Electronic Supplementary Material (ESI) for Analytical Methods.

High Resolution Fourier Transform Ion Trap Enabled by Image Current Splicing: A Theoretical Study

Electronic noise of amplifier

The electronic noise of amplifier includes voltage noise, current noise and thermal noise. OPA 657 was used in the proposed method[1]. The electrical characteristics[2] of OPA 657 related to electronic noise were listed in Table S1.

Table S1 The electrical characteristics of OPA 657 related to electronic noise

Symbol	Parameter	Condition	Min	Typ	Max	Units
GBW	Gain-bandwidth product	G > +40 V/V, T = 25°C		1600		MHz
	Input impedance (differential)	T = 25°C		10^{12}‖0.7		Ω‖pF
	Input impedance (common-mode)	T = 25°C		10^{12}‖4.5		Ω‖pF
e_n	Input voltage noise density	f > 100 kHz, T = 25°C		4.8		$nV/\sqrt{Hz}$
e_{n_L}	Input voltage noise density	f = 10 Hz, T = 25°C		38		$nV/\sqrt{Hz}$
i_n	Input current noise density	f > 100 kHz, T = 25°C		1.3		$fA/\sqrt{Hz}$

The input capacitance of OPA 657 lumped in with differential and common-mode input is

$$C_{opa} = 0.7\text{pF} + 4.5\text{pF} = 5.2\text{pF}. \qquad \text{\\* MERGEFORMAT (1)}$$

The feedback resistance R_f and feedback capacitor C_f are 75 MΩ and 25 fF, respectively. Thus, the zero f_z and pole f_p[3, 4] contained in the transfer function are

$$f_z = \frac{1}{2\pi R_f\left(C_f + C_{opa}\right)} = 406.1\text{Hz} \qquad \text{\\* MERGEFORMAT (2)}$$

$$f_p = \frac{1}{2\pi R_f C_f} = 84.9\text{KHz}. \qquad \text{\\* MERGEFORMAT (3)}$$

Intersection of the voltage noise gain curve with the AOL curve f_i[1] is

$$f_i = \frac{C_f}{C_{opa} + C_f} GBW = 7.7\text{MHz}. \qquad \text{\\* MERGEFORMAT (4)}$$

The start frequency of 1/f noise f_L is 10 Hz obtained in the data sheet, and the 1/f noise voltage corner frequency f_f[1] is

$$f_f = \frac{e_{n\_L}^{2} f_L}{e_n^{2}} = 626.7\text{Hz}. \qquad \text{* MERGEFORMAT (5)}$$

The voltage noise in region 1 (between f_L and f_f) is

$$E_{noe1} = e_n \sqrt{f_f \ln\left(\frac{f_f}{f_L}\right)} = 244.4\text{nV}. \qquad \text{* MERGEFORMAT (6)}$$

The voltage noise in region 2 (between f_z and f_p) is

$$E_{noe2} = \frac{e_n}{f_z}\sqrt{\frac{f_p^{3} - f_z^{3}}{3}} = 168.8\mu\text{V}. \qquad \text{* MERGEFORMAT (7)}$$

The voltage noise in region 3 (between f_p and f_i) is

$$E_{noe3} = e_n \frac{C_{opa} + C_f}{C_f}\sqrt{f_i - f_p} = 2.8\text{mV}. \qquad \text{* MERGEFORMAT (8)}$$

The voltage noise in region 4 (greater than f_i) is

$$E_{noe4} = e_n GWB\sqrt{\frac{1}{f_i}} = 2.8\text{mV}. \qquad \text{* MERGEFORMAT (9)}$$

The voltage noise is

$$E_{noe} = \sqrt{E_{noe1}^{2} + E_{noe2}^{2} + E_{noe3}^{2} + E_{noe4}^{2}} = 3.9\text{mV}. \text{* MERGEFORMAT}$$

(10)

The voltage noise in region 1 and 2 is partially overlapped, which is ignored because the voltage noise in region 1 is much smaller than that in region 2. The current voltage[1] is

$$E_{noi} = i_n R_f \sqrt{K_n f_p} = 35.6\mu\text{V} \qquad \text{* MERGEFORMAT (11)}$$

where $K_n = 1.57$ is the brick wall conversion factor for 1 filter order. The thermal noise[5] is

$$E_{nor} = \sqrt{4kTR_f K_n f_p} = 400.5\mu\text{V} \qquad \text{* MERGEFORMAT (12)}$$

where $k = 1.38 \times 10^{-23}\text{J/K}$ is Boltzmann's constant and $T = 298\text{K}$ is temperature in Kelvin.

