

**SUPPORTING INFORMATION**  
**Graph Convolutional Neural Networks for (QM)ML/MM**  
**Molecular Dynamics Simulations**

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# 1 Model Selection

**Table S1:** Influence of the model depth on the mean absolute error (MAE) on the validation set for the basic GCNN model for the test systems benzene in water and uracil in water. For each property, the model with the lowest MAE is marked in bold.

<b>Benzene</b>			
Depth	$\Delta$ -learning		
	$E$ kJ mol <sup>-1</sup>	$F_{QM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>	$F_{MM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>
2	1.7	<b>20.7</b>	4.0
4	1.2	21.0	3.6
6	<b>1.0</b>	22.9	3.5
8	<b>1.0</b>	24.6	<b>3.4</b>

<b>Uracil</b>			
Depth	$\Delta$ -learning		
	$E$ kJ mol <sup>-1</sup>	$F_{QM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>	$F_{MM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>
2	<b>1.6</b>	<b>49.5</b>	<b>1.2</b>
4	2.0	55.5	<b>1.2</b>
6	2.2	60.2	1.3
8	32.6	62.7	1.3

**Table S2:** Influence of the number of features ( $n_f$ ) on the mean absolute error (MAE) on the validation set for the basic GCNN model for the test systems benzene in water and uracil in water. For each property, the model with the lowest MAE is marked in bold.

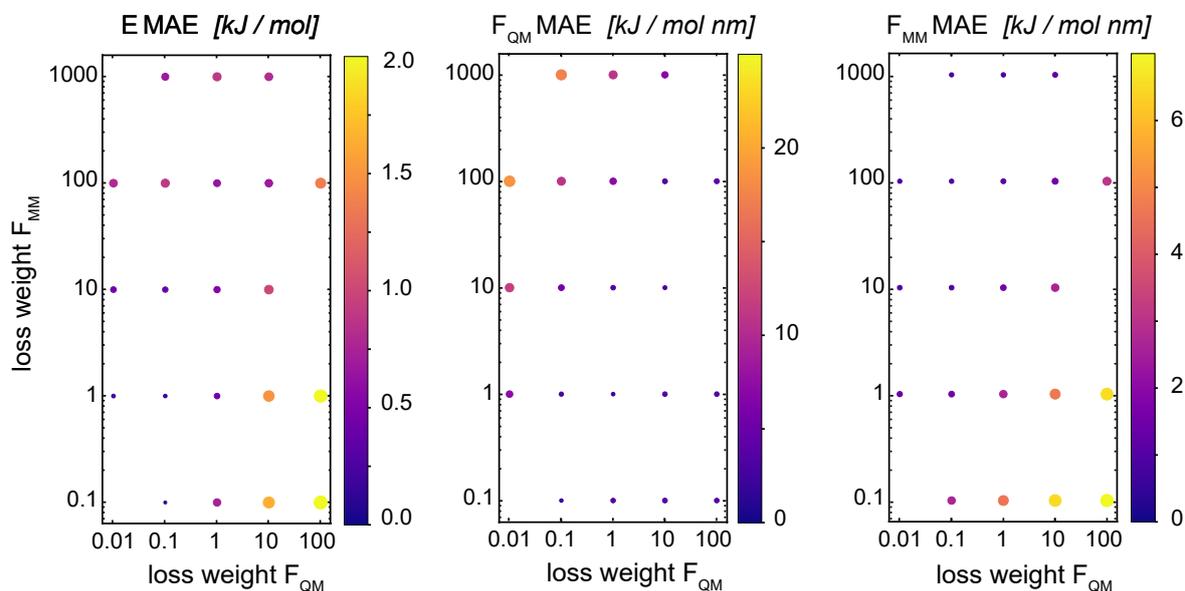
<b>Benzene</b>			
$n_f$	$\Delta$ -learning		
	$E$ kJ mol <sup>-1</sup>	$F_{QM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>	$F_{MM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>
32	1.7	22.1	4.2
64	1.3	<b>21.5</b>	3.9
128	1.0	22.9	3.5
256	<b>0.9</b>	22.9	<b>3.1</b>

