

# Supporting Information:

## A simple fragment-based method for van der Waals corrections over density functional theory

Prasanta Bandyopadhyay, Priya, and Mainak Sadhukhan\*

*Department of Chemistry, Indian Institute of Technology, Kanpur, 208016, India*

E-mail: mainaks@iitk.ac.in

Phone: +123 (0)123 4445556. Fax: +123 (0)123 4445557

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# 1 Polarizability

## 1.1 $\alpha_{xx}$

To calculate the dipole polarizability we should consider all the possible transition with one quanta excitation from ground state. Therefore the possible transitions are  $(0_x 0_y 0_z \rightarrow n_x n_y n_z)$  :

$$000 \rightarrow 100, 000 \rightarrow 010, 000 \rightarrow 001$$

The general expression for calculating the polarizability is:

$$\alpha_{\alpha\beta} = \sum'_{n_x n_y n_z} \frac{\langle 000 | \hat{\mu}_\alpha | n_x n_y n_z \rangle \langle n_x n_y n_z | \hat{\mu}_\beta | 000 \rangle}{E_n - E_0} + \frac{\langle n_x n_y n_z | \hat{\mu}_\alpha | 000 \rangle \langle 000 | \hat{\mu}_\beta | n_x n_y n_z \rangle}{E_n - E_0} \quad (1)$$

Similarly the expression for  $\alpha_{xx}$  is:

$$\alpha_{xx} = \sum'_{n_x n_y n_z} \frac{\langle 000 | \hat{\mu}_x | n_x n_y n_z \rangle \langle n_x n_y n_z | \hat{\mu}_x | 000 \rangle}{E_n - E_0} + \frac{\langle n_x n_y n_z | \hat{\mu}_x | 000 \rangle \langle 000 | \hat{\mu}_x | n_x n_y n_z \rangle}{E_n - E_0} \quad (2)$$

Here  $\mu$  in x-direction is:

$$\hat{\mu}_x = qx$$

The wavefunction for simple harmonic oscillator is:

$$\psi = \frac{1}{\sqrt{2^n n!}} \left[ \frac{mw}{\pi \hbar} \right]^{\frac{1}{4}} e^{-\frac{\alpha^2 r^2}{2}} H_n(\alpha r)$$

$$\alpha_x = \left[ \frac{m\omega_x}{\hbar} \right]^{\frac{1}{2}}, \quad \alpha_y = \left[ \frac{m\omega_y}{\hbar} \right]^{\frac{1}{2}}, \quad \alpha_z = \left[ \frac{m\omega_z}{\hbar} \right]^{\frac{1}{2}}$$

$$\alpha_{x \cdot x} = \frac{H_1(\alpha_x x)}{2}, \quad \alpha_{y \cdot y} = \frac{H_1(\alpha_y y)}{2}$$

For solving the first term in  $\alpha_{xx}$  expression

$$\langle 000 | \hat{\mu}_x | n_x n_y n_z \rangle = q \langle 0_y n_y \rangle \langle 0_z n_z \rangle \langle 0 | x | n_x \rangle$$

This expression can be written as:

$$= q \langle 0_y n_y \rangle \langle 0_z n_z \rangle \frac{\langle 0_x | H_1(\alpha_x x) | n_x \rangle}{2\alpha_x}$$

Out of three possible transition only ( $n_x n_y n_z = 100$ ) is non-zero using orthogonality

$$q \langle 0_y n_y \rangle \langle 0_z n_z \rangle \langle 0 | x | n_x \rangle = q \left[ \int_{-\infty}^{\infty} dy \frac{\alpha_y}{\sqrt{\pi}} e^{-\alpha_y^2 y^2} \int_{-\infty}^{\infty} dz \frac{\alpha_z}{\sqrt{\pi}} e^{-\alpha_z^2 z^2} \int_{-\infty}^{\infty} dx \frac{1}{2\alpha_x} \frac{\alpha_x}{\sqrt{\pi}} e^{-\alpha_x^2 x^2} H_1(\alpha_x x) H_{n_x}(\alpha_x x) \right]$$

$$q \left[ \frac{\alpha_y}{\sqrt{\pi}} \int_{-\infty}^{\infty} dy e^{-\alpha_y^2 y^2} \right] \left[ \frac{\alpha_z}{\sqrt{\pi}} \int_{-\infty}^{\infty} dz e^{-\alpha_z^2 z^2} \right] \left[ \frac{1}{2\alpha_x} \frac{\alpha_x}{\sqrt{\pi}} \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} dx e^{-\alpha_x^2 x^2} H_1(\alpha_x x) H_1(\alpha_x x) \right]$$

$$\int_{-\infty}^{\infty} dy e^{-\alpha_y^2 y^2} = \frac{\sqrt{\pi}}{\alpha_y}$$

similarly for z component

$$\int_{-\infty}^{\infty} dz e^{-\alpha_z^2 z^2} = \frac{\sqrt{\pi}}{\alpha_z}$$

for x part

$$\int_{-\infty}^{\infty} dx e^{-\alpha_x^2 x^2} H_{n_x}(\alpha_x x) H_1(\alpha_x x) = \frac{\sqrt{\pi}}{\alpha_x} 2^{n_x} n_x! \delta_{n_x, 1}$$

Therefore we get

$$q \left( \frac{\alpha_y}{\sqrt{\pi}} \frac{\sqrt{\pi}}{\alpha_y} \right) \left( \frac{\alpha_z}{\sqrt{\pi}} \frac{\sqrt{\pi}}{\alpha_z} \right) \left( \frac{1}{2\alpha_x} \frac{\alpha_x}{\sqrt{\pi}} \frac{1}{\sqrt{2}} \frac{\sqrt{\pi}}{\alpha_x} 2 \right)$$

Final expression for is :

$$\langle 000 | \hat{\mu}_x | n_x n_y n_z \rangle = \frac{q}{\sqrt{2}\alpha_x} \quad (3)$$

$$\langle n_x n_y n_z | \hat{\mu}_x | 000 \rangle = \frac{q}{\sqrt{2}\alpha_x} \quad (4)$$

Therefore the equation 1 becomes

$$\begin{aligned} \alpha_{xx} &= \frac{q^2}{2\alpha_x^2 \hbar \omega} + \frac{q^2}{2\alpha_x^2 \hbar \omega} \\ &= \frac{q^2}{\alpha_x^2 \hbar \omega} \end{aligned}$$

substituting the value of  $\alpha_x$ , we get

$$\alpha_{xx} = \frac{q^2}{mw_x^2}$$

similarly

$$\alpha_{yy} = \frac{q^2}{mw_y^2}$$

and,

$$\alpha_{zz} = \frac{q^2}{mw_z^2}$$

For  $\alpha_{xy}, \alpha_{xz}, \alpha_{zy}$  :

$$\alpha_{xy} = 0, \quad \alpha_{xz} = 0, \quad \alpha_{yz} = 0$$

## 1.2 Dispersion in the Quantum drude oscillator

$$U_{Disp} = - \sum_{m_x A m_y A m_z A \neq 0, n_x B n_y B n_z B \neq 0} \frac{\langle 00 | H' | mn \rangle \langle mn | H' | 00 \rangle}{W_{m0}^A + W_{n0}^B} \quad (5)$$



In general the dispersion energy  $U_{disp}^{\alpha\beta}$  is given by considering,

$$m_{xA}m_{yA}m_{zA} = M, \quad n_{xB}n_{yB}n_{zB} = N$$

$$U_{disp}^{\alpha\beta} = - \sum_{M \neq 0} \sum_{N \neq 0} \frac{\langle 000, 000 | \hat{\mu}_\alpha^A T_{\alpha\beta} \hat{\mu}_\beta^B | M, N \rangle \langle m_{xA}m_{yA}m_{zA}, n_{xB}n_{yB}n_{zB} | \hat{\mu}_\alpha^A T_{\alpha\beta} \hat{\mu}_\beta^B | 000, 000 \rangle}{W_{m0}^A + W_{n0}^B} \quad (6)$$

$$\begin{aligned} U_{disp}^{xy} &= - \sum_{m_{xA}m_{yA}m_{zA} \neq 0} \sum_{n_{xB}n_{yB}n_{zB} \neq 0} \frac{\langle 000, 000 | \hat{\mu}_x^A T_{xy} \hat{\mu}_y^B | m_{xA}m_{yA}m_{zA}, n_{xB}n_{yB}n_{zB} \rangle \langle m_{xA}m_{yA}m_{zA}, n_{xB}n_{yB}n_{zB} \rangle}{W_{m0}^A + W_{n0}^B} \\ &= - \left( \frac{W_{m0}^A W_{n0}^B T_{xy} T_{xy}}{E_A + E_B} \right) \sum_{m_{xA}m_{yA}m_{zA} \neq 0} \frac{|\langle 000 | \hat{\mu}_x^A | m_{xA}m_{yA}m_{zA} \rangle|^2}{W_{m0}^A} \sum_{n_{xB}n_{yB}n_{zB} \neq 0} \frac{|\langle 000 | \hat{\mu}_y^B | n_{xB}n_{yB}n_{zB} \rangle|^2}{W_{n0}^B} \end{aligned}$$

$$\hat{\mu}_x^A = q_A x_A, \hat{\mu}_y^B = q_B y_B$$

$$|T_{xy}|^2 = \frac{(3R_x R_y - R^2 \delta_{ij})^2}{(4\pi\epsilon_0)^2 R^{10}}$$

