

Supplementary Information for

## **The dynamics of freestanding films: predictions for poly(2-chlorostyrene) based on bulk pressure dependence and thoughtful sample averaging**

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### **LCL Equation of State Application to Bulk Poly(2-chlorostyrene): *PVT* Data Fitting for Free Volume Analysis and Modeling of the Local Density Profile**

In order to obtain free volume values,  $V_{\text{free}}(T,P) = V(T,P) - V_{\text{hc}}$ , for application in the Cooperative Free Volume (CFV) rate model's  $\tau(T,V_{\text{free}})$  expression, we first analyze the system's pressure, volume, temperature (*PVT*) data using the Locally Correlated Lattice (LCL) equation of state (EOS)<sup>1</sup> to obtain  $V_{\text{hc}}$ , the system's limiting volume at close packing. The LCL EOS expression is given by

$$\begin{aligned} \frac{P}{k_{\text{B}}T} = & \left(\frac{1}{v}\right) \ln \left[ \frac{V}{V - N_{\text{m}}rv} \right] + \left(\frac{3}{v}\right) \ln \left[ \frac{V - (N_{\text{m}}v/3)(r-1)}{V} \right] \\ & - \left(\frac{3}{v}\right) \left( \frac{(2r+1)^2}{(V/N_{\text{m}}v) - (1/3)(r-1)} \right) \times \left( \frac{\exp[-\varepsilon/k_{\text{B}}T] - 1}{(1/3)(2r+1)\exp[-\varepsilon/k_{\text{B}}T] + (V/N_{\text{m}}v) - r} \right) \end{aligned} \quad (\text{S1})$$

$k_{\text{B}}$  is the Boltzmann constant and  $N_{\text{m}}$  is the number of molecules. In practice, the pressure ( $P$ ) is calculated as a function of two independent variables: temperature ( $T$ ), and either the volume per molecule ( $V/N_{\text{m}}$ ), or the volume per mass ( $V/(N_{\text{m}}M_{\text{w}})$ ) given the molecular weight ( $M_{\text{w}}$ ). The molecular parameters are:  $r$ , the number of segments (occupied lattice sites) per molecule,  $v$ , the volume per lattice site, and  $\varepsilon$ , the segment-segment nonbonded interaction energy. The  $r, v, \varepsilon$  values are obtained by fitting Eqn S1 to *PVT* data, and this then leads to the characteristic volume at close-packing, which is given by  $V_{\text{hc}} = N_{\text{m}}rv$ , or equivalently in volume per gram,  $V_{\text{hc}} = rv/M_{\text{w}}$  where  $M_{\text{w}}$  is the molecular weight in g/mol and where  $v$  is in volume per mol.

We use  $PVT$  data for P2ClS ( $M_w = 58,000$  g/mol) from Roland et al.<sup>2</sup> and fit in a range of  $T = 403$  to  $463$  K and  $P = 0$  to  $100$  MPa, and obtain fitted parameter values of  $r = 5361$ ,  $v = 7.930$  mL/mol, and  $\varepsilon = -2244$  J/mol; this leads to  $V_{hc} = 0.7330$  mL/g. In our procedure (see for example refs<sup>3-6</sup>), when we apply this value of  $V_{hc}$  to calculate free volumes,  $V_{free}(T,P) = V(T,P) - V_{hc}$ , at each  $T,P$  point (e.g. at a chosen dynamics  $T,P$  data point), we use the LCL EOS  $V(T,P)$  value calculated at that  $T,P$ . At  $T,P$  values in or near the fitting range, the difference between the theoretical LCL volume and the actual experimental volume (if available) would of course be very small and so the caution is irrelevant, but when working outside the fitting range (where LCL volumes may start to differ somewhat from experiment), it is best to stay consistent, using the model's theoretical  $V(T,P)$  together with its corresponding model  $V_{hc}$ .

The LCL EOS, in combination with the square gradient approximation,<sup>7-9</sup> was also applied to calculate the local density profile for the P2ClS free surface based on the above characterization of the P2ClS  $PVT$  behavior. Here the Helmholtz free energy density at a given position,  $z$ , near the free surface is given by  $a(\rho(z)) = a_0(\rho(z)) + \kappa(\nabla\rho(z))^2$ .  $a_0(\rho(z))$  is the homogenous contribution, and  $\kappa(\nabla\rho(z))^2$  is the gradient term (with parameter  $\kappa$ ) that accounts for the inhomogeneous contribution coming from the locally varying density. The LCL EOS is used to calculate  $a_0(\rho(z))$  for the given  $\rho$  value. Given a value for the parameter  $\kappa$ , the surface tension,  $\sigma$ , and local density profile,  $\rho(z)$ , can then be calculated by integrations carried out over the variable,  $\rho$ ; see for example the details described in refs<sup>7-9</sup>.

