Theoretical study on stability and ion transport property with halide doping of Na₃SbS₄ electrolyte for all-solid-state battery

Randy Jalem^{a,b,c,*}, Bo Gao,^a Hong-Kang Tian,^a and Yoshitaka Tateyama^{a,c,*}

^aCenter for Green Research on Energy and Environmental Materials (GREEN), National Institute for Materials Science (NIMS), Tsukuba, Japan. E-mail: JALEM.Randy@nims.go.jp; Tel: +81-029-8604636 (ex. 4636).

^bPRESTO, Japan Science and Technology Agency (JST), Saitama, Japan.

^cElements Strategy Initiative for Catalysts & Batteries, Kyoto University, Kyoto, Japan.

Composition	Initial	a1 / Å	a ₂ / Å	a3 / Å	a ₃ /a ₁	a ₃ /a ₂
	structure					
Na ₃ SbS ₄	Tetragonal	7.2207	7.2207	7.3729	1.0211	1.0211
	(unit cell)					
Na _{2.8750} SbS _{3.8750} Cl _{0.1250}	Tetragonal	7.2430	7.2574	7.3682	1.0173	1.0153
	(2x2x2)					
Na _{2.9375} SbS _{3.9375} Cl _{0.0625} *	-	7.176	7.176	7.183	1.0001	1.0001

Table S1. Comparison of DFT lattice parameters with tetragonal Na_3SbS_4 as the initial structure for undoped and doped models.

*Experimental data (F. Tsuji et al., J. Ceram. Soc. Jpn 2020, 128, 641-647).

Table S2. Comparison between DFT results (in this work) and experiment (in parenthesis, based from Ref. 33 of main text) for the structural information of t-Na₃SbS₄.

Atom pair	Bond distance / Å	%diff.
Na1-S(1)	2.91 (2.87)	1.3
Na1-S(2)	2.91 (2.87)	1.3
Na1-S(3)	3.04 (3.02)	0.7
Na1-S(4)	3.04 (3.02)	0.7
Na1-S(5)	3.10 (3.06)	1.3
Na1-S(6)	3.10 (3.06)	1.3
Na2-S(1)	3.04 (3.01)	1.3
Na2-S(2)	3.04 (3.01)	1.3
Na2-S(3)	3.04 (3.01)	1.3
Na2-S(4)	3.04 (3.01)	1.3
Na2-S(5)	3.45 (3.42)	0.8
Na2-S(6)	3.45 (3.42)	0.8
Sb-S	2.36 (2.31)	2.3

Table S3. Variation	of DFT-calculated	lattice constants with	halide anion dop	oing on c-Na ₃ SbS ₄ .
---------------------	-------------------	------------------------	------------------	--

DFT Composition	Ave. lattice constant / Å (GGA-PBE @0 K)	%diff. (undoped vs. doped)
c-Na ₃ SbS ₄ (cubic)	7.2466	-
Na _{2.875} SbS _{3.875} F _{0.125}	7.2775	0.43
Na _{2.875} SbS _{3.875} Cl _{0.125}	7.2900	0.60
Na _{2.875} SbS _{3.875} Br _{0.125}	7.2887	0.58
Na _{2.875} SbS _{3.875} I _{0.125}	7.3100	0.87



Figure S1. Schematic illustration of $2 \times 2 \times 2 \text{ c}$ - Na₃SbS₄ supercell model showing the SbS₄ tetrahedral units (brown) and 2 S sites substituted by halide dopant atoms (blue).



Figure S2. DFT energy difference between cubic and tetragonal structure as a function of number of vacancies. At each cell with a given number of Na vacancies (i.e., 1, and 2), a structure configuration sampling was performed followed by DFT geometry optimization (10 undoped tetragonal structures, 10 undoped cubic structures, 5 Cl-doped tetragonal structures, and 5 Cl-doped cubic structures). The lowest-energy structures for each cases were taken for the calculation of the energy difference between the cubic and tetragonal structure (i.e., $E_{cubic} - E_{tetragonal} / eV/cell$).



