Electronic Supplementary Material (ESI) for Soft Matter.

Supplemental Information for "Axisymmetric membranes with edges under external force"

Leroy L. Jia

Center for Computational Biology, Flatiron Institute,
162 5th Ave, New York, NY 10010, USA

Steven Pei

Department of Physics, Brown University,
182 Hope St, Providence, RI 02912, USA

Robert A. Pelcovits

Theoretical Physics Center and Department of Physics,
Brown University, 182 Hope St, Providence, RI 02912, USA

Thomas R. Powers

Center for Fluid Mechanics and School of Engineering
& Theoretical Physics Center and Department of Physics,
Brown University, 182 Hope St, Providence, RI 02912, USA

I. TENSION VS. EXTENSION

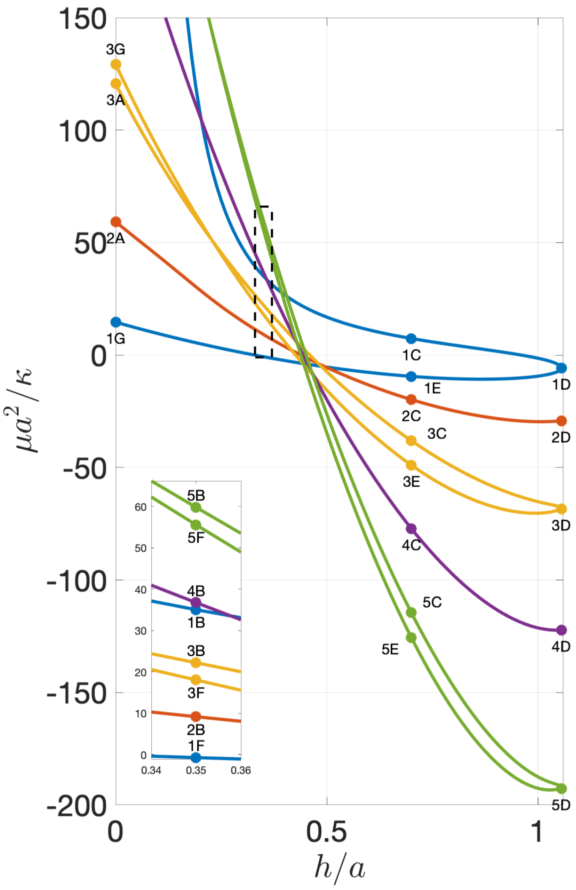

FIG. 1: Dimensionless tension as a function of extension for the first five bending modes of a membrane with $\bar{A} = 1$ and $\bar{\kappa} = 0$. Labeled points correspond to the shapes in Fig. 6 of the main text. At the maximal extension of $h^*/a = 1.0554$, the membrane is a catenoid and its permissible tensions are given by the eigenvalues in Table 1 of the main text. One of the $n = 1$ branches diverges as h approaches zero, while every other branch has finite tension at zero extension. (Inset) Zoomed-in version of the points near $h/a = 0.35$.

II. SELF-INTERSECTING SHAPES WITH NEGATIVE EXTENSION

Much like the classical Euler elastica, we observe that our shapes are mathematically allowed to take negative extensions. In such cases, these surfaces self-intersect and are unstable. Some examples of such shapes follow. Note that some of these shapes are not symmetric about the $z = 0$ plane, even if the mode number is odd.