Therefore we get

$$|T_{xy}|^2 = \frac{(3R_x R_y)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$$E_A = \hbar\omega_{xA}, E_B = \hbar\omega_{yB}$$

Part 1st for  $U_{disp}$  The possible transition for molecule A is  $(m_{xA}m_{yA}m_{zA} = 100, 010, 001)$ ,

similarly for molecule B also, ( $n_{xB}n_{yB}n_{zB} = 100, 010, 001$ )

$$\begin{aligned}\langle 000 | \hat{\mu}_x^A | m_{xA} m_{yA} m_{zA} \rangle &= q_A \langle 0 | x | m_x \rangle_A \langle 0_y n_y \rangle_A \langle 0_z n_z \rangle_A \\ &= q_A \frac{\langle 0_{xA} | H_1(\alpha_{xA} x_A) | m_{xA} \rangle}{2\alpha_{xA}} \langle 0_y n_y \rangle_A \langle 0_z n_z \rangle_A\end{aligned}$$

$$\langle 000 | \hat{\mu}_y^B | n_{xB} n_{yB} n_{zB} \rangle = q_B \langle 0_x n_x \rangle_B \langle 0 | y | n_y \rangle_B \langle 0_z n_z \rangle_B$$

Solving the integration the way we did for polarization

$$q_A \left[ \frac{\alpha_y}{\sqrt{\pi}} \int_{-\infty}^{\infty} dy e^{-\alpha_y^2 y^2} \right] \left[ \frac{\alpha_z}{\sqrt{\pi}} \int_{-\infty}^{\infty} dz e^{-\alpha_z^2 z^2} \right] \left[ \frac{1}{2\alpha_x} \frac{\alpha_x}{\sqrt{\pi}} \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} dx e^{-\alpha_x^2 x^2} H_1(\alpha_x x) H_{n_x}(\alpha_x x) \right]$$

$$\int_{-\infty}^{\infty} dy e^{-\alpha_y^2 y^2} = \frac{\sqrt{\pi}}{\alpha_y}$$

similarly for z component

$$\int_{-\infty}^{\infty} dz e^{-\alpha_z^2 z^2} = \frac{\sqrt{\pi}}{\alpha_z}$$

for x part

$$\int_{-\infty}^{\infty} dx e^{-\alpha_x^2 x^2} H_{n_x}(\alpha_x x) H_1(\alpha_x x) = \frac{\sqrt{\pi}}{\alpha_x} 2^{n_x} n_x! \delta_{n_x 1}$$

For this we get

$$\langle 000 | \hat{\mu}_x^A | m_{xA} m_{yA} m_{zA} \rangle = \frac{q_A}{\sqrt{2}\alpha_{xA}}$$

Similarly for y

$$\langle 000 | \hat{\mu}_y^B | n_{xB} n_{yB} n_{zB} \rangle = \frac{q_B}{\sqrt{2}\alpha_{yB}}$$

Therefore we get

$$-|T_{xy}|^2 \frac{q_A q_B}{2\alpha_{xA}\alpha_{xB}}$$

So,

$$U_{disp} = -|T_{xy}|^2 \frac{q_A^2 q_B^2}{4\alpha_{xA}^2 E_A \alpha_{xB}^2 E_B} \left( \frac{E_A E_B}{E_A + E_B} \right)$$

$$U_{disp} = -|T_{xy}|^2 \left( \frac{q_A^2}{\hbar\omega_{xA}\alpha_{xA}^2} \right) \left( \frac{q_B^2}{\hbar\omega_{yA}\alpha_{yB}^2} \right) \left( \frac{\hbar\omega_{xA}\omega_{yB}}{4(\omega_{xA} + \omega_{yB})} \right)$$

or we can write

$$U_{disp} = -|T_{xy}|^2 (\alpha_{xx}^A \alpha_{yy}^B) \left( \frac{\hbar\omega_{xA}\omega_{yB}}{4(\omega_{xA} + \omega_{yB})} \right)$$

$$|T_{xy}|^2 = \frac{(3xy)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{xx}$

$$U_{disp}^{xx} = -|T_{xx}|^2 (\alpha_{xx}^A \alpha_{xx}^B) \left( \frac{\hbar\omega_{xA}\omega_{xB}}{4(\omega_{xA} + \omega_{xB})} \right)$$

$$|T_{xx}|^2 = \frac{(3x^2 - R^2)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{yy}$

$$U_{disp}^{yy} = -|T_{yy}|^2 (\alpha_{yy}^A \alpha_{yy}^B) \left( \frac{\hbar\omega_{yA}\omega_{yB}}{4(\omega_{yA} + \omega_{yB})} \right)$$

$$|T_{yy}|^2 = \frac{(3y^2 - R^2)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{zz}$

$$U_{disp}^{zz} = -|T_{zz}|^2 (\alpha_{zz}^A \alpha_{zz}^B) \left( \frac{\hbar\omega_{zA}\omega_{zB}}{4(\omega_{zA} + \omega_{zB})} \right)$$

$$|T_{zz}|^2 = \frac{(3z^2 - R^2)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{xz}$

$$U_{disp}^{xz} = -|T_{xz}|^2 (\alpha_{xx}^A \alpha_{zz}^B) \left( \frac{\hbar\omega_{xA}\omega_{zB}}{4(\omega_{xA} + \omega_{zB})} \right)$$

$$|T_{xz}|^2 = \frac{(3xz)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{yz}$

$$U_{disp}^{yz} = -|T_{yz}|^2 (\alpha_{yy}^A \alpha_{zz}^B) \left( \frac{\hbar\omega_{yA}\omega_{zB}}{4(\omega_{yA} + \omega_{zB})} \right)$$

$$|T_{yz}|^2 = \frac{(3yz)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{yx}$

$$U_{disp}^{yx} = -|T_{yx}|^2 (\alpha_{yy}^A \alpha_{xx}^B) \left( \frac{\hbar\omega_{yA}\omega_{xB}}{4(\omega_{yA} + \omega_{xB})} \right)$$

$$|T_{yx}|^2 = \frac{(3yx)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{zx}$

$$U_{disp}^{zx} = -|T_{zx}|^2 (\alpha_{zz}^A \alpha_{xx}^B) \left( \frac{\hbar\omega_{zA}\omega_{xB}}{4(\omega_{zA} + \omega_{xB})} \right)$$

$$|T_{zx}|^2 = \frac{(3zx)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$U_{disp}$  for  $T_{zy}$

$$U_{disp}^{zy} = -|T_{zy}|^2 (\alpha_{zz}^A \alpha_{yy}^B) \left( \frac{\hbar \omega_{zA} \omega_{yB}}{4(\omega_{zA} + \omega_{yB})} \right)$$

$$|T_{zy}|^2 = \frac{(3zy)^2}{(4\pi\epsilon_0)^2 R^{10}}$$

$$U_{disp} = - \sum_{ab}^{xyz} |T_{ab}|^2 (\alpha_{aa}^A \alpha_{bb}^B) \left( \frac{\hbar \omega_{aA} \omega_{bB}}{4(\omega_{aA} + \omega_{bB})} \right) \quad (7)$$

## 2 Monomer Fragmentation

The monomer fragmentation scheme for all the monomers (except Ethene, Water, Ethyne) are given below.

