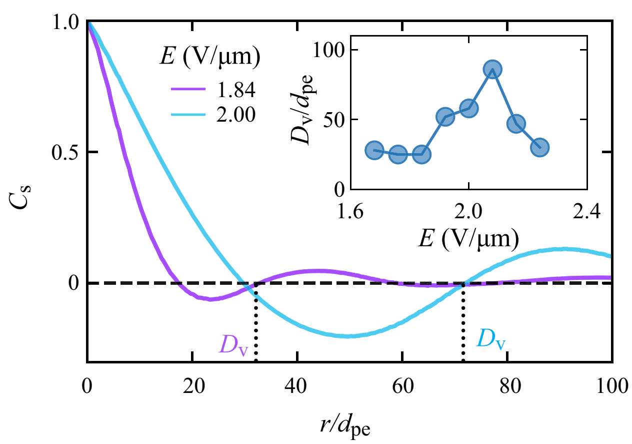
Supplementary Figure 1. Velocity temporal correlation functions C_t of pear-shaped rollers for a set of field strengths. C_t is defined as $C_\mathrm{t}(\tau) = N^{-1} \sum_i \langle \mathbf{v}_i(t) \cdot \mathbf{v}_i(t+\tau) \rangle_t / \langle \mathbf{v}_i^2(t) \rangle_t$, where $\mathbf{v}_i(t)$ is the velocity of a roller i at the moment t; N is the total number of rollers; and $\langle\ \rangle_t$ indicates time average. The persistence time τ_p is defined as the time when C_t decays to $1/e$. The area fraction $\phi = 0.144$.



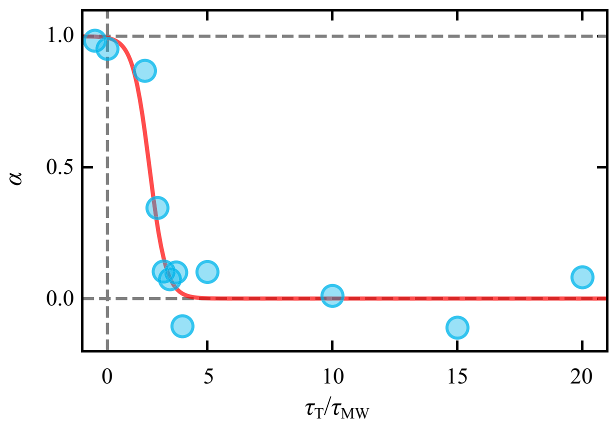
Supplementary Figure 2. Rolling velocity of the pear-shaped particles as a function of the electric field strength E of samples with two different area fractions. The area fractions are 0.001 and 0.144 for red circles and blue squares, respectively. The red dash line is a fit of the high field part of the red curve to $|v| \sim \sqrt{(E/E_\mathrm{c})^2 - 1}$ dependence typical for spherical rollers. $E_\mathrm{c} = 1.96$ V μm^{-1}. Shaded area is a crossover region (β) between two different modes (α, γ) of particles rolling. Error bars are standard deviation of the absolute values of velocities.



Supplementary Figure 3. Phase diagram of spherical Quincke random walkers. Multiple vortices are shown within limited range where $2 \lesssim \tau_T/\tau_{MW} \lesssim 4$.



Supplementary Figure 4. Velocity spatial correlation functions of pear-shaped rollers. The velocity spatial correlation function D_v is defined as $C_s(r) = \langle\langle \mathbf{v}_i(r_0,t) \cdot \mathbf{v}_j(r_0+r,t)\rangle_{i,j}\rangle_t / \langle\langle \mathbf{v}_i^2(r_0,t)\rangle_i\rangle_t$, where $\mathbf{v}_i(r_0,t)$ and $\mathbf{v}_j(r_0+r,t)$ are velocities of roller i and j with a relative distance r at a time t; $\langle\ \rangle_i$ is an ensemble average; and $\langle\ \rangle_t$ is the time average. The vortex size D_v is defined as a second zero crossing in the spatial correlation curve C_s. Insert: Vortex size, D_v, as a function of the field strength E. The area fraction $\phi = 0.144$.



