Electronic Supplementary Information for

First-principles study of bilayer hexagonal structure of SN<sub>2</sub> nanosheet: a

highly stable non-metal platform for the quantum anomalous Hall effect

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Figure S1: The enthalpies of different  $SN_2$  bulk structures as a function of pressures. The geometries of (b)  $CaCl_2$ -like  $SiO_2$ -type, (c)  $TiO_2$ -like  $SiO_2$ -type, and (d) vdw layered bulk structures for the  $SN_2$  system.



Figure S2: (a) The partial charge density of the flat band at  $\Gamma$  point. The isosurface is set to 0.003  $|e|/\dot{A}^3$ . (b) The fat-band analysis of the PBE bands of BHS-SN<sub>2</sub> nanosheet. The contribution of N<sub>out</sub> p<sub>z</sub> orbitals is labelled by colors, and the maximum value is normalized to 1.



Figure S3: The phonon bands of (a)  $S_3N_9$  and (b)  $AsS_3N_8$  nanosheets.



Figure S4: The mean-field theory results for the Curie temperature. (a) The temperature-dependent magnetic moment of hole-doped BHS-SN<sub>2</sub> nanosheet with a compressive strain of -0.05 and without strain. (b) The temperature-dependent magnetic moment of the isolated and fluorographene-supported  $PS_3N_8$  nanosheets. The insets depict the magnetic moment squared near the transition temperature, where a linear fitting is performed to evaluate the T<sub>c</sub> values.



Figure S5: (a) The initial and optimized geometries of  $S_{36}N_{71}P_1$  and  $S_{35}N_{72}P_1$ nanosheets. (b) The variations of Gibbs free energies versus the chemical potential of N elment for the  $S_{36}N_{71}P_1$  and  $S_{35}N_{72}P_1$  nanosheets. (d) The Gibbs free energy values of  $S_{36}N_{71}P_1$  and  $S_{35}N_{72}P_1$  nanosheets under the Npoor and N-rich conditions. Here, the N-poor and N-rich conditions correspond to  $\mu_N = E_{N2}/2$  and  $\mu_N = E_{Natom}$ , respectively.