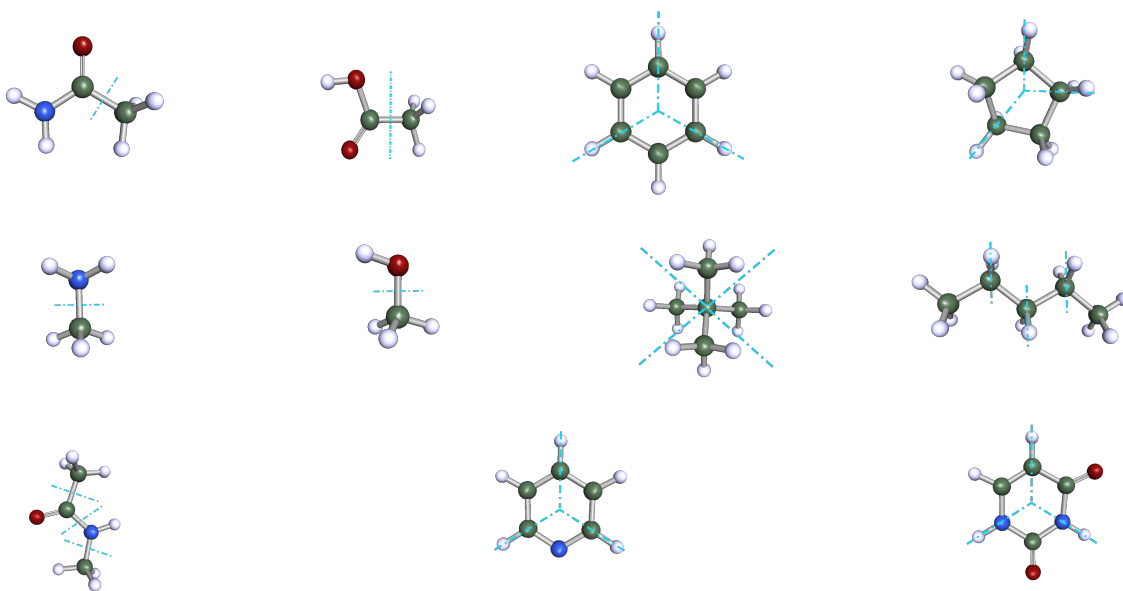
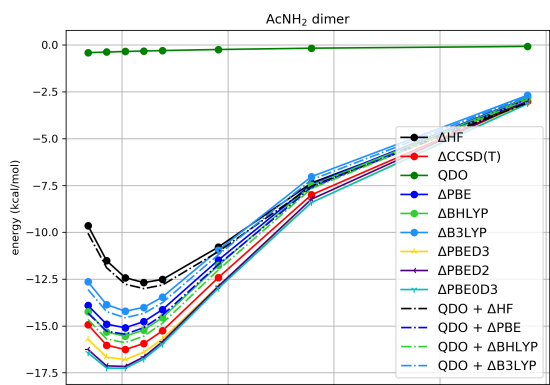


Figure S1: Fragmentation scheme for monomers. Color Code: Green: C, White: H, Red: O, Blue: N

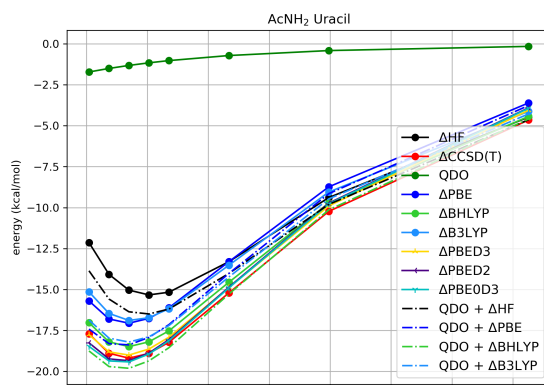
## 3 Interaction energy curves for Complexes

### 3.1 Interaction energy curves for all complexes of S66x8 dataset

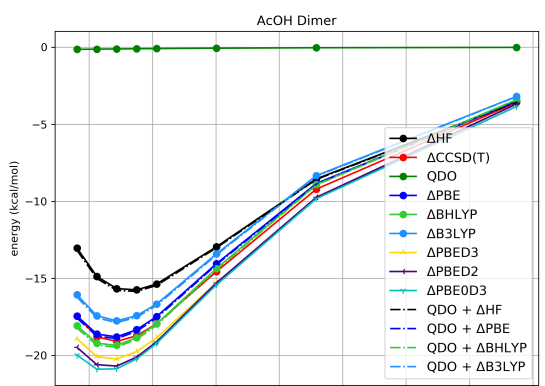
The Following Figures show the interaction energy curve for all the 66 complexes obtained with HF, CCSD(T) and DF methods.



