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Supporting Information

Electro-elastic properties of Piezoelectric $Te_2O(PO_4)_2$ Crystal

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CONTENTS

1. Calculation formula and process of electro-elastic constants.

The dielectric constants can be obtained by the measured capacitance values according to the following formulas with square plate samples k, l, and j, and circumgyrate sample g:

$$\varepsilon_{ij} = \frac{\varepsilon_{ij}^{T}}{\varepsilon_0} = \frac{C_{ij} \times t}{A \times \varepsilon_0} (i = 1, 2, 3) \# (1)$$

$$\varepsilon_{33}^{T'}(\theta) = \varepsilon_{11}^{T} sin^2\theta + 2\varepsilon_{13}^{T} \cos\theta \sin\theta + \varepsilon_{33}^{T} cos^2\theta \# (2)$$

where C is the capacitance, t is the thickness, A is the area of the measured sample, and ε_0 is the vacuum dielectric constant.

$$s^E = \frac{1}{4\rho l f_a^2 (1 - k^2)} \# (3)$$

$$k^2 = \frac{\pi f_r}{2 f_a} \cot \frac{\pi}{2}\left(\frac{f_r}{f_a}\right) \# (4)$$

$$d = k\sqrt{\varepsilon s} \# (5)$$

$$s^E = \frac{1}{4\rho w^2 f_a^2 (1 - k^2)} \# (6)$$

$$s^E = \frac{1}{4\rho t^2 f_a^2 (1 - k^2)} \# (7)$$

$$s^E = \frac{1}{4\rho l^2 f_r^2} \# (8)$$

$$\frac{k^2}{1 - k^2} = \frac{\pi f_a}{2 f_r} \cot \frac{\pi}{2}\left(\frac{f_a}{f_r}\right) \# (9)$$

$$s_{33}^{'}(XZw)\theta = s_{11} sin^4\theta + (2s_{13} + s_{55}) sin^2\theta cos^2\theta + 2s_{15} sin^3\theta cos\theta + s_{33} cos^4\theta + 2s_{33} sin\theta cos^3\theta \# (10)$$

$$s_{22}^{'}(XYt)45° = (s_{22} + 2s_{23} + s_{44} + s_{33})/4 \# (11)$$

$$s_{11}^{'}(XYl)45° = (s_{11} + 2s_{12} + s_{66} + s_{22})/4 \# (12)$$

$$s_{33}^{'}(YZw)45°/-45° = \begin{pmatrix} 4s_{22} + s_{11} + s_{33} + 2s_{15} + 2s_{13} + s_{55} + 2s_{35} \\ + 4s_{23} + 4s_{12} + 4s_{25} + 2s_{44} + 2s_{66} + 4s_{46} \end{pmatrix} \Big/ 16 \# (13)$$

$$s'_{44}(XYl)45° = 0.5s_{44} + s_{46} + 0.5s_{66}\#(14)$$

$$d'_{13}(XYw)\theta = d_{11}\sin^2\theta cos\theta - d_{31}sin^3\theta + d_{13}cos^3\theta + d_{33}sin\theta cos^2\theta + d_{15}sin\theta cos^2\theta - d_{35}sin^2\theta cos\theta\#(15)$$

$$c = s^{-1}\#(16)$$

Elastic compliance constants s_{11} and s_{33} and piezoelectric constants d_{11} and d_{33} were calculated using equations (3)-(5) based on samples a and b. When the electric field was applied along the thickness direction of samples c-f, piezoelectric constants d_{13}, d_{12}, d_{32}, and d_{31} and elastic compliance constants s_{33}, s_{22}, and s_{11} were obtained by equation (5), (8), and (9). Samples c-f were also used to calculate the piezoelectric constants d_{35}, d_{15}, d_{24}, and d_{26} and the elastic constants s_{55}, s_{44}, and s_{66} by equations (4)-(7), where the electric field was applied along the length direction of the samples. Using the sample g-i and combining equations (8) and (9), the elastic compliance constants s_{13}, s_{15}, and s_{35} are obtained. Similarly, elastic compliance constants s_{23}, s_{12}, and s_{25} were obtained in the same way based on samples m-o and equations (8) and (11)-(13). Elastic compliance constant s_{46} was calculated based on the equations (4), (5), (7), and (14), according to the measurement of the thickness-shear vibration mode of sample p with an electric field applied along the length direction. Samples g and i were also used to verify the sign and value of piezoelectric strain constant d_{15} according to equation (15). Stiffness coefficients c_{ij} can be obtained using equation (16).

Table S1. The crystal cuts and vibration modes for the determination of electro-elastic constants of TPO crystal.

Sample	Modes	Constants
X square plate Y square plate Z square plate (XZ*w*)45° bar	-	ε_{11}^{T}, ε_{22}^{T}, ε_{33}^{T}, ε_{13}^{T}
X rod Z rod	longitudinal length extensional vibration mode	d_{11} d_{33}
ZX bar XY bar ZY bar XZ bar	longitudinal length extensional vibration mode	s_{11}, d_{31} s_{22}, d_{12}, d_{32} s_{33}, d_{13}
XZ bar XZ bar	transverse length extensional vibration mod	s_{44}, d_{24} s_{66}, d_{26}
XZ bar ZX bar	thickness shear vibration mod	s_{55}, d_{15}, d_{35}
(XZ*w*)45° bar (XZ*w*)30° bar (XZ*w*)-30° bar	longitudinal length extensional vibration mode	s_{13}, s_{15}, s_{35}
(XY*t*)45° bar (ZX*t*)45° bar (ZX*tw*)45°/-45° bar	longitudinal length extensional vibration mode	s_{12}, s_{23}, s_{25}
(XY*l*)45° bar	thickness shear vibration mod	s_{46}