Figure S3. DFT-calculated total and partial density of states (DOS) of t-Na₃SbS₄ (P42₁c).

Table S4. Various phases in the Na-Sb-S system and their DFT-calculated formation energy (E_f). First entry (in bold letters) is for t-Na₃SbS₄ (cell composition: Na₆Sb₂S₈).

Ce	ell compositi	on	Er / eV/atom	Ce	ll compositi	on	E _f / eV/atom	Ce	ll compositi	on	Ef / eV/atom
Na6	Sb2	S8	-1.09905	Na8			0.010761			S56	0.038013
Na12	Sb4	S12	-1.15974	Na4			0.003017			S32	0.04484
Na2	S4	Sb2	-1.01534	Na3			0.000231			S88	0.028838
Na3	S4	Sb1	-1.09459	Na2			0			S20	0.032116
Na2	Sb2	S4	-0.99488	Na8			0.069048			S28	0.020189
Na8	Sb8		-0.32833	Na1			0.000106			S36	0.016808
Na6	Sb2		-0.43777	Na1			0.002694			S28	0.041008
Na4		S2	-1.19118	Na1			1.072084			S48	0.001343
Na8		S4	-1.25482		Sb2		0.280245			S24	0.015913
Na2		S1	-1.29437		Sb2		0.065751			S80	0.014414
Na6		S6	-1.16219		Sb1		0.324607			S104	0.028873
Na1		S1	-0.79163		Sb4		0.261254			S72	0.020778
Na4		S4	-1.16472		Sb4		0.114852			S72	0.037362
Na1		S1	-0.66411		Sb1		0.311715			S18	0.355
	Sb8	S12	-0.65163		Sb1		0.234278			S4	1.122607
	Sb8	S20	-0.43184		Sb1		0.048567			S4	0.342059
	Sb17	S27	-0.58276		Sb2		0			S4	0.343461
	Sb11	S18	-0.56467		Sb14		0.315406			S6	0.06543
	Sb2	S4	-0.31157			S32	0			S1	0.621523
[]	Sb2	S2	-0.36271			S32	0.000961			S4	0.509846
Na4			0.128501			S18	0.442006			S9	0.04123
Na1			0.0026			S8	0.010616			S1	1.298147



Figure S4. Na-Sb-S thermodynamic phase diagram. Na $_3$ SbS $_4$ is indicated by a red square.

Table S5. Detailed chemical reactions and critical chemical potentials from grand potential phase diagram analysis for open transition metal (TM) species for different layered-type Na[TM]O₂ (TM = {V, Cr, Mn, Fe, Co, Ni}) cathode compounds. Note: $\mu_V = 0$, $\mu_{Cr} = 0$, $\mu_{Mn} = 0$, $\mu_{Fe} = 0$, $\mu_{Co} = 0$, and $\mu_{Ni} = 0$ are referenced to the chemical potentials of V metal ($\mu_V^0 = -9.0866 \text{ eV}$), Cr metal ($\mu_{Cr}^0 = -9.6367 \text{ eV}$), Mn metal ($\mu_{Mn}^0 = -9.1592 \text{ eV}$), Fe metal ($\mu_{Fe}^0 = -8.4598 \text{ eV}$), Co metal ($\mu_{Co}^0 = -7.1100 \text{ eV}$), and Ni metal ($\mu_{Ni}^0 = -5.7811 \text{ eV}$), respectively.