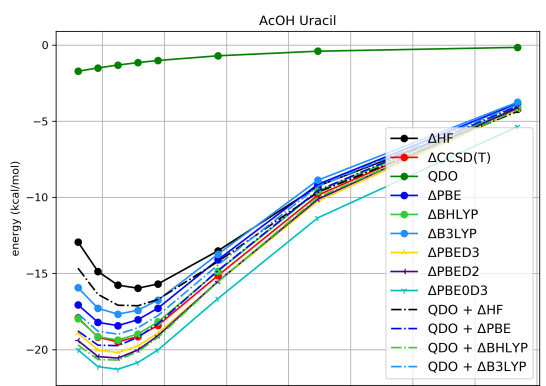
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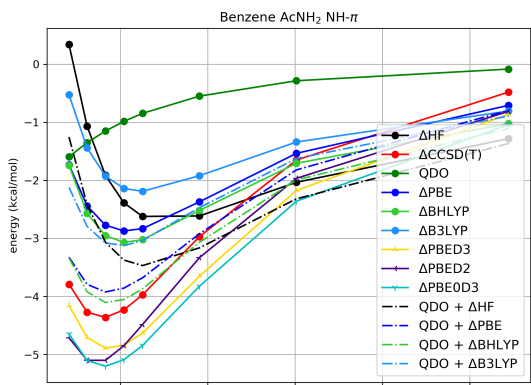
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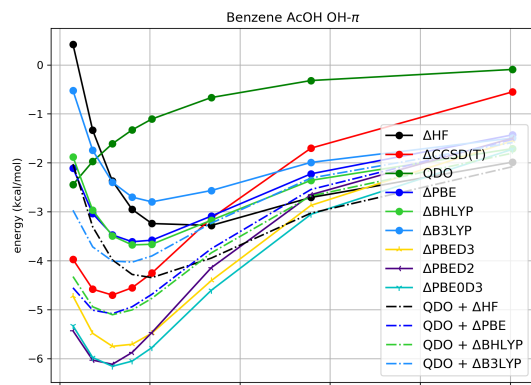
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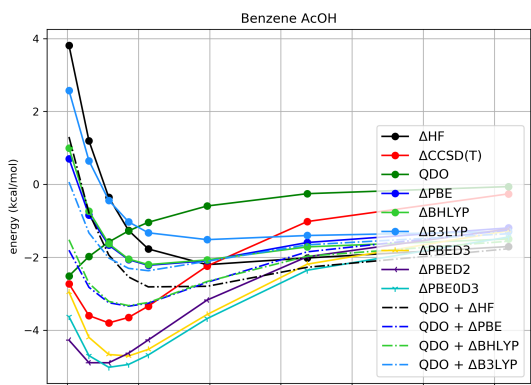
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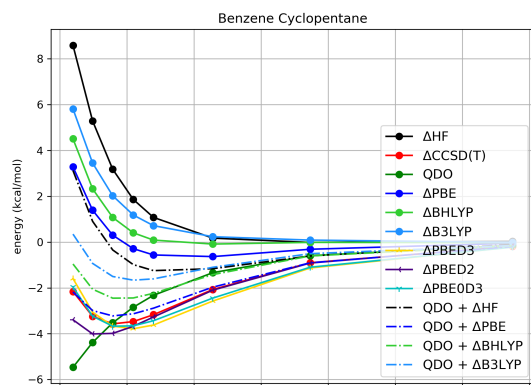
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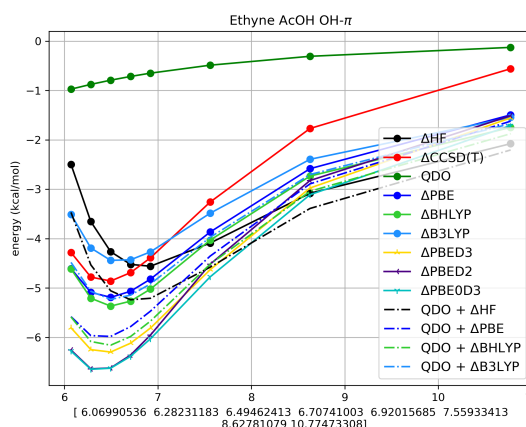
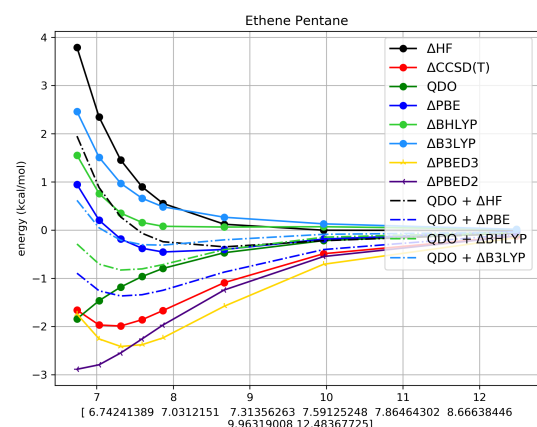
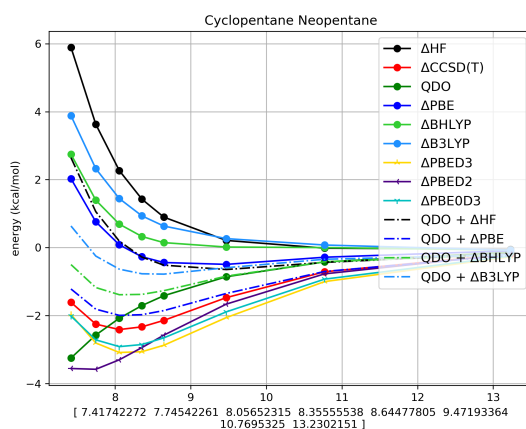
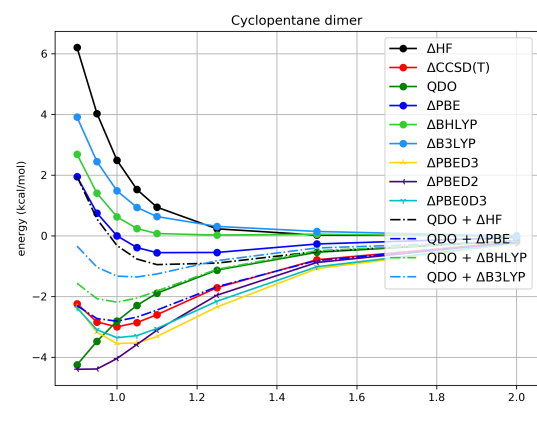
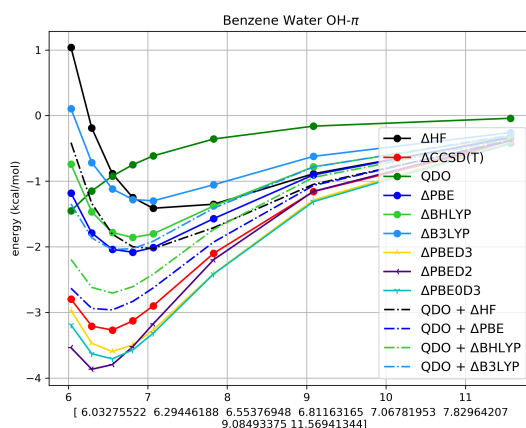
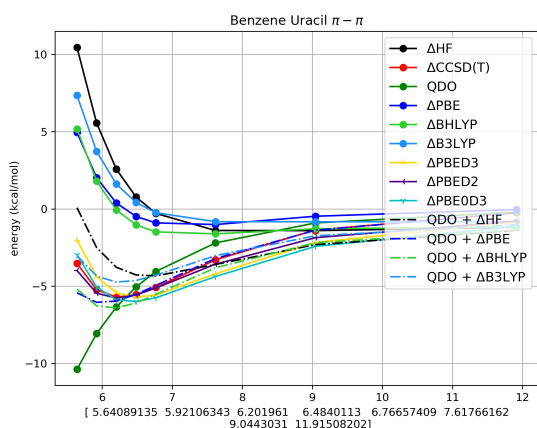
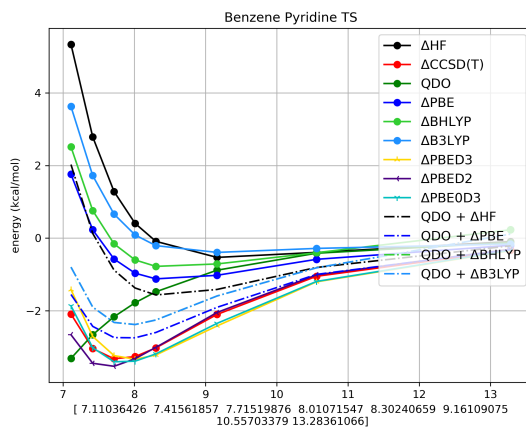
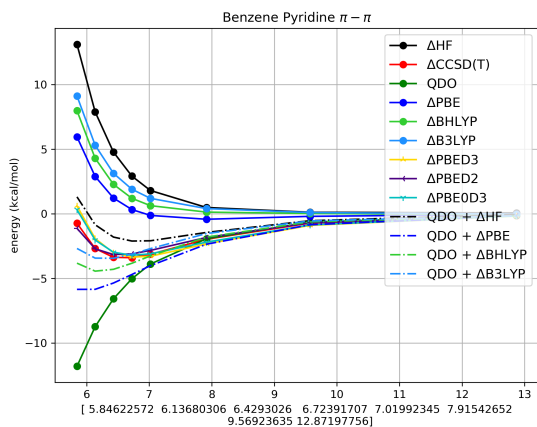
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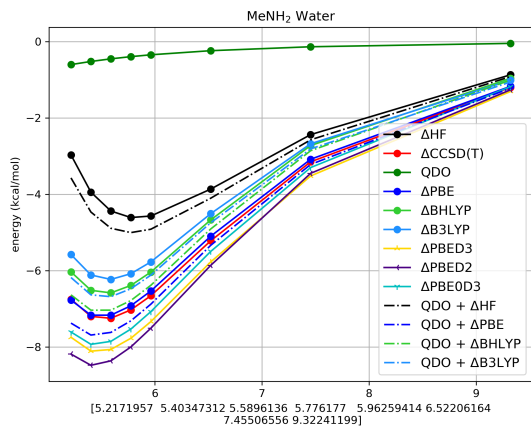
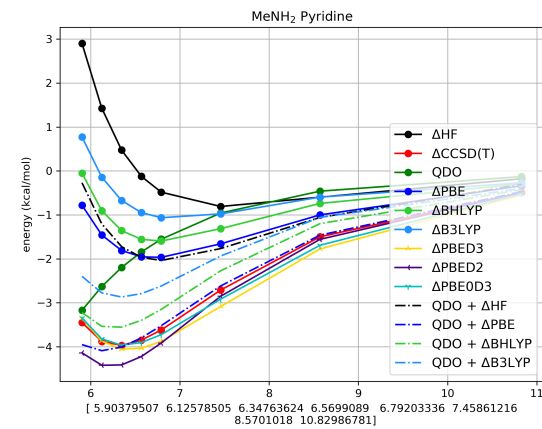
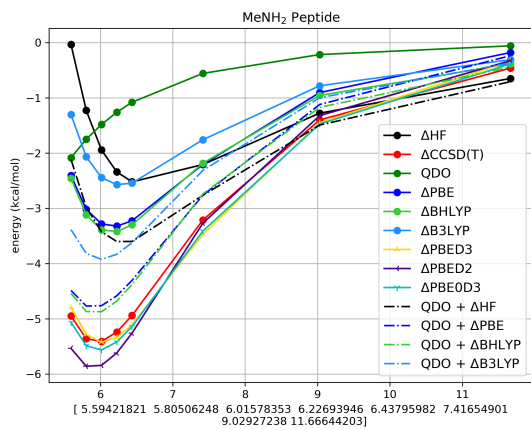
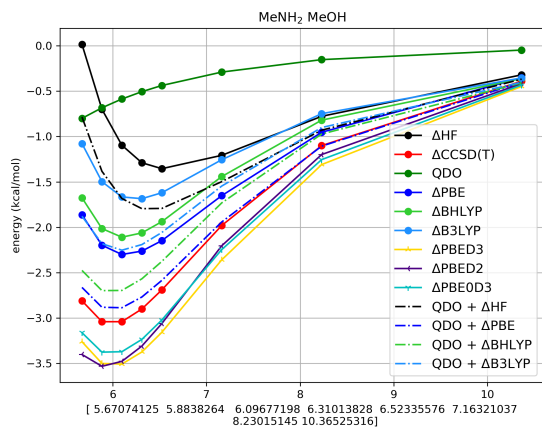
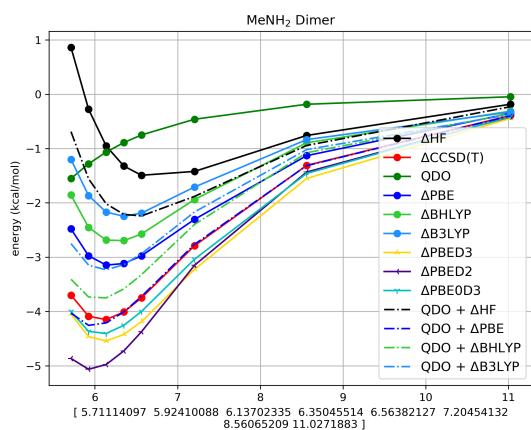
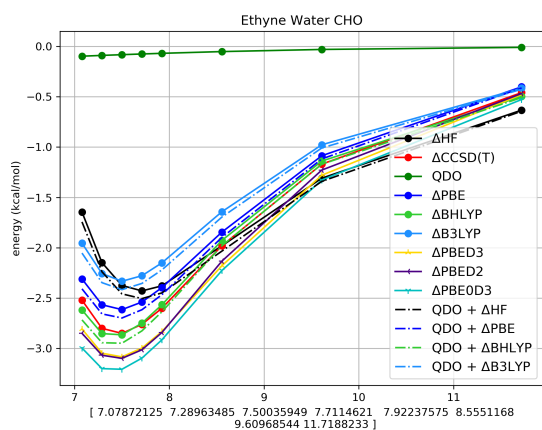
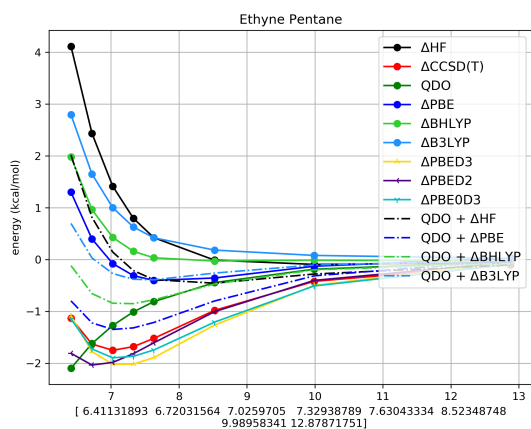
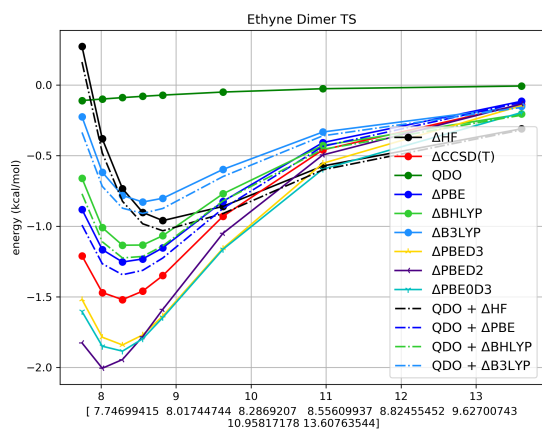