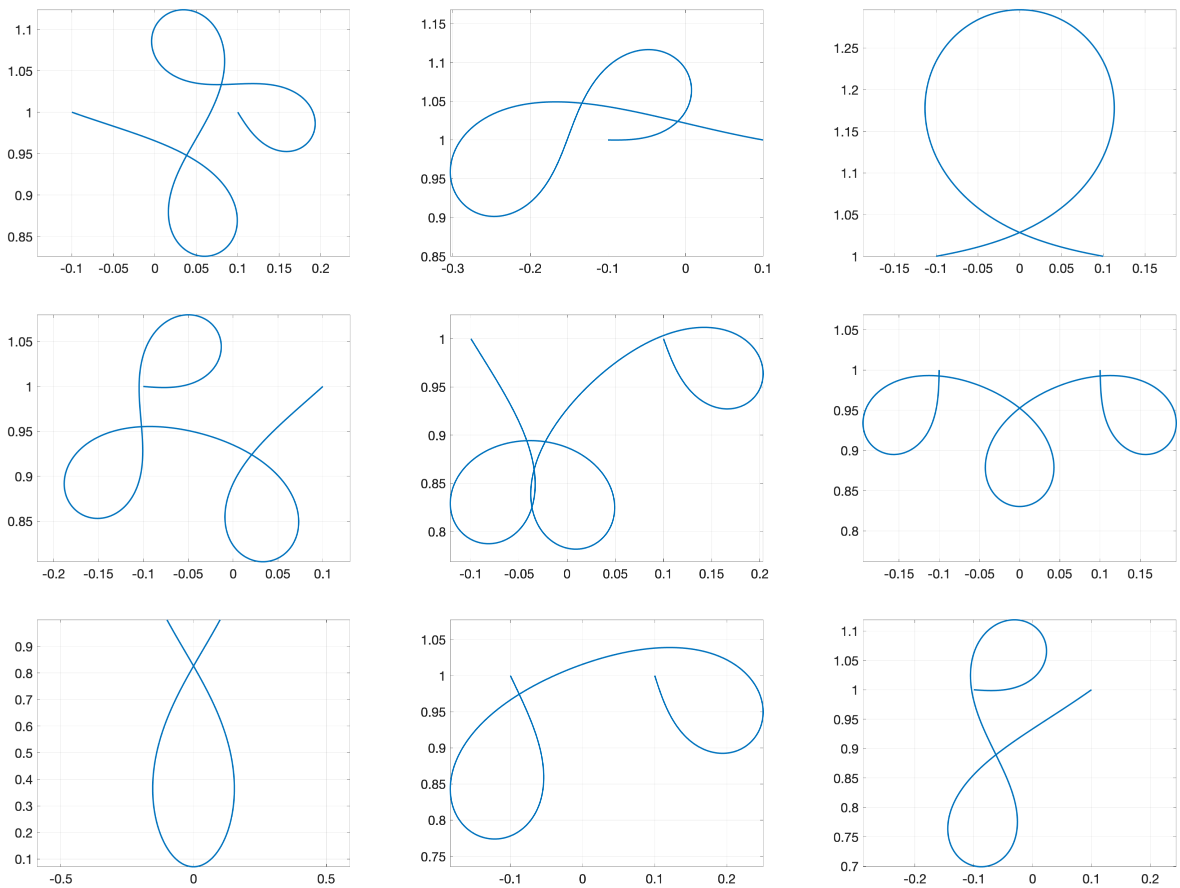

FIG. 2: A selection of self-intersecting numerically computed solutions of various mode numbers to the membrane shape equation with negative extension. For each plot, $h/a = -0.2$, $\bar{A} = 1$, $\bar{\kappa} = 0$, the horizontal axis is z/a, the vertical axis is r/a, and the axisymmetric surface is produced by revolving the curve about the line $r = 0$.

III. SINGULAR PERTURBATION THEORY FOR THE CASE OF $\bar{\kappa} \neq 0$

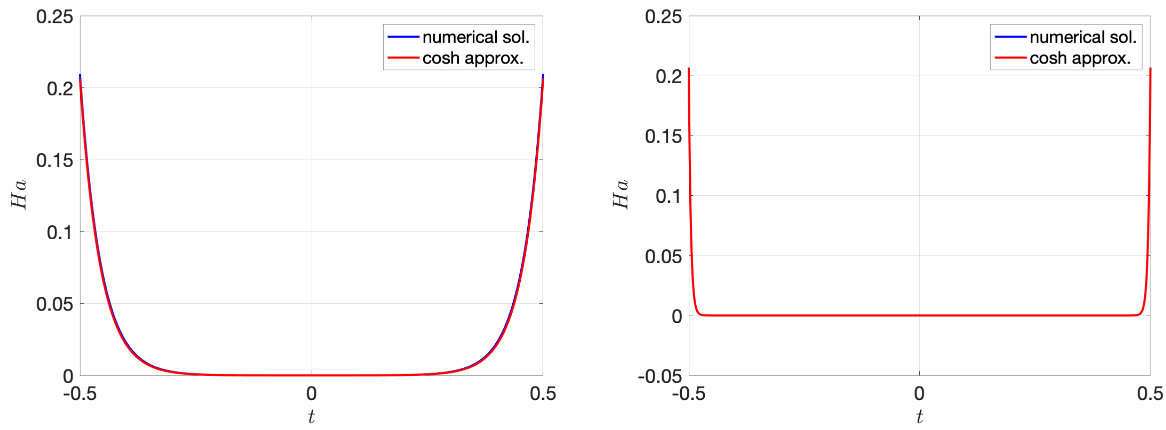

FIG. 3: Numerically computed (blue) dimensionless mean curvature and leading order asymptotic approximation (red, eqn (37) of the main text) for a catenoid-like shape with $\bar{\kappa}/\kappa = 0.5$ and $\bar{A} = 1$. Outside of boundary layers of width $O(\sqrt{\kappa/\mu})$, the mean curvature is exponentially small. (Left) A membrane with extension $h/(2a) = 0.52769$ and $\mu a^2/\kappa = 402$. (Right) A membrane with $h/(2a) = 0.52769739$ and $\mu a^2/\kappa = 3.85 \times 10^4$ (the two curves are nearly indistinguishable).