Supplementary Figure 5. Persistence index of spherical Quincke random walkers under different the polarization memory. The red line is a fitting curve using Supplementary Eq (2). The gray dash lines are guide lines.

SUPPLEMENTARY NOTE 1

Persistence length for Quincke Random-Walkers

Shown in Supplementary Figure 3 is the phase diagram which summarizes the range of existence of different phases. For $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} < 2$, particles keep full memory of their direction of motion and eventually form a giant single vortex, similar to the one previously reported for persistently running particles in the DC limit, i.e. $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} \to 0$ [1]. Therefore, although particles are continuously running in the DC-driven case while they periodically do runs punctuated by full stop during τ_{T} period in the modulated AC cases, as long as the polarization of particles is fully retained, they both result in equivalent individual and collective behaviors. On the other end of the spectrum, when particles' polarization is fully reset at τ_{T} period of the signal, i.e. $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} > 4$, particles perform random motion without forming any coherent structures.

For the intermediate values of $2 \lesssim \tau_{\mathrm{T}}/\tau_{\mathrm{MW}} \lesssim 4$, particles partially keep the memory of their direction of motion. Upon applying the pulsed-modulated field, neighboring particles interact locally and form small vortices. However, due to the lack of full memory, before the neighboring vortices find time to merge and form larger ones, the particles forget their memory and as a result, they form pinned vortices.

For given τ_{R} and τ_{T} values, as the activity level (i.e. velocity) increases by the field magnitude, neighboring particles interact to larger distances and consequently the average number of pinned vortices that form in the field of view decreases. In order to fully characterize the role of relevant kinematic parameters on the structure of the vortex lattice, we recast the parameters in terms of the kinematic persistence length scale $L_{\mathrm{p}} = \langle|v|\rangle \cdot \tau_{\mathrm{p}}$, where $\langle|v|\rangle$ and τ_{p} are the mean velocity and the persistence time of the particles. In a correlated random-walk, the persistence time τ_{p} can be calculated from the corresponding Mean-Squared Displacement MSD as [2, 3]:

$$\tau_{\mathrm{p}} = \tau_{\mathrm{R}}(1 + 2\frac{\alpha}{1-\alpha}), \tag{1}$$

where $0 \leq \alpha = \langle\cos(\theta)\rangle \leq 1$ is the persistence index, which quantifies the degree of memory in the correlated run. $\langle ... \rangle$ is the average over all turning events and θ is the change in the direction of motion between two consecutive runs. In the context of Quincke random walkers, it has been shown that the persistence index α is controlled through the degree of polarization/depolarization of the particles, i.e. $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}}$ [4]. The functional form can be well approximated by an inverted logistic function (Supplementary Figure 5):

$$\alpha = \frac{1}{1 + e^{k(\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} - x_0)}}, \tag{2}$$

where x_0 is the sigmoid's midpoint and k controls the growth/decay rate of the curve. Our previous experimental measurements [4] suggest that $k = 3$ and $x_0 = 2.68$ can well capture the measured persistence index. Combining Eqs. 2 and 1 with the mean velocity v_{m} gives the functional form of the kinematic persistence length L_{p} for a given degree of depolarization $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}}$:

$$L_{\mathrm{p}} = \langle|v|\rangle\tau_{\mathrm{R}}(1 + \frac{2}{e^{k(\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} - x_0)}}). \tag{3}$$

For a fully uncorrelated runs, i.e. $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} \gg x_0$, persistence index α approaches zero and $L_{\mathrm{p}} = \langle|v|\rangle \cdot \tau_{\mathrm{R}}$. However, in fully correlated runs, i.e. $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}} \ll x_0$, persistence index α is one and kinematic persistence length scale of the particles $L_{\mathrm{p}} \to \infty$. In other words, in a bounded domain, persistent (in DC driven) and fully correlated runners (in pulse-modulated case) can explore the corresponding characteristic length scale of the domain, leading to the formation of a giant single vortex.

As Eq. 3 suggests, the kinematic length can be set separately with the activity level via changing the applied field magnitude, run time τ_{R}, and degree of depolarization $\tau_{\mathrm{T}}/\tau_{\mathrm{MW}}$.

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