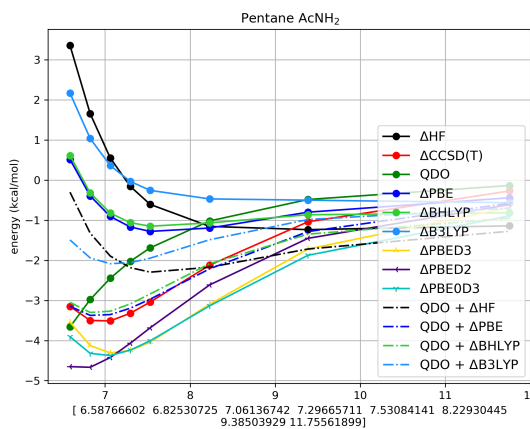
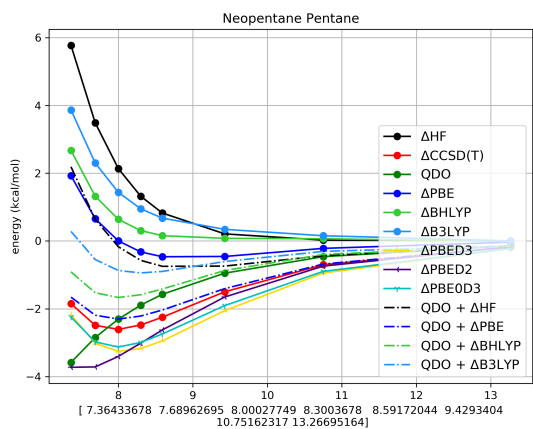
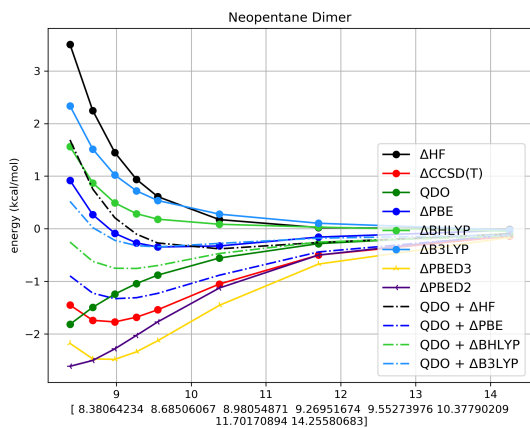
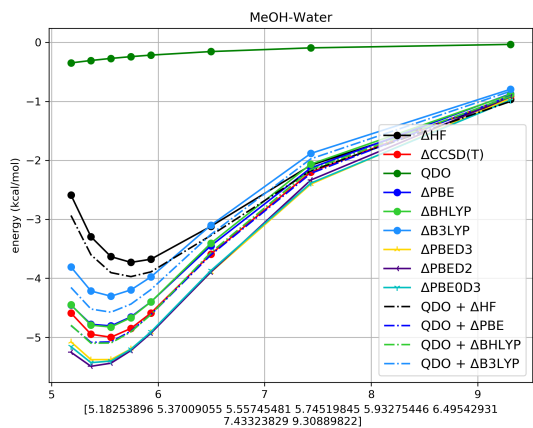
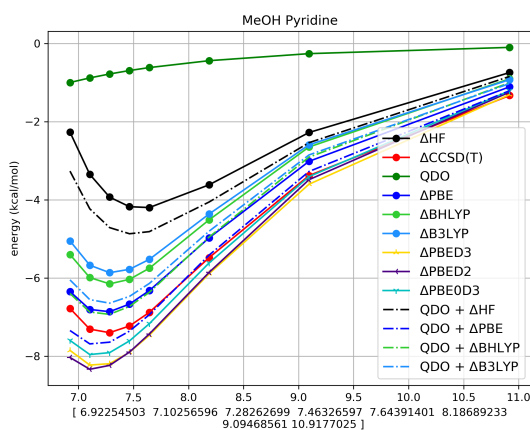
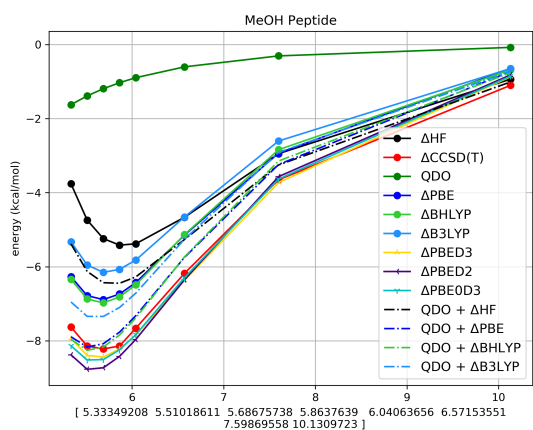
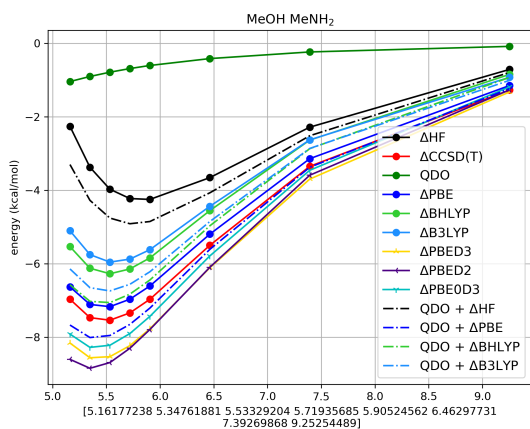
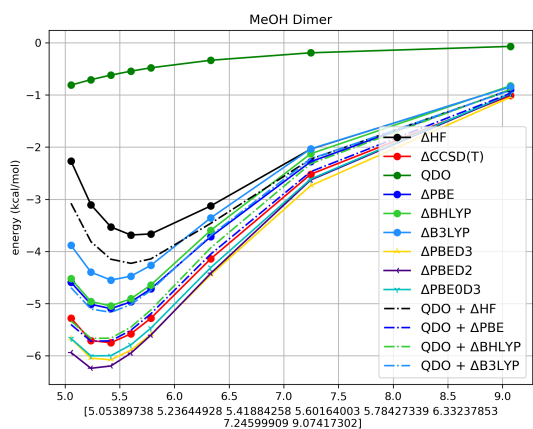
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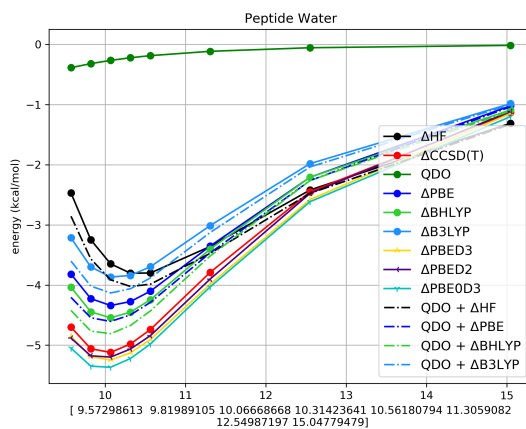
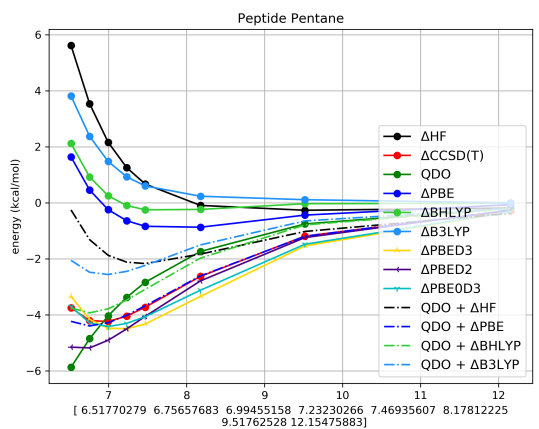
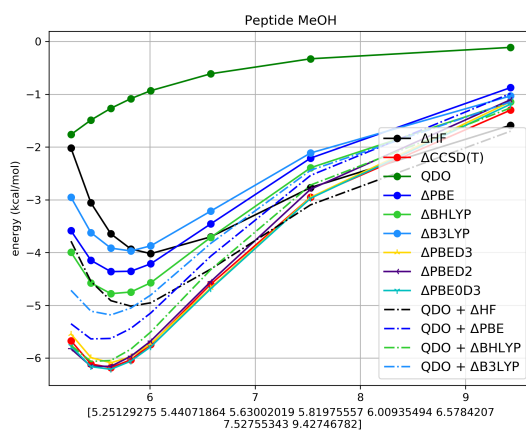
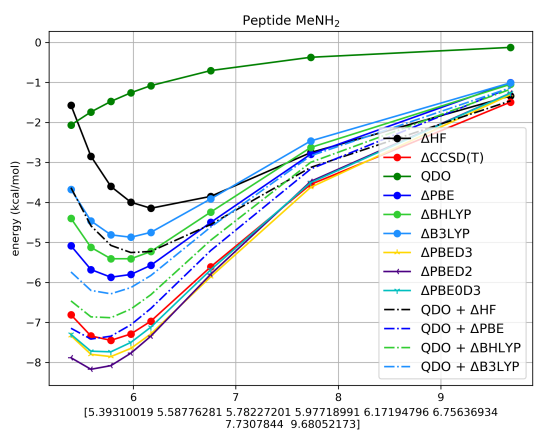
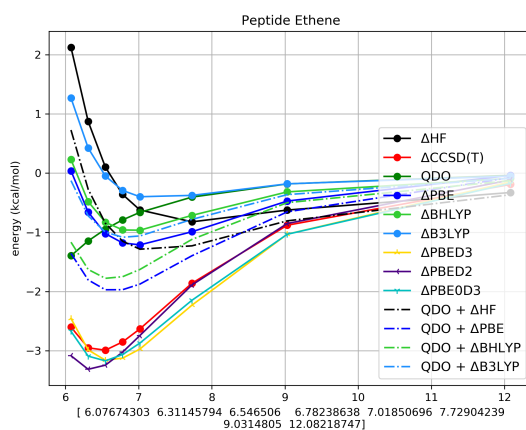
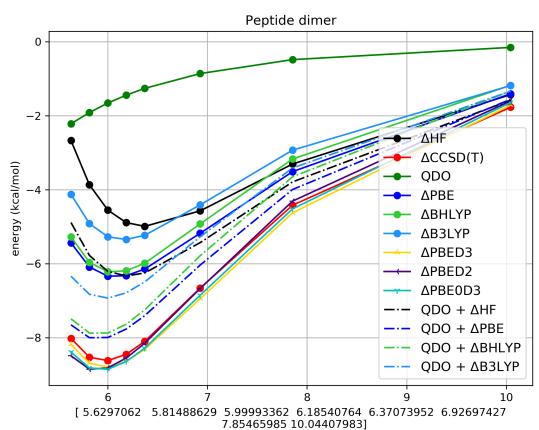
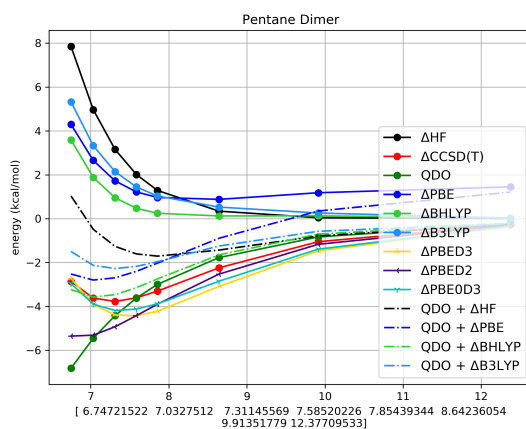
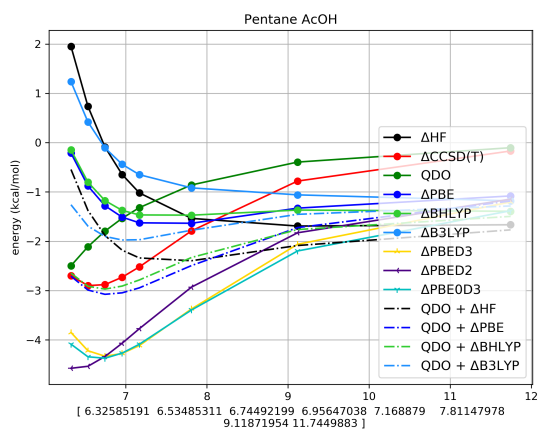


