## Soft Matter

## ARTICLE TYPE

# Electronic Supplementary Information for "Rotational diffusion and rotational correlations in frictional amorphous disk packings under shear." 

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This supplemental material provides additional data. We include examples of microrotation fields in Fig. 1. We show an overview of micro-rotation data for the three selected densities for the $\mu_{l}, \mu_{m}, \mu_{h}$ in Fig. 2 , Also the statistics of microrotations for a few example data sets is shown, both as $\theta_{i}^{m}(\gamma)$ trajectories at particle level in Fig. 33, and their probability distribution functions in Fig. 4. The robustness of power law fits for smaller strain is included in Fig. 5. We reveal some examples of the spatial $\theta_{i}$ field and the spatial distribution of $S_{n}$ in Fig. 6. The supplementary videos are described in the next section.

## Supplementary Videos

The animations included as the supplementary videos are grouped into three sets, according to the grains' friction coefficients (i.e., high ( $\mu_{h}$ ), medium ( $\mu_{m}$ ), and low $\left(\mu_{l}\right)$ friction). There are three animations in each set, which correspond to three different packing fractions. Packing fractions choses are highlighted in Fig. 5 from this ESI. The name of each file denotes the friction coefficient and packing fraction.
Each animation file contains two panels, which record the micro-rotation (left) and the local Moran's I (right) of the grains in the first 21 frames of a run of an experiment, which correspond to the period when the shear strain $\gamma$ increases from 0 to 0.05 . A grain is represented as a circle, and the links between them represent the neighborhood relationships. The sizes of the circles are proportional to the sizes of the grains. The colors of the circles are their micro-rotation and local Moran's $I_{i}$ values as quantified by the included color bars. Rotations are in radians, Moran's $I$ is dimensionless. A negative $I_{i}$ (red color) means that for that particle, its neighborhood has a counter rotating tendency, consistent with gear-like motion. For a positive (blue) $I_{i}$, the local neighborhood is rotating in the same direction. Note that sometimes, positive and negative $I_{i}$ are observed in adjacent particles; this can be due to the adjacent particles being outside the distance cutoff or simply because other particles in the neighborhood
contribute more to the value of $I_{i}$.
In the videos we can find the following qualitative features:

- In video $\mu_{l}, \phi_{l}$ we see that correlations are both positive and negative and fluctuate strongly in space and time. The mean Moran's $I$ is thus only weakly negative.
- In video $\mu_{l}, \phi_{h}$ we see that in contrast to $\phi_{l}$ for the low friction particles at high density persistent clusters do emerge, yet are amorphous in structure and contain positive and negative $I_{i}$.
- In video $\mu_{m}$, $\phi_{l}$ we observe very few clusters and the ones that emerge are located close to a wall and have no obvious shape.
- In video $\mu_{m}, \phi_{h}$ we see that initially, correlations fluctuate in space and time and are also positive and negative, yet after some amount of strain, persistent stringlike clusters can be observed to grow.
- In video $\mu_{h}, \phi_{l}$, clusters emerge earlier than for $\mu_{m}, \phi_{l}$, yet the ones that emerge are also located close to a wall and have no obvious shape.
- In video $\mu_{h}$, $\phi_{h}$ we see that clusters of strongly negative correlations emerge throughout the packing very early in the shear process, long before any such clustering is visible in the microrotation signal itself.


Figure 1 Examples of micro-rotation at shear strain $\gamma=0.15$ for (a) $\mu_{l}$ and (b) $\mu_{h}$ particles.


Figure 2 Mean $\left(\theta_{i}^{m}\right)$ and standard deviation $\left(\sigma_{m}\right)$ values of the micro-rotation, and the value of rigid rotation $\left(\theta^{R}\right)$ for: (a) $\mu_{l}$ particles with packing fraction $\phi=0.783$, (b) $\mu_{l}$ particles with $\phi=0.81$, (c) $\mu_{l}$ particles with $\phi=0.828$, (d) $\mu_{m}$ particles with $\phi=0.692$, (e) $\mu_{m}$ particles with $\phi=0.758$, (f) $\mu_{m}$ particles with $\phi=0.816$, (g) $\mu_{h}$ particles with $\phi=0.713$, (h) $\mu_{h}$ particles with $\phi=0.744$, and (i) $\mu_{h}$ particles with $\phi=0.807$. Each panel shows data for 5 repeats with the same particle type and $\phi$.


Figure 3 Micro-rotation $\theta_{i}^{m}$ of ten randomly selected grains and the minimum and maximum values of $\theta_{i}^{M}$ in the packing as a function of shear strain $\gamma$ for (a) $\mu_{l}$ with packing fraction $\phi=0.828$, (b) $\mu_{m}$ with packing fraction $\phi=0.816$, and (c) $\mu_{h}$ with packing fraction $\phi=0.744$.


Figure 4 Frequency distribution of normalized micro-rotation at several given shear strains $\gamma$ indicated by different colors for (a) $\mu_{l}$ particles with packing fraction $\phi=0.828$, (b) $\mu_{m}$ particles with $\phi=0.816$, and (c) $\mu_{h}$ particles with $\phi=0.807$.


Figure 5 Standard deviation of rotations as a function of imposed shear strain $\gamma$ in three different packing fractions for the (a) $\mu_{l}$, (b) $\mu_{m}$, and (c) $\mu_{h}$ particles. All experiments are repeated five times. (d) and (e) show, respectively, the variation of the parameters $D$ and $n$ as a function of packing fraction of the sets in $\mu_{l}, \mu_{m}, \mu_{h}$ where the fitting is performed for $\gamma$ from 0 to 0.06 . The highlighted data in yellow are the data used in panels a-c.


Figure 6 (a) Micro-rotation ( $\theta_{i}^{m}$ ) and (b) local Moran's I $\left(I_{i}\right)$ at low packing fractions $\phi$ for $\mu_{l}$ and $\mu_{h}$ particles at shear strain $\gamma=0.15$. For both panels, left: $\mu_{l}$ particles with packing fraction $\phi=0.783$; right: $\mu_{h}$ particles with $\phi=0.713$.