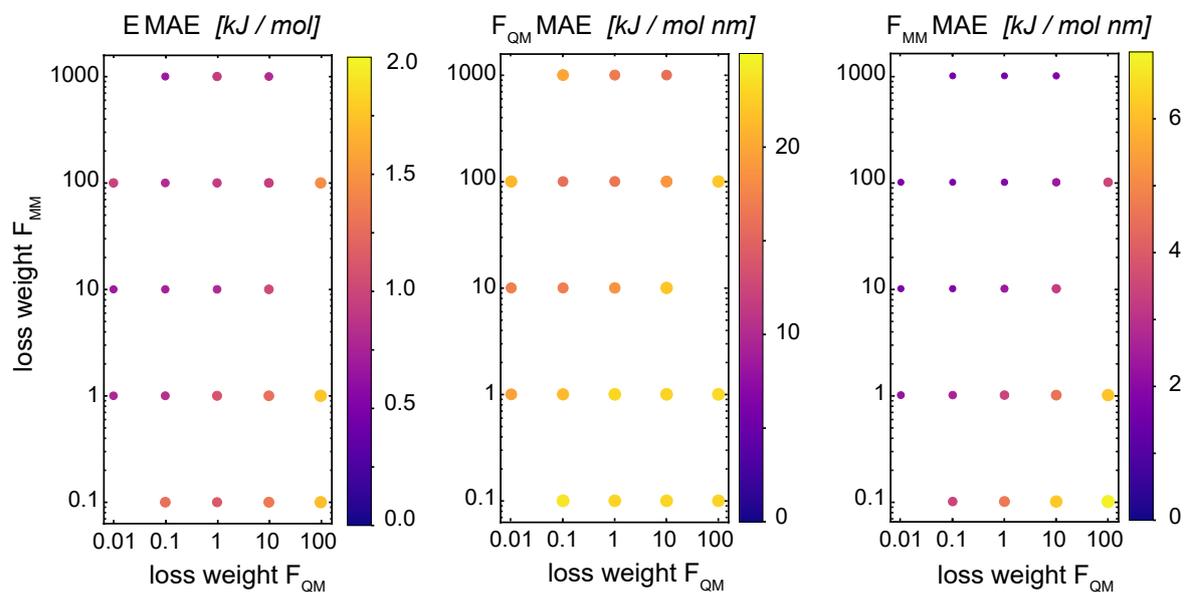
<b>Uracil</b>			
$n_f$	$\Delta$ -learning		
	$E$ kJ mol <sup>-1</sup>	$F_{QM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>	$F_{MM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>
32	2.5	54.3	<b>1.2</b>
64	<b>1.9</b>	<b>53.5</b>	1.3
128	2.2	60.2	1.3
256	2.5	60.8	1.3

