Supplementary Information – Torsional instability of constant viscosity elastic liquid bridges

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1. Numerical mesh and time step convergence tests

Fig. S1(a) and (b) demonstrate the numerical mesh and time step convergence, respectively. In the mesh convergence study, we use three consecutively doubled meshes (see Table below) to probe the spatial accuracy of the numerical scheme. The mesh is highly packed around the midplane of the liquid bridge ($z = 1.5$ mm) to ensure high resolution of the free surface curvature and the stresses when indent formation arises (see ESI, Video S1 and S2). In the time step convergence study, we employ three consecutively halved dimensionless time steps ($dt = 0.02\Omega, 0.01\Omega, 0.005\Omega$) to evaluate the temporal accuracy of the simulations. The simulations are performed for $\Omega = 1$ rad/s and the minimum liquid bridge radius (neck radius) R is monitored. The results are mesh-independent and converge with time step refinement. Mesh M2 was chosen for all simulations due to its slightly superior accuracy over mesh M1 but lower computational cost compared with mesh M3. In the same spirit, $dt = 0.01\Omega$ was chosen for the rest of the simulations. Each simulation with M2 and $dt = 0.01\Omega$ takes about 4 hours when performed in parallel using 128 cores.

2. Distributions of stress along the liquid bridge free surface

Fig. S2-S4 contain the simulated profile of the liquid bridge free surface, as well as the distributions of the dimensionless polymeric radial normal stress Σ_{rr}^*, axial normal stress Σ_{zz}^*, azimuthal normal stress $\Sigma_{\theta\theta}^*$ and shear stresses Σ_{rz}^* and $\Sigma_{r\theta}^*$ along the liquid bridge free surface, at different times t and rotational speeds Ω. The symbol Σ is used here to distinguish the polymeric stress tensor $\mathbf{\Sigma} = G(\mathbf{C} - \mathbf{I})$ from the total stress tensor $\mathbf{T} = -P\mathbf{I} + \mathbf{\Sigma} + \eta_s\dot{\boldsymbol{\gamma}}$, where G is the elastic modulus, $\mathbf{C}$ is the conformation tensor, $\mathbf{I}$ is the unit tensor, P is the thermodynamic pressure, η_s is the solvent viscosity and $\dot{\boldsymbol{\gamma}}$ is the deformation rate tensor of the fluid.

Mesh	# elements	# nodes	Min. element edge
M1	38168	19065	$4\cdot10^{-5}\ R_p$
M2	150900	75411	$2\cdot10^{-5}\ R_p$
M3	603123	301482	$1\cdot10^{-5}\ R_p$

3. Effect of the imposed rotational speed on the liquid bridge thinning speed

Functional relationship between the thinning speed of the liquid bridge and the imposed rotational speed can be obtained if we assume that the time evolution of the neck radius R can be approximated as a power-law decay $R \propto t^{-\beta}$. Such assumption is true for 0.5 mm $\lesssim R \lesssim$ 0.7 mm, as shown in Fig. S5(a). Locally at any point in that range of R, the elastocapillary number Ec is of the same value over all imposed rotational speeds Ω. Hence, the power-law index β can be treated as a function of the Tanner number Tn alone. Fig. S5(b) shows β as a function of the characteristic Tanner number $\mathrm{Tn_c} \equiv \mathrm{Tn}|_{R=R_\mathrm{p}}$. The relation between β and $\mathrm{Tn_c}$ can be fitted empirically as $\beta = 0.12\ln(\mathrm{Tn_c}) + 0.097$ (see black broken line). Such dependence of β on $\ln(\mathrm{Tn_c})$ agrees with our previous study [1], except that the numerical values are different.

4. Caption of video S1

Time evolution of the mesh for $\Omega = 10$ rad/s. The video start at $\Omega t = 90$ and each frame corresponds to a time interval of $\Omega\Delta t = 0.1$.

5. Caption of video S2

Zoomed-in view of the time evolution of the mesh for $\Omega = 10$ rad/s. The video start at $\Omega t = 90$ and each frame corresponds to a time interval of $\Omega\Delta t = 0.1$.

6. Caption of video S3

Time evolution of a viscoelastic liquid bridge subjected to a $\Omega = 50$ rad/s rotation, which corresponds to Fig. 2(c) in the main manuscript.

7. Caption of video S4

Time evolution of a viscoelastic liquid bridge subjected to a $\Omega = 200$ rad/s rotation, which corresponds to Fig. 2(d) in the main manuscript.