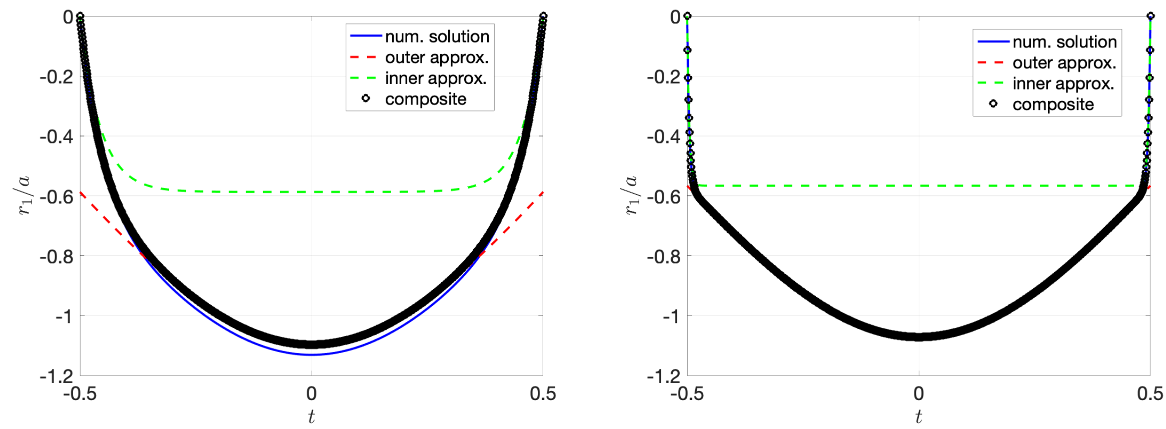

FIG. 4: Numerically computed perturbation to height profile r_1/a (blue) along with approximations valid outside the boundary layers (red, eqn (40) of the main text), inside the boundary layers (green, eqn (42) of the main text), and in the whole domain (black, eqn (44) of the main text) for the $\mu > 0$ case. Parameters are the same as in the previous figure. The area under the curve approaches $-a^2\bar{A}/L_0$ and constrains μ.

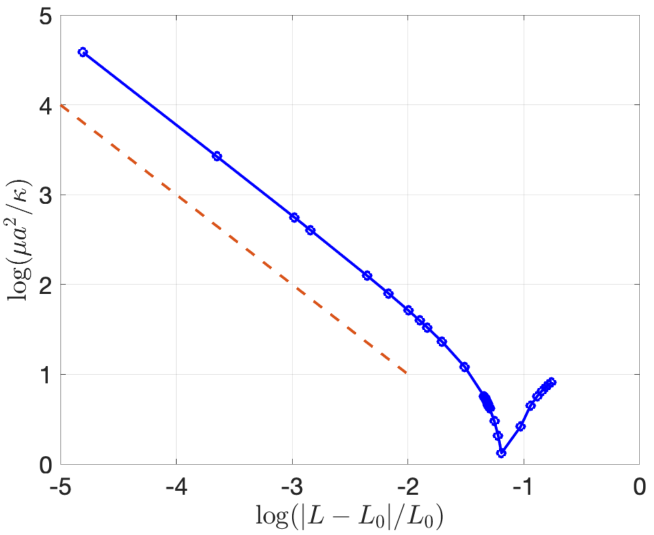

FIG. 5: Numerically computed dimensionless tension as a function of $\epsilon = |L - L_0|/L_0$ (as defined after eqn (27) of the main text) on a log-log scale (blue), for the positive tension case with $\bar{A} = 1$ and $\bar{\kappa}/\kappa = 0.5$. The slope of the dashed red line is -1. The logarithms have base 10.