Compound	Open TM	Critical reactions (μ_{TM} relative to μ_{TM}^{0})
	element	
NaVO ₂ (@full	V	$NaVO_2 \rightarrow NaVO_2 (0 eV)$
sodiation)		NaVO ₂ → $0.2222 V_2O_3 + 0.3333 Na_3VO_4 + 0.2222 V (-1.7950 eV)$
VO ₂ (@full	V	$2 \operatorname{VO}_2 \rightarrow 2 \operatorname{VO}_2 (-3.071 \text{ eV})$
desodiation)		2 VO ₂ → 0.5714 V ₃ O ₇ + 0.2857 V (-4.7314 eV)
NaCrO ₂ (@full	Cr	$NaCrO_2 \rightarrow NaCrO_2 (0 eV)$
sodiation)		NaCrO ₂ → 0.5 Na ₂ CrO ₄ + 0.5 Cr (-4.0055 eV)
CrO ₂ (@full	Cr	$2 \operatorname{CrO}_2 \rightarrow 2 \operatorname{CrO}_2 (-4.9986 \text{ eV})$
desodiation)		2 CrO ₂ → 0.3333 Cr ₅ O ₁₂ + 0.3333 Cr (-5.5896 eV)
NaMnO ₂ (@full	Mn	$2 \text{ NaMnO}_2 \rightarrow 2 \text{ NaMnO}_2 (-0.2598 \text{ eV})$
sodiation)		2 NaMnO ₂ → 0.75 Na ₂ MnO ₃ + 0.25 Na ₂ Mn ₃ O ₇ + 0.5 Mn (-3.5522 eV)
MnO ₂ (@full	Mn	$4 \operatorname{MnO}_2 \rightarrow 4 \operatorname{MnO}_2 (-3.9869 \text{ eV})$
desodiation)		$4 \operatorname{MnO}_2 \rightarrow 4 \operatorname{O}_2 + 4 \operatorname{Mn} (-5.4311 \text{ eV})$
NaFeO ₂ (@full	Fe	2 NaFeO ₂ → 2 NaFeO ₂ (0 eV)
sodiation)		2 NaFeO ₂ → 2 NaO ₂ + 2 Fe (-5.0297 eV)
FeO ₂ (@full	Fe	2 FeO ₂ + 0.6667 Fe → 1.333 Fe ₂ O ₃ (-0.9179 eV)
desodiation)		$2 \text{ FeO}_2 \rightarrow 2 \text{ O}_2 + 2 \text{ Fe} (-4.7154 \text{ eV})$
NaCoO ₂ (@full	Со	$2 \operatorname{NaCoO}_2 \rightarrow 2 \operatorname{NaCoO}_2 (-0.0769 \text{ eV})$
sodiation)		2 NaCoO ₂ → 0.4 Na ₃ (CoO ₂) ₄ + 0.2 Na ₄ CoO ₄ + 0.2 Co (-2.7802 eV)
CoO ₂ (@full	Co	$4 \operatorname{CoO}_2 \rightarrow 4 \operatorname{CoO}_2 (-3.0933 \text{ eV})$
desodiation)		$4 \operatorname{CoO}_2 \xrightarrow{} 4 \operatorname{O}_2 + 4 \operatorname{Co} (-3.4132 \text{ eV})$
NaNiO ₂ (@full	Ni	2 NaNiO ₂ → 2 NaNiO ₂ (-0.3627 eV)
sodiation)		2 NaNiO ₂ → 2 NaO ₂ + 2 Ni (-2.1525 eV)
NiO ₂ (@full	Ni	$NiO_2 + 0.5 Ni \rightarrow 0.5 Ni_3O_4 (-1.7025 eV)$
desodiation)		$NiO_2 \rightarrow O_2 + Ni (-1.9185 \text{ eV})$

Table S6. Detailed chemical reactions and critical chemical potentials from grand potential phase diagram analysis for open transition metal (TM) species for different electrolyte/electrolyte-related compounds. Note: $\mu_V = 0$, $\mu_{Cr} = 0$, $\mu_{Mn} = 0$, $\mu_{Fe} = 0$, $\mu_{Co} = 0$, and $\mu_{Ni} = 0$ are referenced to the chemical potentials of V metal ($\mu_V^0 = -9.0866 \text{ eV}$), Cr metal ($\mu_{Cr}^0 = -9.6367 \text{ eV}$), Mn metal ($\mu_{Ni}^0 = -9.1592 \text{ eV}$), Fe metal ($\mu_{Fe}^0 = -8.4598 \text{ eV}$), Co metal ($\mu_{Co}^0 = -7.1100 \text{ eV}$), and Ni metal ($\mu_{Ni}^0 = -5.7811 \text{ eV}$), respectively.