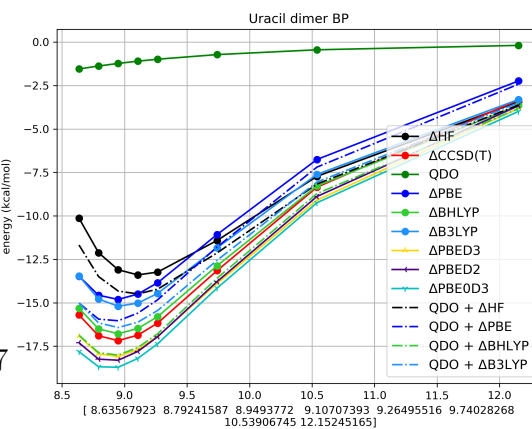
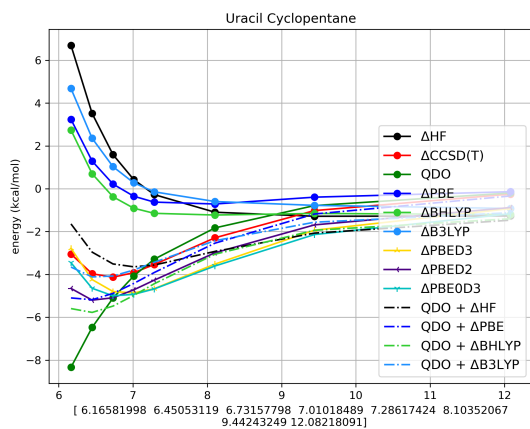
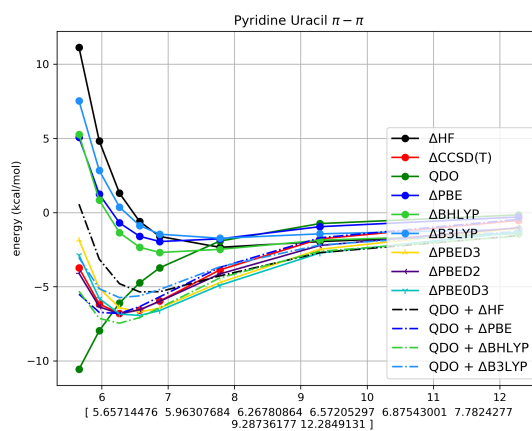
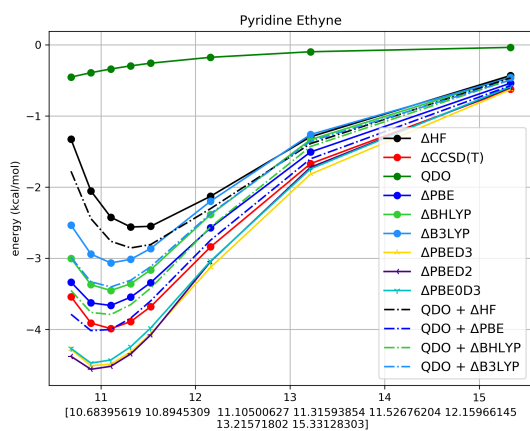
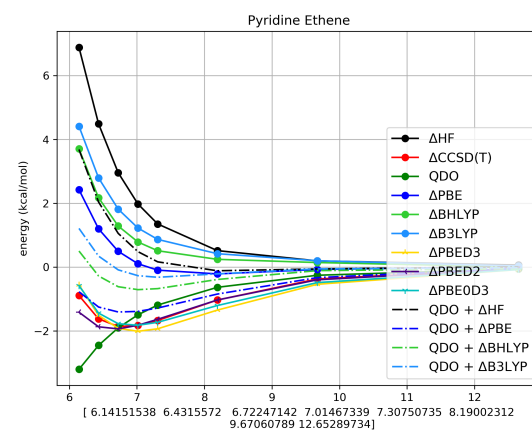
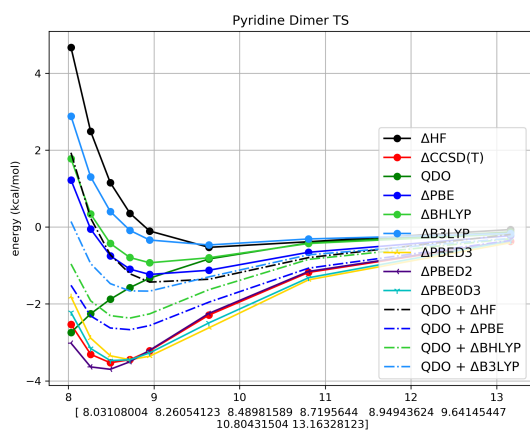
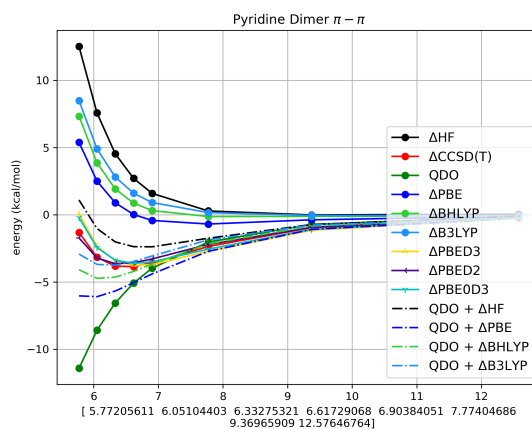
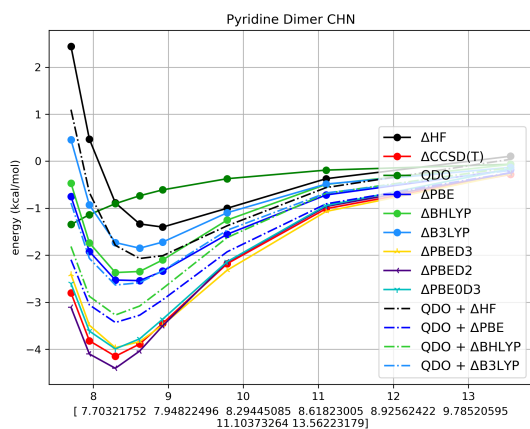


