I. ANALYSIS FOR THE YY- AND YZ-COMPONENT OF THE DIFFUSION COMPONENT

In the main document of the manuscript, our analysis focuses on the z-component of the diffusivity tensor. Here, it is extended to two additional components \( D_{yy} \) and \( D_{yz} \). The components related to the advection direction, \( x \), have been excluded, due to the non-affine displacements could not simply be isolated by subtracting the displacement that results from integrating the individual particle velocities. Here, we could argue that the non-constant shear rate introduces a Taylor dispersion that we were not able to eliminate using the mentioned procedure.

As a first step, we examine the particles’ mean squared displacements in the \( yy \)- and \( yz \)-directions (MSD\(_{yy}\) and MSD\(_{yz}\)) as a function of the time-lag \( \tau \). The expressions that describe MSD\(_{yy}\) and MSD\(_{yz}\) are respectively,

\[
\text{MSD}_{yy}(\tau) = \frac{1}{N_k} \sum_{i} (y_i(\tau + t) - y_i(t))^2
\]

\[
\text{MSD}_{yz}(\tau) = \frac{1}{N_k} \sum_{i} (y_i(\tau + t) - y_i(t))(z_i(\tau + t) - z_i(t))
\]

averaged over specific particle ensembles with \( N_k \) particles. The rest of the procedure is carried out as in the manuscript. That means the system is split in vertical direction into 20 blocks of width \( \Delta H = L \) in terms of the particle’s longest size \( L \). Moreover, the particles are tracked for a total time elapsed of \( \tau_T = 120 \) s with a resolution \( \delta \tau = 0.05 \) s.

Fig. 1 shows examples for both a,c) MSD\(_{yy}\)(\( \tau \)) and b,d) MSD\(_{yz}\)(\( \tau \)) outcomes (in log–log scale) for a subset of the sections labeled with \( H \) representing the region height \( z = H \). In addition, the same data but using linear scale are displayed in the insets. The results correspond to the systems a,b) aspect ratio \( \xi = 1.3 \) and inclination \( \alpha = 27^\circ \) and c,d) aspect ratio \( \xi = 2.5 \) and inclination \( \alpha = 30^\circ \). As a guide to the eye, the graphs include continuous lines representing power-law functions with exponent 2 and 1 which stand for the ballistic and diffusive limit, respectively. Similarly to the diffusivity in the \( z \) direction, MSD curves change their behavior close to \( \tau = 1 \) s. In detail, for \( \tau < 1 \) s, the movement of the particles is ballistic; however, although there is reduction of the slopes, the movement remains superdiffusive for \( \tau > 1 \) s. This result is more noticeable in the Fig. 1a) and differs from the obtained ones in the \( z \)-direction. For this reason, in these cases, we expect \textit{a priori} no well-defined diffusivity values.

In any case, Fig. 2a and Fig. 2b exemplify the profiles of \( D_{yy}(z) \) and \( \dot{\gamma}(z) \) (and \( D_{yz}(z) \) and \( \dot{\gamma}(z) \)) normalized by the maximum diffusion coefficient \( D_{max} \) and the maximum shear rate \( \dot{\gamma}_{max} \) for \( \xi = 1.3 \) and \( \xi = 2.5 \), respectively. In both cases, we obtain that local diffusivity values do not correlate with the local shear rate values. Consequently, the simple scaling relation \( D \sim d^2 \dot{\gamma} \) does not seem to be a good choice.
FIG. 1: The mean squared displacements (MSD) vs. $\tau$ for the cases a,b) $\xi = 1.3$, $\alpha = 27^\circ$ and c,d) $\xi = 2.5$, $\alpha = 30^\circ$ (both in log-log scales) in the a,c) yy- and b,d) yz-direction. Each curve represents the MSD calculated for each block of base equivalent to the system and width $L$ whereas lines show the powers 1 and 2. In both cases, the inset shows the same but on linear scale.
FIG. 2: Plots of a) $D_{yy}/D_{yy}^{max}$, b) $D_{yz}/D_{yz}^{max}$ (markers) and $\dot{\gamma}/\dot{\gamma}_{max}$ (curves) against the height $z$ for $\xi = 1.3$ and several inclinations, $\alpha = [26.0, 26.5, 27.0, 27.5, 28.0]^{\circ}$. c) and d) respectively display the same quantities but $\xi = 2.5$ and $\alpha = [29.0, 29.5, 29.0, 29.5, 30.0]^{\circ}$.