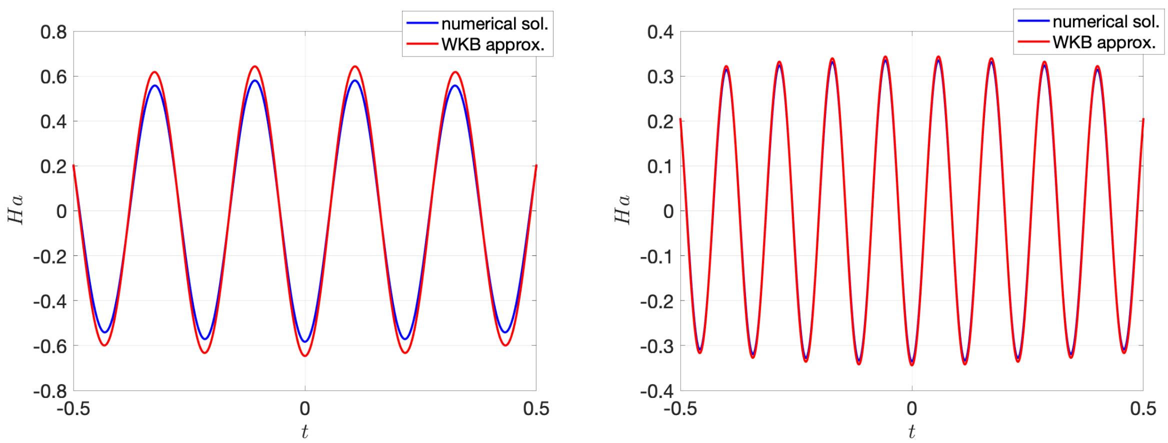

FIG. 6: Numerically computed (blue) dimensionless mean curvature and leading order WKB asymptotic approximation (red, eqn (37) of the main text) for a catenoid-like shape with $\bar{\kappa}/\kappa = 0.5$ and $\bar{A} = 1$. As $|\mu|$ increases, the solution oscillates more and more. (Left) A membrane with extension $h/(2a) = 0.5274$ and $\mu a^2/\kappa = -660$. (Right) A membrane with $h/(2a) = 0.52767$ and $\mu a^2/\kappa = -2.36 \times 10^3$. The two curves are nearly indistinguishable when μ is this large. Note that for both plots, $\cos(L_0\sqrt{\mu/\kappa}/2)$ is not close to zero.

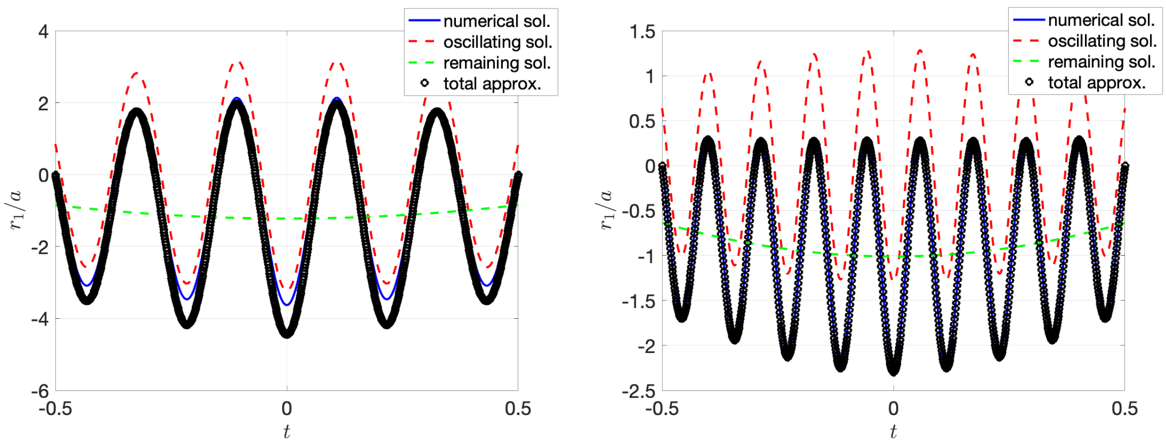

FIG. 7: Numerically computed perturbation to height profile r_1/a (blue) and leading order WKB approximation (black) composed of two parts: a response to a rapidly oscillatory forcing (red, eqn (51) of the main text) and a remainder (green, eqn (52) of the main text), for the $\mu < 0$ case. Parameters are the same as in the previous figure. The area under the curve constrains μ.