Compound	Open TM	Critical reactions (μ_{TM} relative to μ_{TM}^{0})
	element	
Na ₃ SbS ₄	V	2 Na ₃ SbS ₄ + 0.8 V → 0.8 Na ₃ SbS ₃ + 1.2 NaSbS ₂ + 0.8 Na ₃ VS ₄ (-1.5665 eV)
		$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-3.2691 \text{ eV})$
	Cr	2 Na ₃ SbS ₄ + 1.333 Cr → 1.333 NaCrS ₂ + 0.6667 NaSbS ₂ + 1.333 Na ₃ SbS ₃ (-
		1.0402 eV)
		$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-2.0617 \text{ eV})$
	Mn	$2 \operatorname{Na_3SbS_4} + \operatorname{Mn} \rightarrow 2 \operatorname{Na_3SbS_3} + \operatorname{MnS_2} (-0.8262 \text{ eV})$
		$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-1.6204 \text{ eV})$
	Fe	$2 \operatorname{Na_3SbS_4} + \operatorname{Fe} \rightarrow 2 \operatorname{Na_3SbS_3} + \operatorname{FeS_2} (-0.5356 \text{ eV})$
		$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-1.4968 \text{ eV})$
	Со	$2 \operatorname{Na_3SbS_4} + \operatorname{Co} \rightarrow \operatorname{CoS_2} + 2 \operatorname{Na_3SbS_3} (-0.9348 \text{ eV})$
		$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-1.0114 \text{ eV})$
	Ni	2 Na ₃ SbS ₄ + 1.5 Ni → 2 Na ₃ SbS ₃ + 0.5 Ni ₃ S ₄ (-0.4050 eV)
		$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-0.8448 \text{ eV})$
Na ₃ SbS ₃	V	$4 \text{ Na}_{3}\text{SbS}_{3} + 2.4 \text{ V} \rightarrow 2.4 \text{ Na}_{3}\text{VS}_{4} + 2.4 \text{ Na}_{2}\text{S} + 4 \text{ Sb} (-1.0179 \text{ eV})$
		$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-1.3927 \text{ eV})$
	Cr	$4 \operatorname{Na_3SbS_3} + 4 \operatorname{Cr} \rightarrow 4 \operatorname{NaCrS_2} + 4 \operatorname{Na_2S} + 4 \operatorname{Sb} (0 \text{ eV})$
		$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-0.8664 \text{ eV})$
	Mn	$4 \text{ Na}_3\text{SbS}_3 + 6 \text{ Mn} \rightarrow 2 \text{ Na}_6\text{MnS}_4 + 4 \text{ MnS} + 4 \text{ Sb} (-0.0674 \text{ eV})$
		$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-0.4704 \text{ eV})$
	Fe	$4 \operatorname{Na_3SbS_3} + 6 \operatorname{Fe} \rightarrow 2 \operatorname{FeS} + 4 \operatorname{FeSbS} + 6 \operatorname{Na_2S} (0 \text{ eV})$
		$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-0.2034 \text{ eV})$
	Со	4 Na ₃ SbS ₃ + 8.083 Co → 0.75 Co ₉ S ₈ + 1.333 CoSb ₃ + 6 Na ₂ S (-0.1089 eV)
		$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-0.2358 \text{ eV})$
	Ni	$4 \text{ Na}_3\text{SbS}_3 + 13 \text{ Ni} \rightarrow 3 \text{ Ni}_3\text{S}_2 + 4 \text{ NiSb} + 6 \text{ Na}_2\text{S} (-0.0374 \text{ eV})$
		$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-0.1900 \text{ eV})$
NaSbS ₂	V	2 NaSbS ₂ + 1.667 V → 0.6667 Na ₃ VS ₄ + 0.3333 V ₃ S ₄ + 2 Sb (-1.0179 eV)
		$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-1.4963 \text{ eV})$
	Cr	$2 \operatorname{NaSbS}_2 + 2 \operatorname{Cr} \rightarrow 2 \operatorname{NaCrS}_2 + 2 \operatorname{Sb} (0 \text{ eV})$