8. Reference

[1]: S. T. Chan, F. P. van Berlo, H. A. Faizi, A. Matsumoto, S. J. Haward, P. D. Anderson and A. Q. Shen, *Proc. Natl. Acad. Sci. U.S.A.*, 2021, **118**, e2104790118.

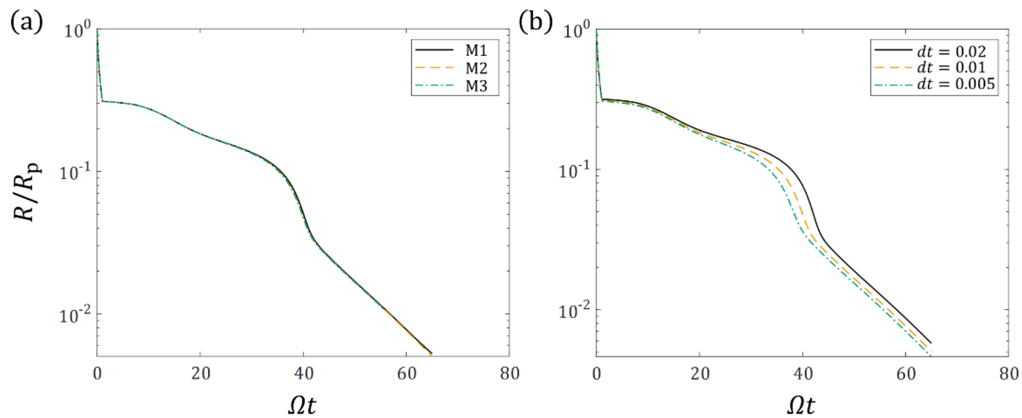

Fig. S1: Simulated time evolution of the dimensionless neck radius R/R_p of the viscoelastic liquid bridge of height $H = 3$ mm subjected to a rotational speed of $\Omega = 1$ rad/s with different meshes and time steps used. (a) Simulation results with mesh M1, M2 and M3. (b) Simulation results with dimensionless time step $dt = 0.02\Omega$, 0.01Ω and 0.005Ω.