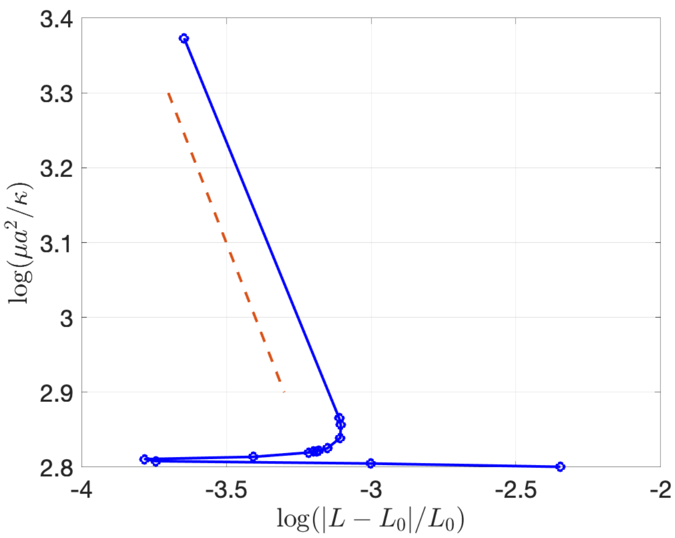

FIG. 8: Numerically computed dimensionless tension as a function of $\epsilon = |L - L_0|/L_0$ (as defined after eqn (27) of the main text) on a log-log scale (blue) for the negative tension case with $\bar{A} = 1$ and $\bar{\kappa}/\kappa = 0.5$. The slope of the dashed red line is -1. The logarithms have base 10.

IV. MERIDIANS OF THE $\bar{\kappa} = 0$ SURFACES

Here, we plot the meridians $r(z)$ of the zero Gaussian curvature modulus axisymmetric surfaces shown in Figs. 6, 8, 9, and 11 of the main text.

