# Supplementary Material: Conformational Behavior and Self-Assembly of Disjoint Semi-Flexible Ring Polymers Adsorbed on Solid Substrates 

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## Overlap Density

Since in the present article, we frequently refer to the overlap density, we provide in this supplementary a rationale and values of the overlap density, $\rho^{*}$ in the case of uniform substrates. In the case of polymer chains, the overlap density in two dimensions is typically defined as $\rho^{*} \approx 1 / R_{g}^{2}$, where $R_{g}$ is the radius of gyration of the polymer in the dilute regime. However, since $R_{g}$ is smaller than the real size of the molecule, we use a more accurate definition of the size of the ring polymer defined as the ensemble average of the maximum distance between monomers of a ring polymer and its center of mass in the dilute regime. This length scale, $R_{\text {max }}$ is proportional to the radius of gyration of the polymer. The overlap density, defined as $\rho^{*} \approx 1 / R_{\max }^{2}$ is shown versus the bending rigidity of the polymers, $\kappa$, in Fig. S1. This figure shows that the overlap density decreases with increasing $\kappa$ up to about $\kappa \approx 5 k_{B} T$, which is inline with the fact that the degree of anisotropy of the polymers increases with $\kappa$ (i.e., a decrease in $\Sigma$ ) for low values of $\kappa$ as shown by Fig. 3. However, for $\kappa \gtrsim 5 k_{B} T, \rho^{*}$ weakly increases with $\kappa$, which is also inline with the decrease in the degree of anisotropy (or increases in $\Sigma$ ) for large values of $\kappa$ as shown by Fig. 3.


Figure S1: Solid circles correspond to the overlap density of the ring polymers versus their bending rigidity in the case of a uniform substrate. The solid line is a guide to the eye.

## Curvature Distribution

Fig. S2 shows the distribution of the local curvature, $D(c)$ for the case of $\kappa=100 k_{B} T$ at different densities. Notice that the peak is at a finite value of $c$ for densities lower than $0.01 b^{-2}$. However, for densities higher that $0.01 b^{-2}$, the peak shifts to $c \approx 0$, and a shoulder emergence at large positive values of $c$. The change in the shape of the distribution is correlated with the change of the morphology of the polymers from mainly obround to biconcave.


Figure S2: Normalized curvature distribution for the case of $\kappa=100 k_{B} T$. Data correspond to different densities shown in the legend. Notice the emergence of a shoulder at high curvatures for $\rho>0.01 b^{-2}$ and a shift of the main peak's position to the left as $\rho$ is increased.

## Positional and Nematic Correlations

Fig. S3 depicts the positional pair correlation, calculated using Eq. (13) of the main text, in the case of low bending rigidity $\left(\kappa=5 k_{B} T\right)$ and high bending rigidity $\left(\kappa=80 k_{B} T\right)$.


Figure S3: Positional correlation function $G_{p}(r)$ versus $r$ for different values of the density. (A) and (B) corresponds to $\kappa=5 k_{B} T$ and $80 k_{B} T$, respectively.

Fig. S4 depicts the orientational pair correlations in the case of low bending rigidity ( $\kappa=5 k_{B} T$ ) and high bending rigidity $\left(\kappa=80 k_{B} T\right)$. The orientational pair correlations is calculated using

$$
\begin{equation*}
G_{n}(r)=\left\langle\cos 2\left[\theta\left(\mathbf{R}_{k}\right)-\theta\left(\mathbf{R}_{l}\right)\right] \delta\left(\mathbf{r}-\mathbf{R}_{k l}\right)\right\rangle \tag{1}
\end{equation*}
$$

where $\mathbf{R}_{k}$ and $\mathbf{R}_{l}$ are the coordinates of the centers of mass of polymers $k$ and $l$, and $\theta\left(\mathbf{R}_{k}\right)$ is the angle between the director of polymer $k$ and an arbitrary fixed axis.


Figure S4: Nematic correlation function $G_{n}(r)$ versus $r$ for different values of the density. (A) and (B) corresponds to $\kappa=5 k_{B} T$ and $80 k_{B} T$, respectively.

