Supplementary Information

Bonding Heterogeneity in Mixed-Anion Compounds Realizes Ultralow Lattice Thermal Conductivity

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Figure S1. The total charge density of (a) MnSbS$_2$Cl and (b) MnBiS$_2$Cl with isosurface value of 0.05 e bohr$^{-3}$, and the electron localization function (ELF) of (c) MnSbS$_2$Cl and (d) MnBiS$_2$Cl with isosurface value of 0.80 for (c) and 0.72 for (d), respectively. The black, red, purple, yellow, and green atoms represent Mn, Sb, Bi, S, and Cl, respectively.
Figure S2. The calculated potential energy of each atomic site as a function of displacements along cartesian $x$-, $y$-, and $z$-directions for MnSb$_2$S$_2$Cl, MnBi$_2$S$_2$Cl, and CuTaS$_3$ with fitting curves using only quadratic term.
Figure S3. The calculated potential energy of each atomic site as a function of displacements along cartesian $x$-, $y$-, and $z$-directions for MnSbS$_2$Cl, MnBiS$_2$Cl, and CuTaS$_3$. 


**Figure S4.** The calculated mean square displacement as a function of temperature along cartesian x-, y-, and z-directions for MnSbS$_2$Cl, MnBiS$_2$Cl, and CuTaS$_3$. 
Figure S5. The calculated phonon group velocity as a function of the phonon frequency for MnSbS$_2$Cl, MnBiS$_2$Cl, and CuTaS$_3$. 
Figure S6. The calculated spectral lattice thermal conductivity, $\kappa_{\text{spec}}$, as a function of the phonon frequency for MnSbS$_2$Cl, MnBiS$_2$Cl, and CuTaS$_3$. 
Figure S7. The XRD patterns of the polycrystalline bulk samples prepared by SPS for MnSbS$_2$Cl, MnBiS$_2$Cl, and CuTaS$_3$, together with their simulated patterns. Black arrows denote unknown impurity phases.

Table S1. Lattice parameters of the relaxed primitive cell of MnSbS$_2$Cl, MnBiS$_2$Cl, and CuTaS$_3$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$a$ (Å)</th>
<th>$b$ (Å)</th>
<th>$c$ (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MnSbS$_2$Cl</td>
<td>9.1800</td>
<td>3.7704</td>
<td>12.039</td>
</tr>
<tr>
<td>MnBiS$_2$Cl</td>
<td>9.1672</td>
<td>3.8346</td>
<td>12.111</td>
</tr>
<tr>
<td>CuTaS$_3$</td>
<td>9.3918</td>
<td>3.4628</td>
<td>11.688</td>
</tr>
</tbody>
</table>