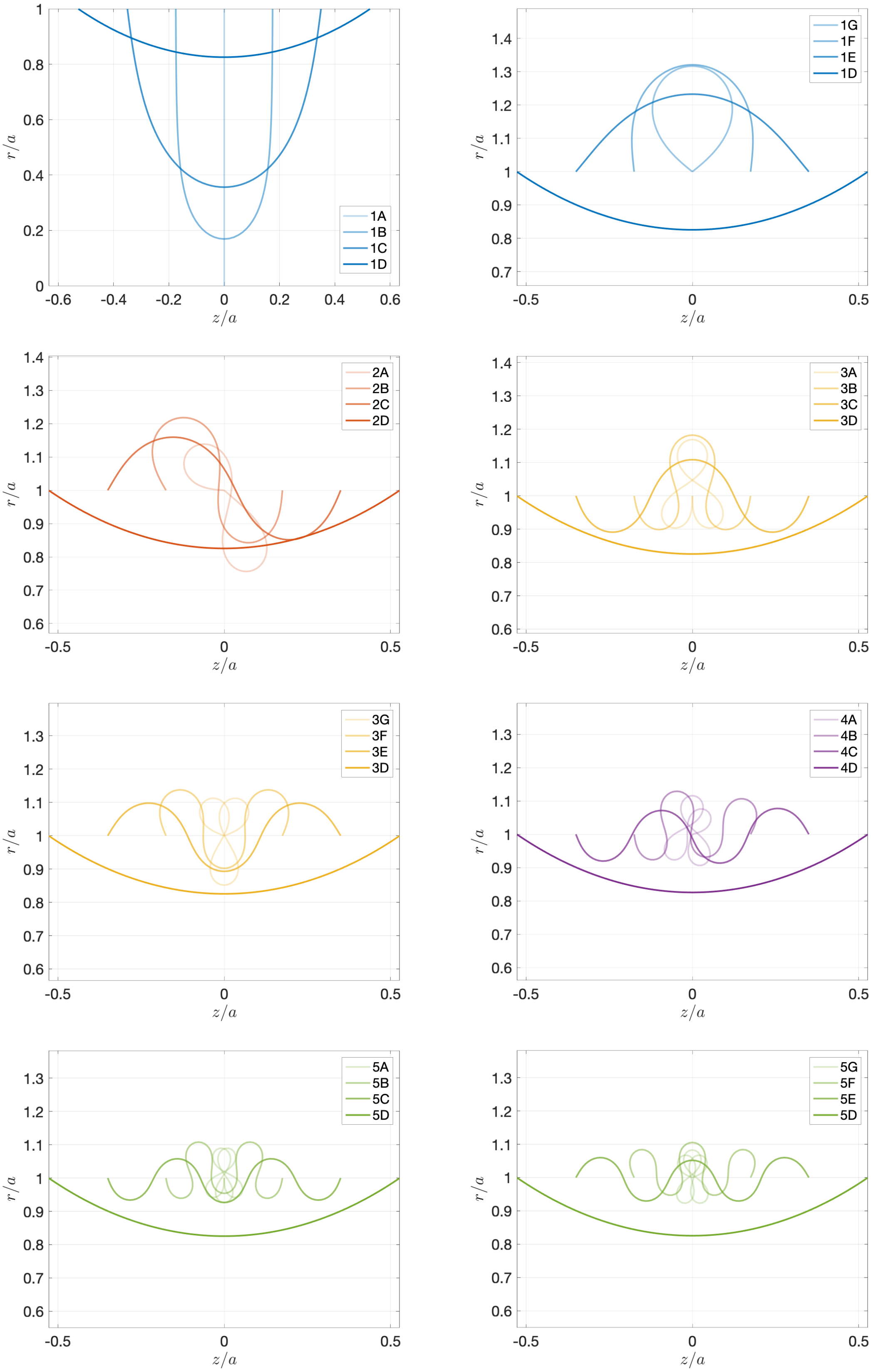

FIG. 9: Meridians of the surfaces with $\bar{A} = 1$ and $\bar{\kappa} = 0$ (cf. Figs. 6 and 13a of the main text).