The total noise is

$$E_{no} = \sqrt{E_{noe}^{2} + E_{noi}^{2} + E_{nor}^{2}} = 3.9\text{mV}. \qquad \text{* MERGEFORMAT (13)}$$

Validation using FT-ICR data

Figure S1a plotted the raw signal in time domain which was the image current induced by cytochrome C (16+) ions at 1×10^{-10} Torr with nitrogen as the buffer gas. Compared with Figure S1b, the signal decay in Figure 1 was not apparent. Figure S2b showed the raw signal with only the 186-187 kHz frequency band retained, which is the signal shown in Figure 1 filtered by a bandpass filter with a passband of 186-187 kHz. The signal in Figure S2b was exponentially decaying.

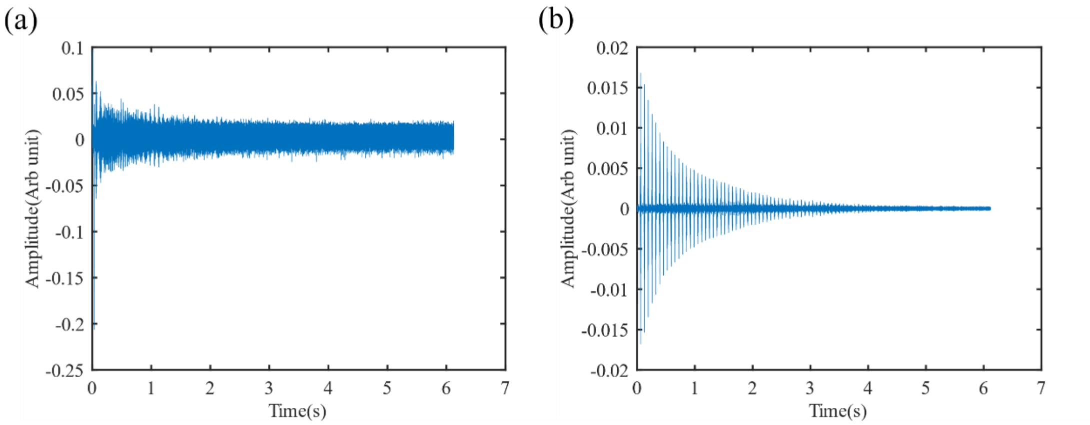

Figure S1 (a). The raw signal in time domain which was the image current induced by cytochrome C (16+) ions at 1×10^{-10} Torr with nitrogen as the buffer gas. **(b).** The raw signal filtered by a bandpass filter with passband of 186-187 kHz.

Due to the previous process such as envelope normalization and the splicing, the reconstructed signal didn't decay, which resulted in a poor overlap between the frequency spectrum of the reconstructed signal and these 3 sets of raw signals. When the reconstructed signal was decayed as the raw signals, the correlation coefficients between the reconstructed spectrum and 3 sets of raw spectra were increased to 0.98-0.99, which meant that they were about the same. Figure S2 was the frequency spectra of the reconstructed signal with the same decay ratio and these 3 sets of raw signals.

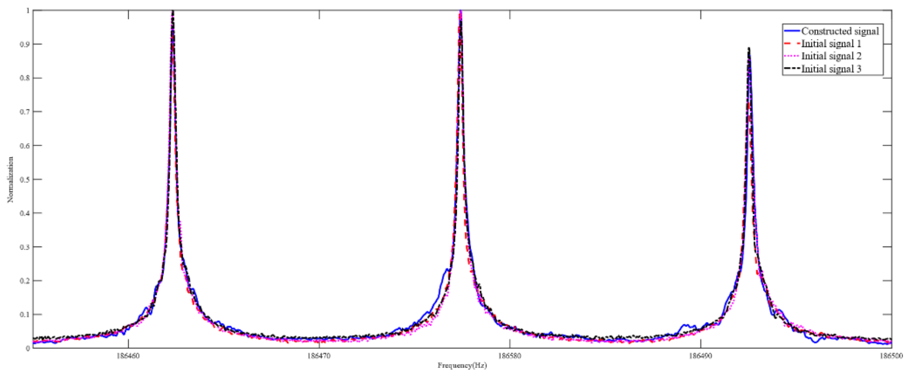

Figure S2. The frequency spectra of the reconstructed signals with decay and these 3 sets of raw signals.

References:

1. K. Art, *Operational Amplifier Noise*, 2012.
2. J. M. Bryant, *Electron Eng*, 1996, **68**, 47-&.
3. A. V. Kretinin and Y. Chung, *Rev Sci Instrum*, 2012, **83**.
4. A. J. Peyton, Y. Aboufakher and B. Wilson, *Iee P-Circ Dev Syst*, 1994, **141**, 210-214.
5. F. N. Trofimenkoff and O. A. Onwuachi, *Ieee T Educ*, 1989, **32**, 12-17.