		$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-1.0402 \text{ eV})$
	Mn	2 NaSbS ₂ + 2 Mn → 0.6667 Na ₃ SbS ₃ + 2 MnS + 1.333 Sb (-0.4704 eV)
		$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-0.5422 \text{ eV})$
	Fe	2 NaSbS ₂ + 2 Fe → 0.6667 Na ₃ SbS ₃ + 0.6667 FeS + 1.333 FeSbS (-0.2034 eV)
		$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-0.3771 \text{ eV})$
	Со	2 NaSbS ₂ + 2.694 Co → 0.25 Co ₉ S ₈ + 0.6667 Na ₃ SbS ₃ + 0.4444 CoSb ₃ (-0.2358
		eV)
		$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-0.3648 \text{ eV})$
	Ni	2 NaSbS ₂ + 4.333 Ni → 0.6667 Na ₃ SbS ₃ + Ni ₃ S ₂ + 1.333 NiSb (-0.1900 eV)
		$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-0.2702 \text{ eV})$
Na ₂ S	V	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$
	Cr	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$
	Mn	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$
	Fe	$2 \operatorname{Na_2S} \rightarrow 2 \operatorname{Na_2S} (0 \text{ eV})$
	Со	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$
	Ni	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$

Table S7. Detailed chemical reactions and critical chemical potentials from grand potential phase diagram analysis for open X species (X = {O, Sb, S}) for different layered-type Na[TM]O₂ (TM = {V, Cr, Mn, Fe, Co, Ni}) cathode compounds. Note: $\mu_0 = 0$, $\mu_{Sb} = 0$ and $\mu_S = 0$ are referenced to the chemical potentials of 1/2O₂ gas ($\mu_0^0 = -4.9355 \text{ eV}$), Sb metal ($\mu_{Sb}^0 = -4.1278 \text{ eV}$), and S solid ($\mu_S^0 = -4.1279 \text{ eV}$), respectively.