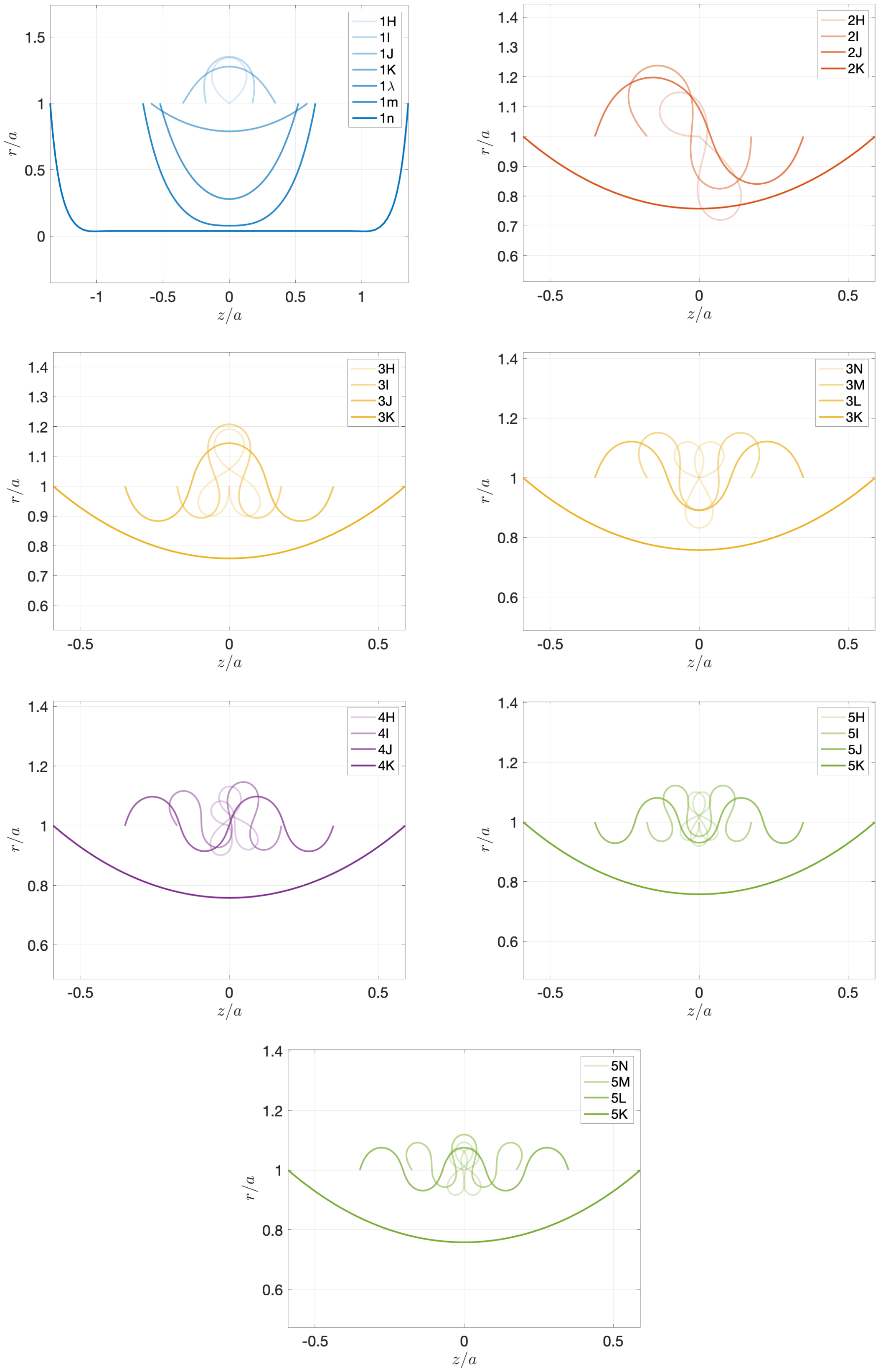

FIG. 10: Meridians of the thick surfaces with $\bar{A} = 1.1$ and $\bar{\kappa} = 0$ (cf. Fig. 8 of the main text).

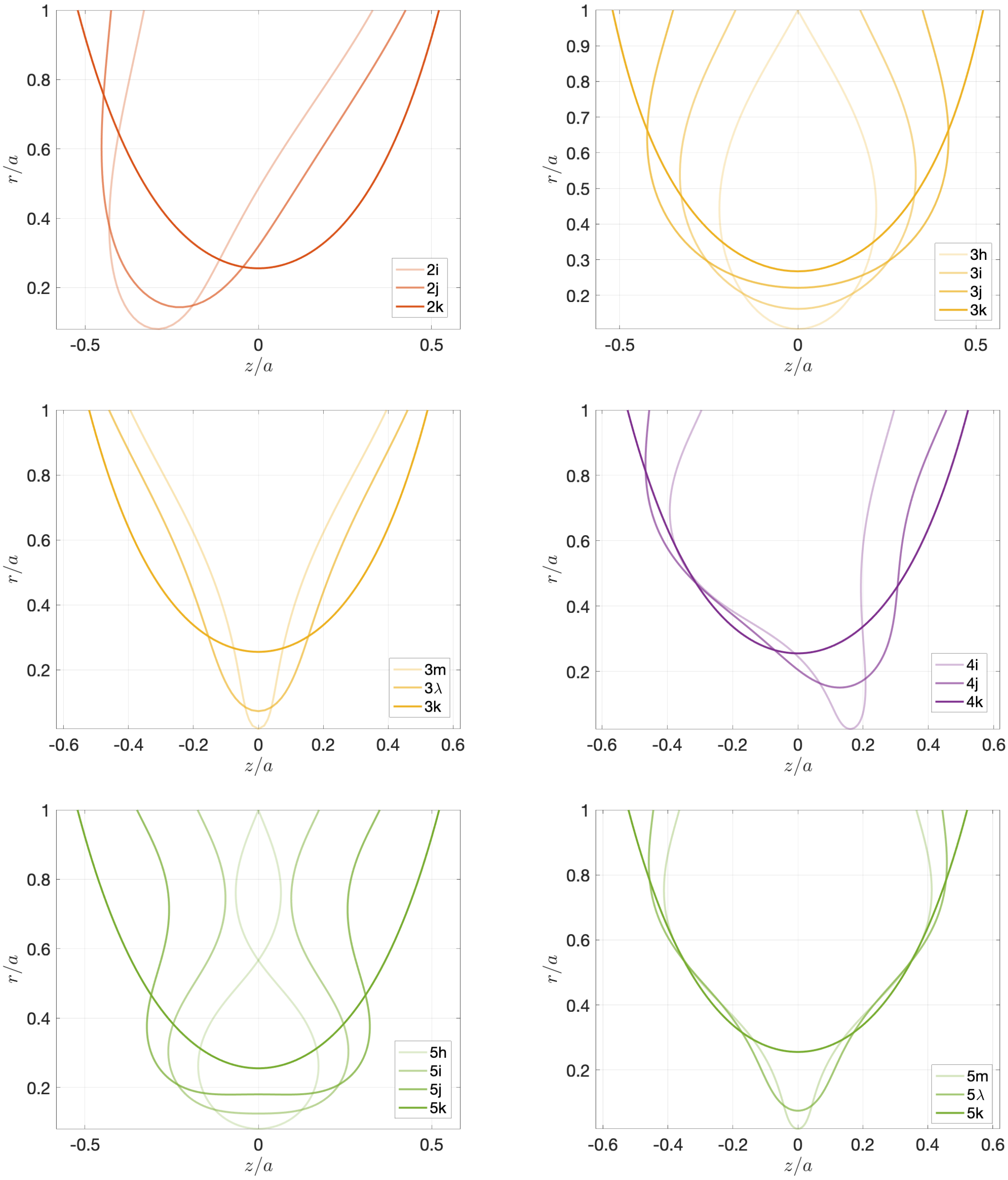

FIG. 11: Meridians of the thin surfaces with $\bar{A} = 1.1$ and $\bar{\kappa} = 0$ (cf. Fig. 9 of the main text).