Here for P2ClS, we first fit the value of  $\kappa$  such that it leads to agreement with the experimental surface tension, and once this  $\kappa$  value is obtained we then use it to calculate the corresponding P2ClS density profile, as shown for example in Fig. 3 in the main article (for  $T = 425$  K). The experimental surface tension for P2ClS was approximated based on values for polymer critical surface tension that have been tabulated in Lee.<sup>10</sup> The value for P2ClS (42 mN/m) is found to be somewhat higher than that for PS (36 mN/m), an observation that correlates with the former having a somewhat higher cohesive energy and  $T_g$  value. The critical surface tension (measured using solvent contact angles) is typically similar to, but still distinct from, the surface tension measured on the melt e.g. by the pendant drop technique. Therefore we obtained an estimate of latter for P2ClS by taking the available experimental PS surface tension (30.6 mN/m

at  $T = 433\text{K}$ )<sup>11</sup> and scaling it by the ratio of the P2CLS to PS critical surface tension (42/36) which thus gives the approximate P2CLS surface tension of about 35.7 mN/m at 433K which is the value that was used to obtain  $\kappa$ .

## References

- (1) White, R. P.; Lipson, J. E. G. Polymer Free Volume and Its Connection to the Glass Transition. *Macromolecules* **2016**, *49* (11), 3987–4007. <https://doi.org/10.1021/acs.macromol.6b00215>.
- (2) Roland, C. M.; McGrath, K. J.; Casalini, R. Dynamic Heterogeneity in Poly(Vinyl Methyl Ether)/Poly(2-Chlorostyrene) Blends. *Macromolecules* **2006**, *39* (10), 3581–3587. <https://doi.org/10.1021/ma060315k>.
- (3) White, R. P.; Lipson, J. E. G. The Cooperative Free Volume Rate Model for Segmental Dynamics: Application to Glass-Forming Liquids and Connections with the Density Scaling Approach. *Eur. Phys. J. E* **2019**, *42* (8), 100. <https://doi.org/10.1140/epje/i2019-11862-3>.
- (4) White, R. P.; Lipson, J. E. G. Connecting Pressure-Dependent Dynamics to Dynamics under Confinement: The Cooperative Free Volume Model Applied to Poly(4-Chlorostyrene) Bulk and Thin Films. *Macromolecules* **2018**, *51* (20), 7924–7941. <https://doi.org/10.1021/acs.macromol.8b01392>.
- (5) White, R. P.; Lipson, J. E. G. The Cooperative Free Volume Rate Model Applied to the Pressure Dependent Segmental Dynamics of Natural Rubber and Polyurea. *Rubber Chem. Technol.* **2019**, in press.
- (6) White, R. P.; Lipson, J. E. G. A Simple New Way To Account for Free Volume in Glassy Dynamics: Model-Free Estimation of the Close-Packed Volume from PVT Data. *J. Phys. Chem. B* **2021**, *125* (16), 4221–4231. <https://doi.org/10.1021/acs.jpcc.1c01620>.
- (7) Sanchez, I. C. *Physics of Polymer Surfaces and Interfaces*; Butterworth: Greenwich, CT, 1992.
- (8) Poser, C. I.; Sanchez, I. C. Surface-Tension Theory of Pure Liquids and Polymer Melts. *J. Colloid Interface Sci.* **1979**, *69* (3), 539–548. [https://doi.org/10.1016/0021-9797\(79\)90142-5](https://doi.org/10.1016/0021-9797(79)90142-5).
- (9) Dee, G.; Sauer, B. The Surface Tension of Polymer Liquids. *Adv. Phys.* **1998**, *47* (2), 161–205. <https://doi.org/10.1080/000187398243546>.
- (10) Lee, L. Relationships Between Surface Wettability and Glass Temperatures of High Polymers. *J. Appl. Polym. Sci.* **1968**, *12* (4), 719–730. <https://doi.org/10.1002/app.1968.070120410>.
- (11) Wu, S. Surface and Interfacial Tensions of Polymer Melts .2. Poly(Methyl Methacrylate), Poly(Normal-Butyl Methacrylate), and Polystyrene. *J. Phys. Chem.* **1970**, *74* (3), 632-. <https://doi.org/10.1021/j100698a026>.