Compound	Open X	Critical reactions (μ_X relative to μ_X^0)
	element	
NaVO ₂ (@full	0	$NaVO_2 \rightarrow NaVO_2 (-3.0210 \text{ eV})$
sodiation)		$NaVO_2 \rightarrow V + Na + O_2 (-4.6840 \text{ eV})$
	Sb	$NaVO_2 \rightarrow NaVO_2(0 eV)$
	S	$NaVO_2 \rightarrow NaVO_2(-1.1630 \text{ eV})$
VO ₂ (@full	0	$2 \operatorname{VO}_2 \rightarrow 2 \operatorname{VO}_2 (-1.3608 \text{ eV})$
desodiation)		2 VO ₂ → 0.6667 V ₃ O ₅ + 0.3333 O ₂ (-2.1908 eV)
	Sb	$2 \operatorname{VO}_2 \xrightarrow{} 2 \operatorname{VO}_2 (-1.5958 \text{ eV})$
	S	$2 \operatorname{VO}_2 \xrightarrow{} 2 \operatorname{VO}_2 (-1.7449 \text{ eV})$
NaCrO ₂ (@full	0	$NaCrO_2 \rightarrow NaCrO_2 (-1.2456 \text{ eV})$
sodiation)		$NaCrO_2 \rightarrow Cr + Na + O_2 (-4.5194 \text{ eV})$
	Sb	$NaCrO_2 \rightarrow NaCrO_2 (0 eV)$
	S	$NaCrO_2 \rightarrow NaCrO_2 (-1.0065 \text{ eV})$
CrO ₂ (@full	0	$2 \operatorname{CrO}_2 \xrightarrow{} 2 \operatorname{CrO}_2 (-0.2880 \text{ eV})$
desodiation)		$2 \text{ CrO}_2 \rightarrow \text{Cr}_2\text{O}_3 + 0.5 \text{ O}_2 (-0.5836 \text{ eV})$
	Sb	$2 \operatorname{CrO}_2 \rightarrow 2 \operatorname{CrO}_2 (-5.4118 \text{ eV})$
	S	$2 \operatorname{CrO}_2 \rightarrow 2 \operatorname{CrO}_2 (-6.4591 \text{ eV})$
NaMnO ₂ (@full	0	$2 \text{ NaMnO}_2 \rightarrow 2 \text{ NaMnO}_2 (-1.1808 \text{ eV})$
sodiation)		2 NaMnO ₂ → Na ₂ Mn ₂ O ₃ + 0.5 O ₂ (-3.7410 eV)
	Sb	$2 \text{ NaMnO}_2 \rightarrow 2 \text{ NaMnO}_2 (-0.7291 \text{ eV})$
	S	$2 \text{ NaMnO}_2 \rightarrow 2 \text{ NaMnO}_2 (-1.8060 \text{ eV})$
MnO ₂ (@full	0	$4 \operatorname{MnO}_2 \rightarrow 4 \operatorname{MnO}_2 (0 \text{ eV})$
desodiation)		$4 \operatorname{MnO}_2 \xrightarrow{} 2 \operatorname{Mn}_2 \operatorname{O}_3 + \operatorname{O}_2 (-0.7220 \text{ eV})$
	Sb	$4 \operatorname{MnO}_2 \rightarrow 4 \operatorname{MnO}_2 (-4.8282 \text{ eV})$
	S	$4 \operatorname{MnO}_2 \rightarrow 4 \operatorname{MnO}_2 (-6.1673 \text{ eV})$
NaFeO ₂ (@full	0	$2 \text{ NaFeO}_2 \rightarrow 2 \text{ NaFeO}_2 (0 \text{ eV})$
sodiation)		2 NaFeO ₂ → 0.6667 Na ₃ FeO ₃ + 1.333 Fe + O ₂ (-3.6000 eV)
	Sb	$2 \text{ NaFeO}_2 \rightarrow 2 \text{ NaFeO}_2 (0 \text{ eV})$
	S	$2 \text{ NaFeO}_2 \rightarrow 2 \text{ NaFeO}_2 (-1.3806 \text{ eV})$

FeO ₂ (@full	0	$2 \operatorname{FeO}_2 \xrightarrow{} 2 \operatorname{Fe} + 2 \operatorname{O}_2 (-3.3572 \text{ eV})$
desodiation)	Sb	$2 \text{ FeO}_2 \rightarrow \text{Fe}_2\text{O}_3 + 0.5 \text{ O}_2 (-6.8397 \text{ eV})$
	S	$2 \text{ FeO}_2 \rightarrow \text{Fe}_2\text{O}_3 + 0.5 \text{ O}_2 (-8.3005 \text{ eV})$
NaCoO ₂ (@full	0	$2 \operatorname{NaCoO}_2 \rightarrow 2 \operatorname{NaCoO}_2 (-0.6634 \text{ eV})$
sodiation)		2 NaCoO ₂ → 1.333 CoO + 0.6667 Na ₃ CoO ₃ + 0.3333 O ₂ (-2.5832 eV)
	Sb	$2 \operatorname{NaCoO}_2 \rightarrow 2 \operatorname{NaCoO}_2 (-2.7485 \text{ eV})$
	S	$2 \operatorname{NaCoO}_2 \rightarrow 2 \operatorname{NaCoO}_2 (-3.6036 \text{ eV})$
CoO ₂ (@full	0	$4 \operatorname{CoO}_2 \xrightarrow{} 4 \operatorname{CoO}_2 (0 \text{ eV})$
desodiation)		4 CoO ₂ → 1.333 Co ₃ O ₄ + 1.333 O ₂ (-0.1600 eV)
	Sb	$4 \operatorname{CoO}_2 \rightarrow 4 \operatorname{CoO}_2 (-6.2564 \text{ eV})$
	S	$4 \operatorname{CoO}_2 \xrightarrow{} 4 \operatorname{CoO}_2 (-7.5514 \text{ eV})$
NaNiO ₂ (@full	0	2 NaNiO ₂ → 2 NaNiO ₂ (-0.0040 eV)
sodiation)		2 NaNiO ₂ → 0.4 Na ₅ NiO ₄ + 1.6 NiO + 0.4 O ₂ (-1.5018 eV)
	Sb	2 NaNiO ₂ → 2 NaNiO ₂ (-6.1942 eV)
	S	2 NaNiO ₂ → 2 NaNiO ₂ (-7.3372 eV)
NiO ₂ (@full	0	$NiO_2 \rightarrow Ni + O_2 (-3.7290 \text{ eV})$
desodiation)	Sb	NiO ₂ → 0.3333 Ni ₃ O ₄ + 0.3333 O ₂ (-6.8918 eV)
	S	NiO ₂ → 0.3333 Ni ₃ O ₄ + 0.3333 O ₂ (-8.4912 eV)