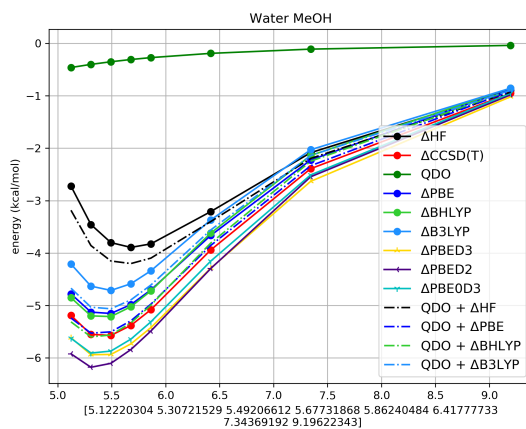
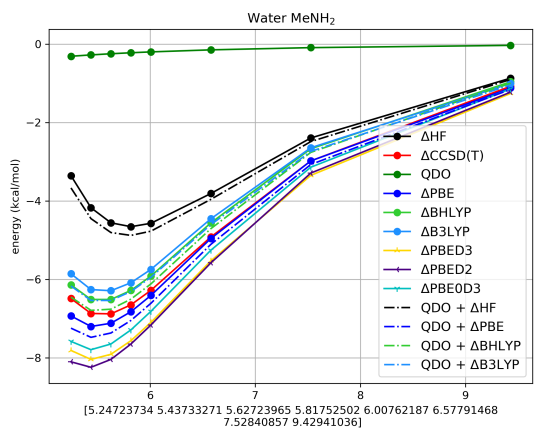
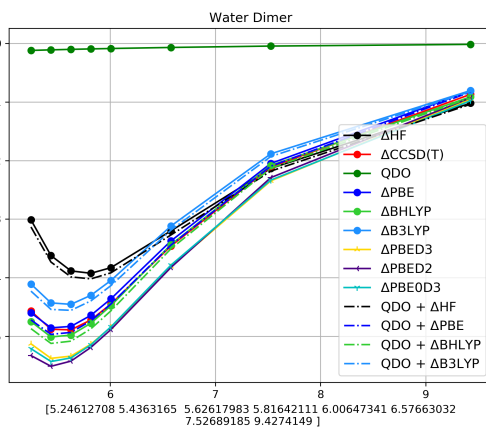
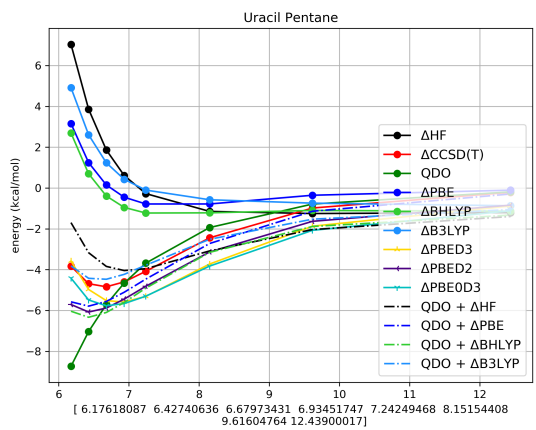
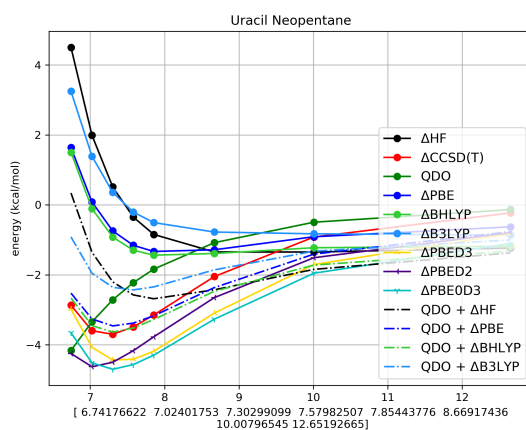
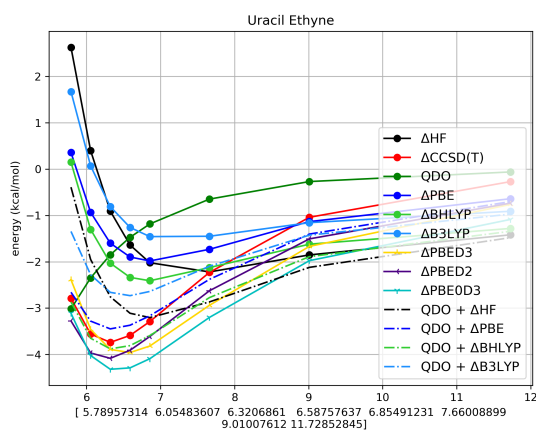
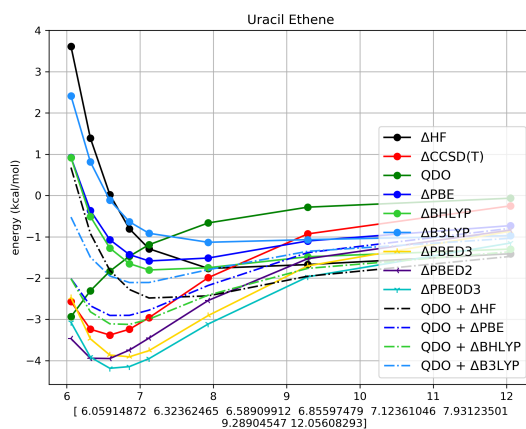
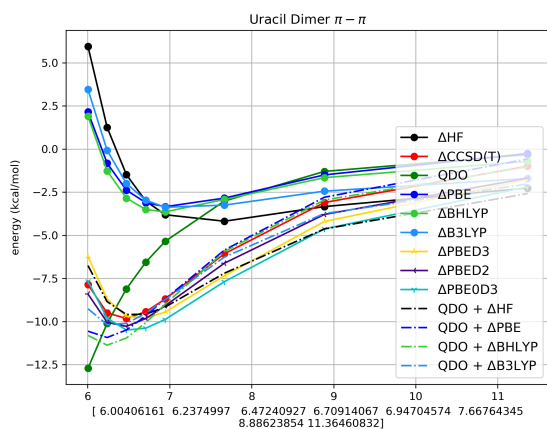