## 2 Force Loss Contribution

training-set

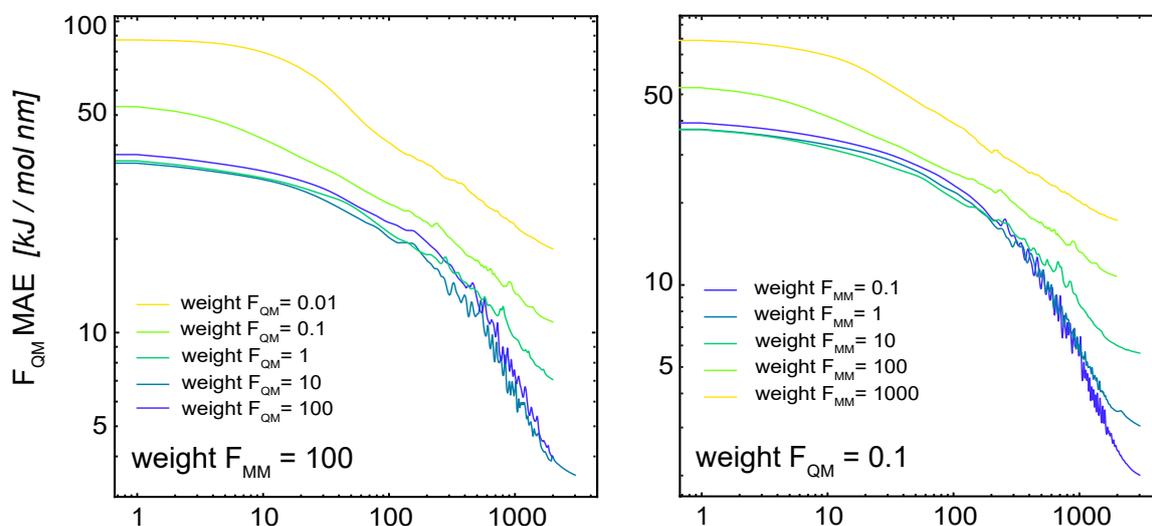


validation-set

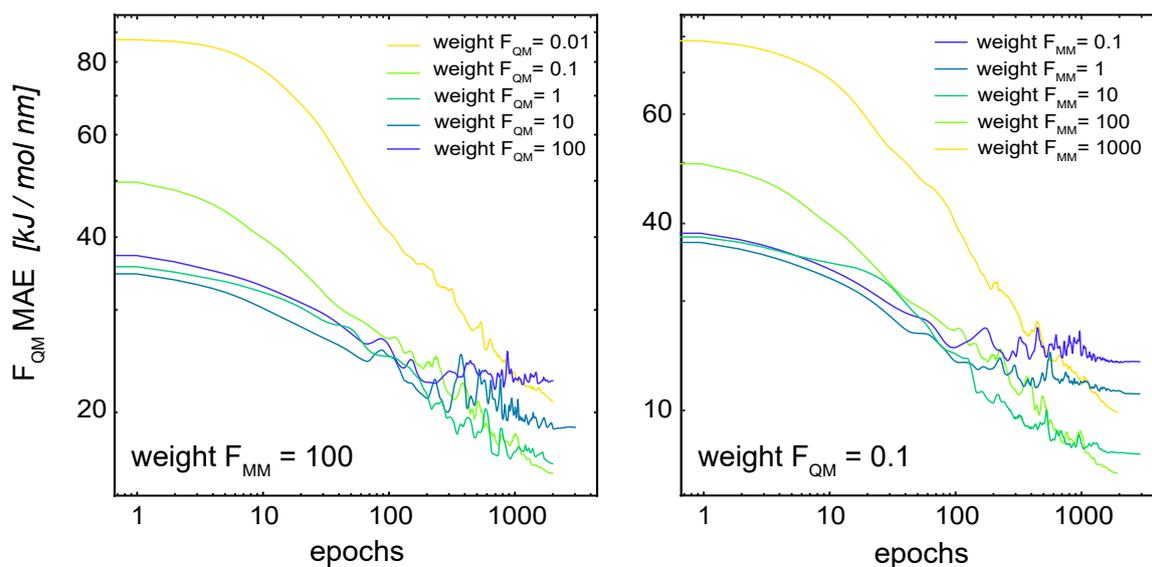


**Figure S1:** Influence of relative weights ( $wF_{QM}$  and  $wF_{MM}$ ) for the different loss terms (QM forces, MM forces) on the mean absolute error (MAE) of the energy (**left**), forces on QM particles (**middle**), and forces on the MM particles from the QM zone (**right**). The  $\Delta$ -learning GCNN was used with the test system benzene in water. The weight of the energy loss ( $wE$ ) was kept constant at 1.0. (**Top**): MAE for the training set. (**Bottom**): MAE for the validation set. The color map and the relative size of the points indicate the MAE.

### training-set



### validation-set



**Figure S2:** Learning curves for  $F_{QM}$  in the training set (**top**) and validation set (**bottom**) when varying the relative loss weightings  $wF_{QM}$  (**left**) and  $wF_{MM}$  (**right**). The  $\Delta$ -learning GCNN was used with the test system benzene in water. The weight of the energy loss ( $wE$ ) was kept constant at 1.0.

### 3 Neighborhood Reduction

**Table S3:** Influence of the neighborhood reduction schemes on the mean absolute error (MAE) on the validation set for the  $\Delta$ -learning GCNN model with the test systems benzene in water and uracil in water. For each property, the model with the lowest MAE is marked in bold.

<b>Benzene</b>			
Scheme	$\Delta$ -learning		
	$E$ kJ mol <sup>-1</sup>	$F_{QM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>	$F_{MM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>
Complete	<b>0.7</b>	<b>17.0</b>	<b>1.8</b>
KNN-8	2.2	28.4	5.1
KNN-12	1.5	26.3	4.3
Voronoi	2.1	34.8	3.1

<b>Uracil</b>			
Scheme	$\Delta$ -learning		
	$E$ kJ mol <sup>-1</sup>	$F_{QM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>	$F_{MM}$ kJ mol <sup>-1</sup> nm <sup>-1</sup>
Complete	<b>1.6</b>	<b>52.8</b>	<b>0.8</b>
KNN-8	2.6	60.8	1.0
KNN-12	2.0	57.6	1.0
Voronoi	3.1	73.9	0.9