Table S8. Detailed chemical reactions and critical chemical potentials from grand potential phase diagram analysis for open X species (X = {O, Sb, S}) for different electrolyte/electrolyte-related compounds. Note: $\mu_0 = 0$, $\mu_{Sb} = 0$ and $\mu_S = 0$ are referenced to the chemical potentials of 1/2O₂ gas ($\mu_0^0 = -4.9355 \text{ eV}$), Sb metal ($\mu_{Sb}^0 = -4.1278 \text{ eV}$), and S solid ($\mu_S^0 = -4.1279 \text{ eV}$), respectively.

Compound	Open X	Critical reactions (μ_X relative to μ_X^0)
	element	
Na ₃ SbS ₄	0	$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-3.0102 \text{ eV})$
	Sb	$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (-1.0216 \text{ eV})$
		$2 \text{ Na}_3\text{SbS}_4 \rightarrow 3 \text{ Na}_2\text{S} + 5 \text{ S} + 2 \text{ Sb} (-2.9677 \text{ eV})$
	S	$2 \operatorname{Na_3SbS_4} \rightarrow 2 \operatorname{Na_3SbS_4} (0 \text{ eV})$
		$2 \text{ Na}_3\text{SbS}_4 \rightarrow 2 \text{ Na}_3\text{SbS}_3 + 2 \text{ S} (-0.6742 \text{ eV})$
Na ₃ SbS ₃	0	$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-3.0512 \text{ eV})$
	Sb	$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (0 \text{ eV})$
		4 Na ₃ SbS ₃ → 2.4 Na ₃ SbS ₄ + 2.4 Na ₂ S + 1.6 Sb (-1.2822 eV)
	S	$4 \operatorname{Na_3SbS_3} \rightarrow 4 \operatorname{Na_3SbS_3} (-0.6742 \text{ eV})$
		$4 \operatorname{Na_3SbS_3} \rightarrow 6 \operatorname{Na_2S} + 4 \operatorname{Sb} + 6 \operatorname{S} (-1.5290 \text{ eV})$
NaSbS ₂	0	$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (-2.8492 \text{ eV})$
	Sb	$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (0 \text{ eV})$
		2 NaSbS ₂ → 0.6667 Na ₃ SbS ₄ + 1.333 S + 1.333 Sb (-1.6958 eV)
	S	$2 \text{ NaSbS}_2 \rightarrow 2 \text{ NaSbS}_2 (0 \text{ eV})$
		2 NaSbS ₂ → 0.6667 NaSbS ₂ + 1.333 Sb + 2 S (-1.3553 eV)
Na ₂ S	0	$2 \operatorname{Na_2S} \rightarrow 2 \operatorname{Na_2S} (-3.0946 \text{ eV})$
	Sb	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$
	S	$2 \operatorname{Na}_2 S \rightarrow 2 \operatorname{Na}_2 S (0 \text{ eV})$
		$2 \operatorname{Na_2S} \rightarrow 4 \operatorname{Na} + 2 \operatorname{S} (-3.8831 \text{ eV})$



Figure S5. a-b) Schematic illustration summary for $X = \{O, Sb, S\}$ species tendency to migrate across the Na[*TM*]O₂-SE interface (SE: solid electrolyte).