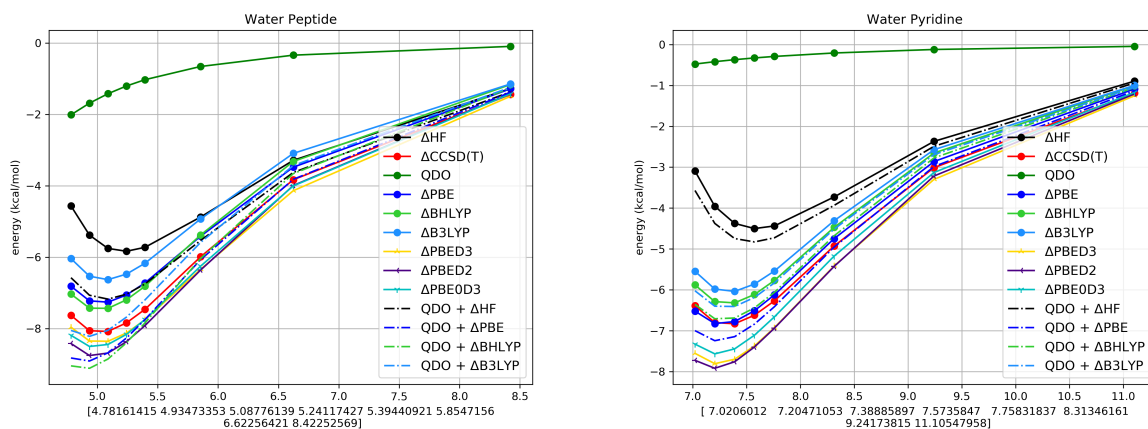


Figure S2: Interaction energy curves for 66 complexes with 66x8 data points.

### 3.2 Interaction energy curves for different fragmentation scheme of Uracil dimer base-pair and AcNH<sub>2</sub> dimer

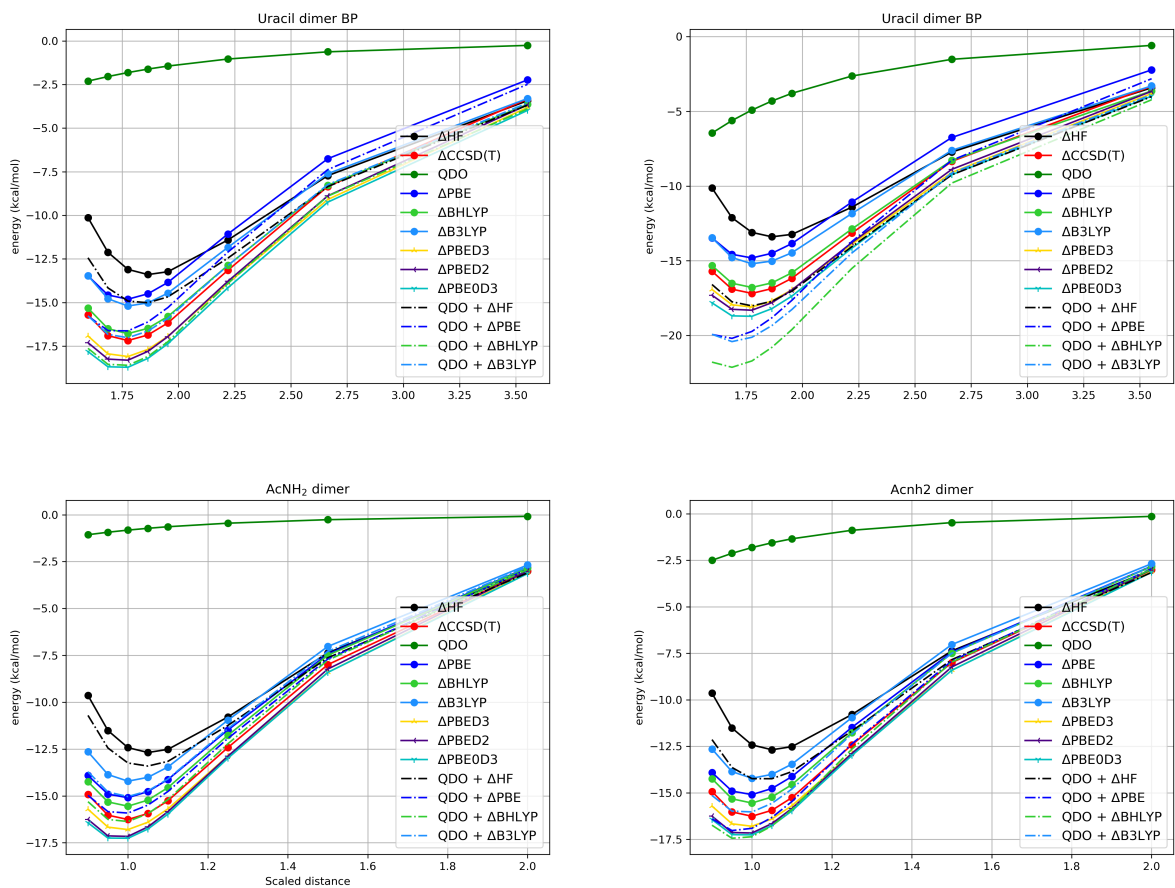


Figure S3: Interaction energy curves for Uracil dimer base-pair and AcNH<sub>2</sub> dimer. Top-left: Uracil spliced into 3 fragments, top-right: Uracil spliced into 6 fragments, bottom-left: AcNH<sub>2</sub> spliced into 2 fragments and bottom-right: AcNH<sub>2</sub> spliced into 3 fragments