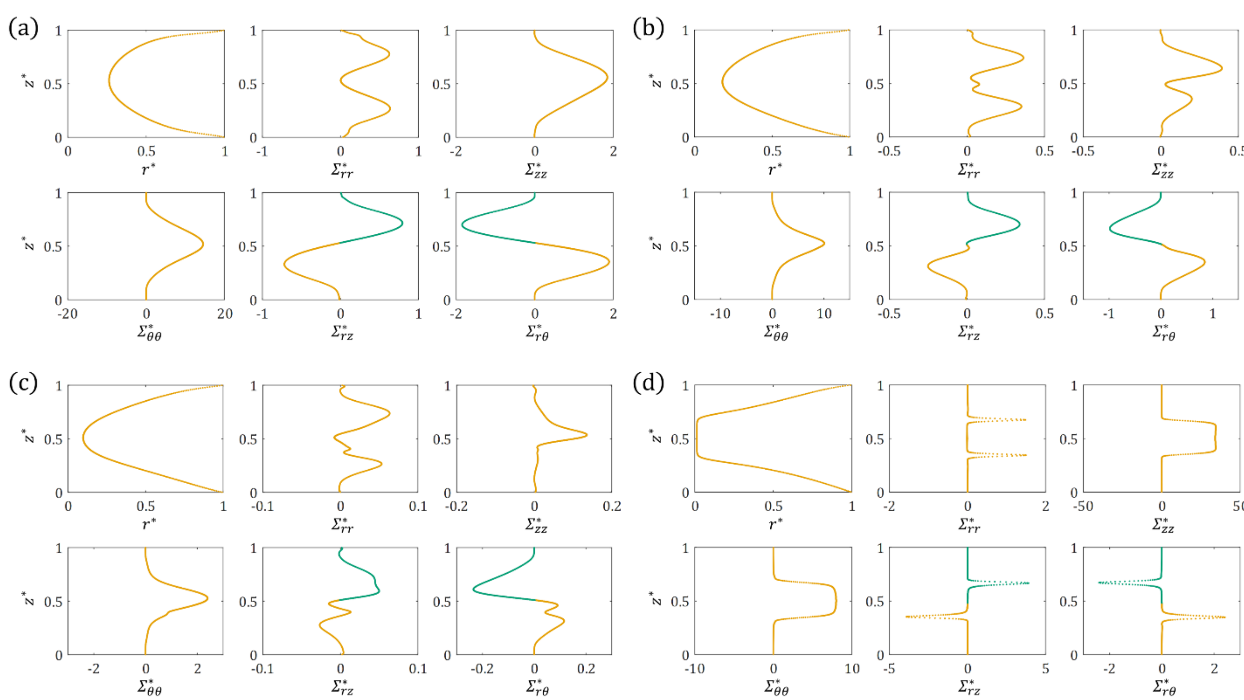

Fig. S2: Simulated profile of the liquid bridge free surface and stress distributions along the free surface. The imposed rotational rate is $\Omega = 1$ rad/s. (a) $t = 5.85$ s, (b) $t = 15.85$ s, (c) $t = 31.85$ s, (d) $t = 47.85$ s.

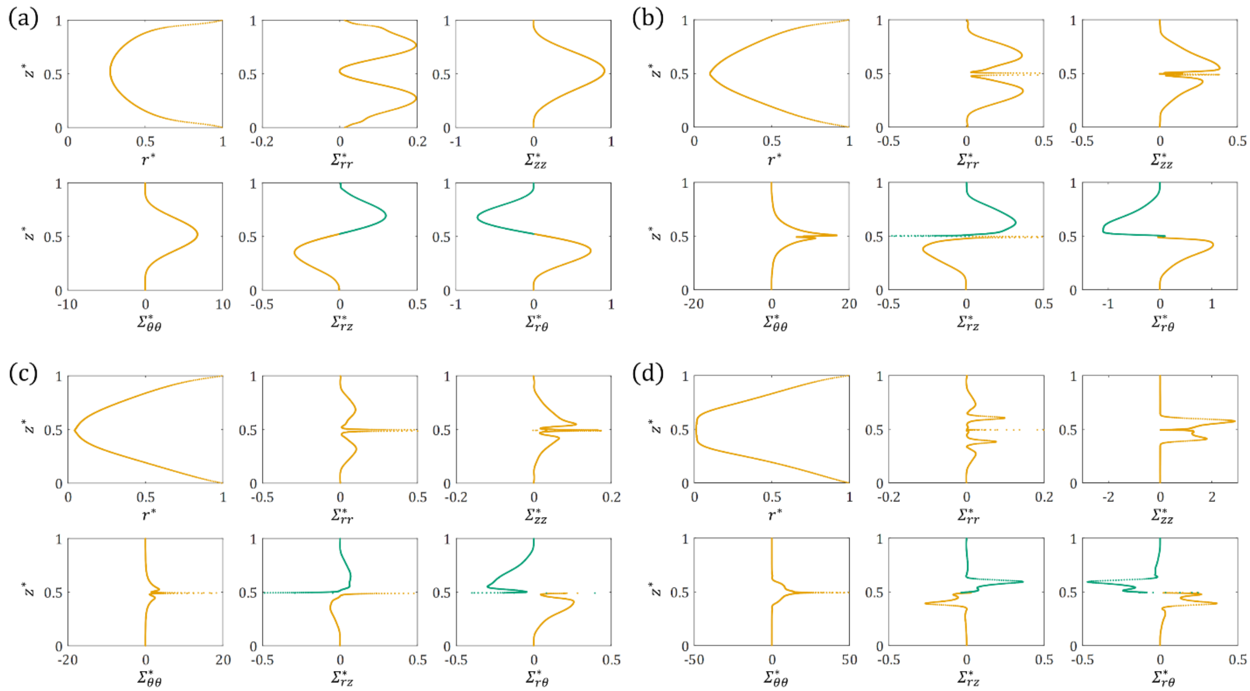

Fig. S3: Simulated profile of the liquid bridge free surface and stress distributions along the free surface. The imposed rotational rate is $\Omega = 5$ rad/s. (a) $t = 1.05$ s, (b) $t = 10.25$ s, (c) $t = 20.25$ s, (d) $t = 30.25$ s.