#### 3.1 Data Set Ordering

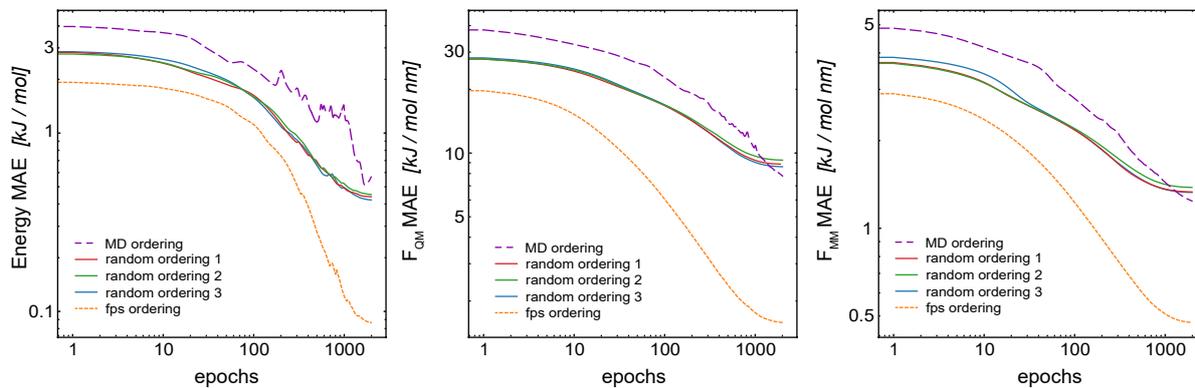
For the farthest-point-sampling (fps), we used a simple fingerprint together with a hash function. For each frame from the training set, we calculated all atom-atom distances ( $r_{ij}$ ) rounded to 0.001 nm and stored these together with the corresponding atom types as  $[Z_i, Z_j, r_{ij}]$ . Next, we hashed these objects to 64 bit and counted the occurrence of each bit within the hash of each frame. Finally, we compared the resulting bit counts of different frames using the Jaccard/Tanimoto index [1, 2],

$$T = \frac{\sum_k C_k}{\sum k(U_k - C_k)} \quad (1)$$

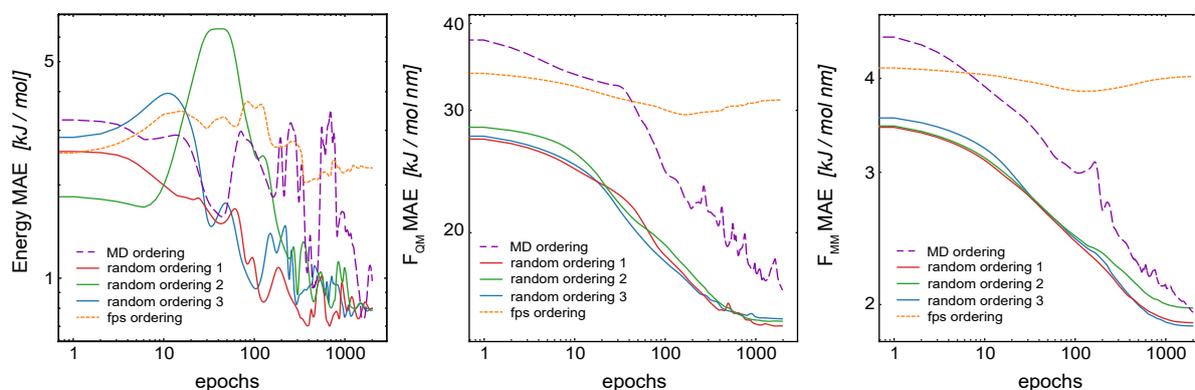
where  $C_k$  is the intersection of the counts at bit  $k$  between two frames and  $U_k$  is the union of the counts at bit  $k$  between two frames.

The fps-ordering was initialized at the first MD frame. At each step, the frame with the least summed similarity to all already chosen frames was added to the fps-ordered set until all training frames were contained.

### training-set



### validation-set



**Figure S3:** Learning curves for the training set (**top**) and validation set (**bottom**) when varying the order of the data points in the training set. The  $\Delta$ -learning GCNN for benzene in water is shown.

## References

- [1] Paul Jaccard. The distribution of the flore in the alpine zone. 1. *New Phytologist*, 11:37–50, 1912.
- [2] T T Tanimoto. *An Elementary Mathematical Theory of Classification and Prediction*. International Business Machines Corporation, 1958.