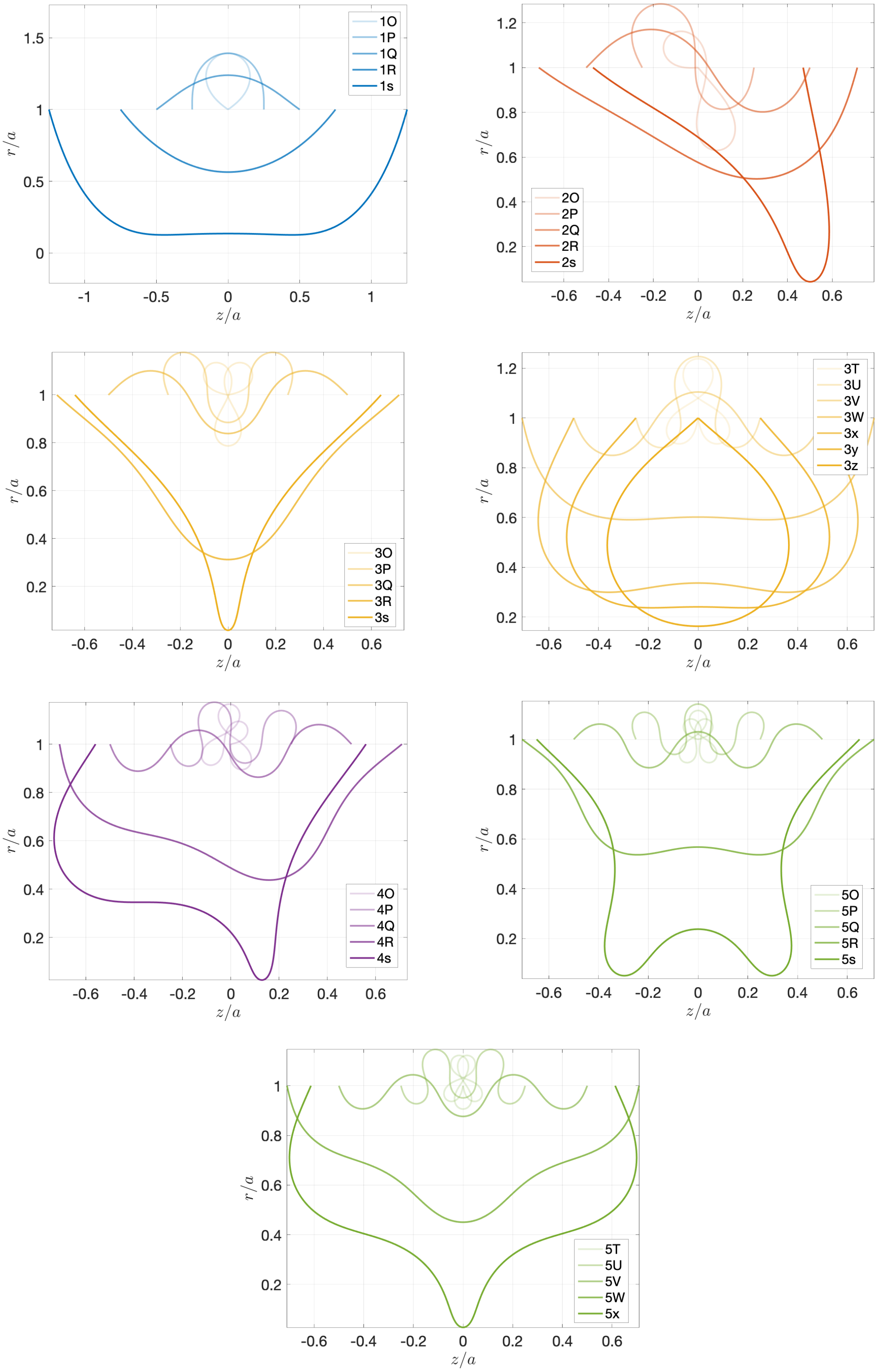

FIG. 12: Meridians of the surfaces with $\bar{A} = 1.3$ and $\bar{\kappa} = 0$ (cf. Fig. 11 of the main text).