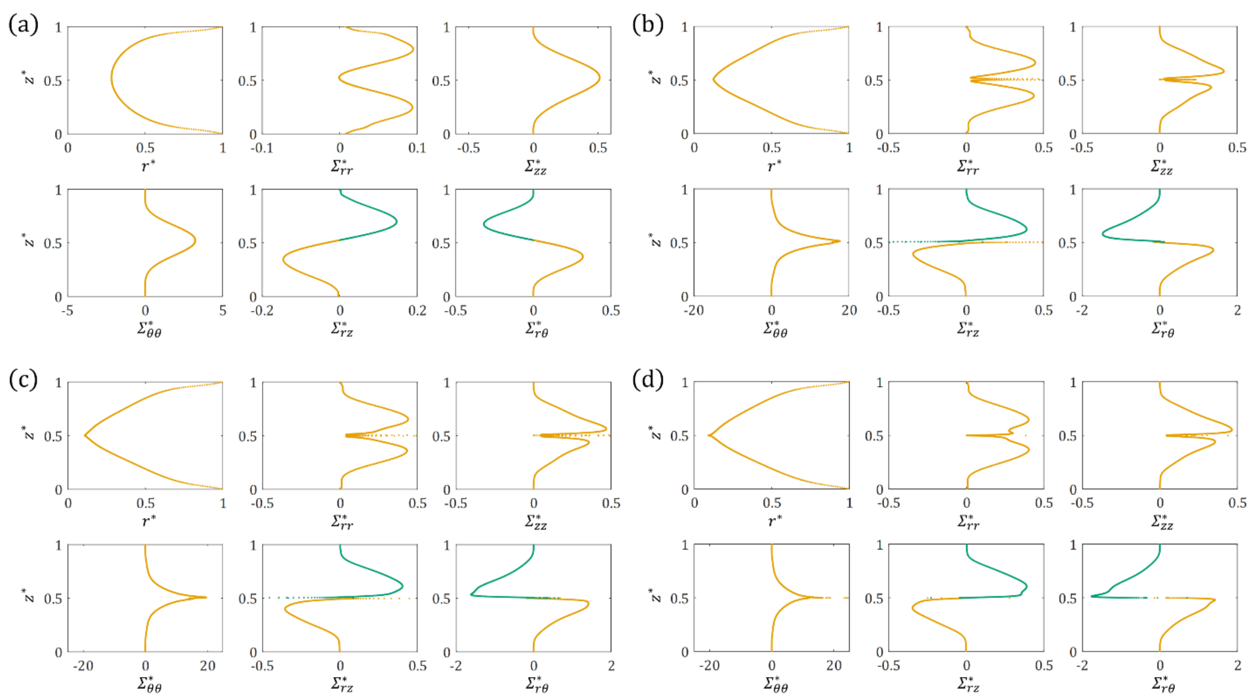

Fig. S4: Simulated profile of the liquid bridge free surface and stress distributions along the free surface. The imposed rotational rate is $\Omega = 10$ rad/s. (a) $t = 0.45$ s, (b) $t = 2.75$ s, (c) $t = 3.95$ s, (d) $t = 5.15$ s.

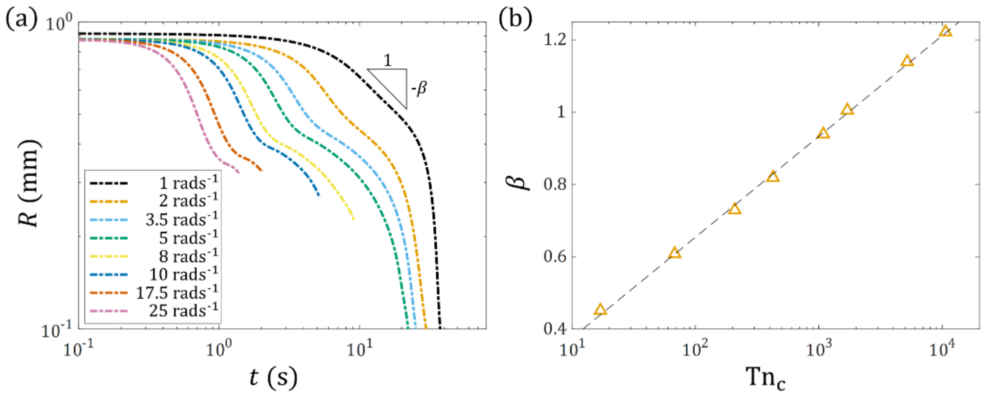

Fig. S5: (a) Simulated time evolution of the neck radius R of the viscoelastic liquid bridge of height $H = 3$ mm subjected to different rotational speeds Ω. For 0.5 mm $\lesssim R \lesssim$ 0.7 mm the time evolution can be approximated as a power-law decay $R \propto t^{-\beta}$. (b) Power-law index β as a function of the characteristic Tanner number $\mathrm{Tn_c} \equiv \mathrm{Tn}|_{R=R_\mathrm{p}}$. The function relation can be fitted empirically as $\beta = 0.12\ln(\mathrm{Tn_c}) + 0.097$ (black broken line).