Figure S6. Mean square displacement (*MSD*) plots derived from DFT-MD calculations: a) 1-vacancy undoped (1v-u), b) 2-vacancy F-doped (2v-F), c) 2-vacancy Cl-doped (2v-Cl), d) 2-vacancy Br-doped (2v-Br), e) 2-vacancy I-doped (2v-I) model, and f) highlighted MSD plots at 373-K for a-e.



Figure S7. Li-atom mean square displacement (*MSD*) plot of c-NSS from DFT-MD calculation under NVT ensemble condition at 473 K.

Diffusion barrier calculation

Nudged elastic band (NEB) supercell (2 x 2 x 2) method was used to estimate ion diffusion barrier values.⁵⁹ For the Na local pathway, 7 intermediate images were interpolated linearly between fully optimized initial and final structure coordinate data. Energy barriers were then obtained in the dilute vacancy limit (or one vacancy/supercell) with cell edges of at least 14 Å.



Figure S8. DFT-NEB Na ion migration energy comparison for Na1-Na1 pathway for Na_3SbS_4 and related compounds.



Figure S9. Visualization of local Na pathways calculated by DFT-NEB for SE-related compounds: a) Na₃SbS₃, b) NaSbS₂, c) Na₂S.

Voltage (V vs. Na/Na ⁺)	Decomposition reaction
0	$2Na_{23}Sb_8S_{31}Cl$ + 128Na → 16Na ₃ Sb + 62Na ₂ S + 2NaCl
0.53	$2Na_{23}Sb_8S_{31}Cl + 96Na \rightarrow 16NaSb + 62Na_2S + 2NaCl$
0.65	$2Na_{23}Sb_8S_{31}Cl + 80Na → 62Na_2S + 2NaCl + 16Sb$
1.17	$2Na_{23}Sb_8S_{31}Cl + 32Na \rightarrow 16Na_3SbS_3 + 14Na_2S + 2NaCl$
1.54	$2Na_{23}Sb_8S_{31}Cl + 18Na \rightarrow 14NaS + 16Na_3SbS_3 + 2NaCl$
1.56	$2Na_{23}Sb_8S_{31}Cl + 11Na \rightarrow 7NaS_2 + 16Na_3SbS_3 + 2NaCl$
1.65	$2\mathrm{Na}_{23}\mathrm{Sb}_8\mathrm{S}_{31}\mathrm{Cl} \rightarrow 15\mathrm{Na}\mathrm{S}_2 + 16\mathrm{Na}\mathrm{Sb}\mathrm{S}_2 + 2\mathrm{Na}\mathrm{Cl} + 13\mathrm{Na}$
1.82	$2Na_{23}Sb_8S_{31}Cl \rightarrow 16NaSbS_2 + 6Na_2S_5 + 2NaCl + 16Na$
2.18	$2Na_{23}Sb_8S_{31}Cl \rightarrow 7.6Na_2S_5 + 8Sb_2S_3 + 2NaCl + 28.8Na$
2.22	$2Na_{23}Sb_8S_{31}Cl \rightarrow 6.733Na_2S_5 + 7.667Sb_2S_3 + 0.6667SbS_8Cl_3 +$
	32.53Na
3.38	$2Na_{23}Sb_8S_{31}Cl \rightarrow 7.667Sb_2S_3 + 0.6667SbS_8Cl_3 + 33.67S + 46Na$

Table S9. Summary of DFT-predicted decomposition reactions related to voltage stability window for Na with Cl-doped Na₃SbS₄.