Electronic Supplementary Information (ESI):

Aerator bubble size estimation

To enable a detailed discussion on the effects of different aeration systems in enzyme solutions, mainly the aeration-coupled parameters, bubble size d_b µm, residence time and introduced interfacial area, are of interest and estimated in the following undersection. Based on the criteria of adjusting a constant k_La equal to 160 h^{-1}, the calculation of bubble characteristic parameters was performed.

With the assumption that ideal spherical shaped bubbles are generated, the shape factor φ (equation (7)) is equal to one (m) [72]. In respect to this, the bubble size d_b is equal to the Sauter mean diameter d_{32} and the volume-specific surface area SV (m^{-1}) is calculated as written in equation (7).

$$SV = \frac{6 \cdot \phi}{d_{32}} = \frac{6 \cdot \phi}{d_b}, \quad \phi = 1 \rightarrow d_{32} = d_b \, , \qquad\qquad (\ 7\ )$$

The aerator pore size of the sintered material on the one hand and the opening size of the open pipe on the other hand is resulting in huge differences in bubble generation behaviour as there is an expansion in size by rising, Young Laplace pressure etc. [42].

Bubble size for open pipe with 5 mm opening

Using a 5 mm (single opening) open pipe, Weber number We of >> 2 were calculated for the tested parameter. In consequence, a jet-gassing regime is present under the investigated flow rate of 16.67 vvm resulting in a k_La of 160 /h. According to Mersman, secondary bubble formation is present, when the critical criterion of We^2/Fr >> 675 is reached. This critical ratio is fulfilled for the open pipe aeration under previous named aeration conditions. Therefore, the correlation from Mersmann (1977) is applicable for the open pipe and is used as follows [43].

$$d_{AV} = d_{b,max} = 3 \cdot \sqrt{\frac{\sigma_L}{\Delta\rho \cdot g}} \qquad\qquad (\ 8\ )$$

Mersmann is assuming that the buoyancy force is equal to the surface tension, which is leading to $d_{b,max}$ (maximal bubble diameter). With this assumption and out of force balance the maximal stable bubble size $d_{b,max}$ is estimated [42]. This equation is exclusively valid for low viscosity liquids up to η_L = 3 mPa s[43]. In the underlying system, the viscosity is assumed to be in the same magnitude as water with 1 mPa*s (1.013 bar, 20°C) [73]. The equation (9) from Mersmann is applied to estimate the d_{32} of the 5 mm open pipe aeration for low-viscosity liquids [43].

$$d_{32} = 0.6 \cdot d_{b,max} \qquad\qquad (\ 9\ )$$

Bubble size for sintered frit with 10 µm average pore size

The bubble diameter d_b, which is generated by the 10 µm sintered frit, is estimated using the correlation from Welty in equation (10) [74]. To apply this equation, following assumptions have to be taken into account: the buoyancy is equal to the force resulting from the surface tension at the pore of the aerator, the bubbles are present as separate individual bubbles and a low gas volume flow is set [74].

$$d_{AV} = d_b = \left(\frac{6 \cdot d_p \cdot \sigma_L}{g\,(\rho_L - \rho_G)}\right) \qquad\qquad (\ 10\ )$$

Adjusting a constant k_La of 160 h^{-1} with the 10 µm sintered frit, a gassing rate of 1.36 vvm had to be applied. With equation (10) and assuming φ = 1, an d_{32} of 768.77 µm is calculated for the 10 µm sintered frit.

Bubble size for sintered frit with 2 µm average pore size

The Sauter mean diameter and bubble size distribution using the 2 µm pore size sintered frit is reported in a previous contributions of our group[9] . In respect to this, the d_{32} is equal to 320 µm and this value is used in the underlying contribution. Further, out of the reported bubble size distribution, a lower fraction of submillibubbles and a high fraction of bubbles in the microbubble range were generated [9]. The influence of the larger submillibubble

fraction according to the high volume cavity is resulting in an increased d_{32} value of 320 µm. Nevertheless, the 2 µm pore size sintered frit is suitable for generation of high fractions of microbubbles, which is in good agreement with the observed behaviour of the experiments.

Estimation of terminal rising velocity and interfacial area
As the bubble diameter decreases, the terminal rising velocity is decreased, which leads to a longer residence time τ (s). The Stokes' law is the classic approach to calculate the terminal rising velocity of a gas bubble [75,76]. The assumption of an immobile bubble surface with low Reynolds numbers Re << 1 is required for using the Stokes equation, which fine bubbles fulfil [75]. Another interpretation of the Naiver-Stokes equation is introduced by Hadamard and Rybczynsk in 1911, whereby the viscosity η (N s m^{-2}) and the disperse phase and the height h (m) of the liquid phase are taken into account [76,77]. Accordingly, the terminal residence time τ (s) of a bubble is calculated using equation (11).

$$\tau = \frac{d_{32}^2 \cdot g \cdot (\rho_{Gas} - \rho_{Liquid})}{18 \cdot \eta \cdot h} \qquad\qquad (\ 11\ )$$

The produced bubble sizes from 2 µm sintered frit results in significantly longer residence time as compared with the 10 µm sintered frit (submillibubble) and the 5 mm open pipe (macro bubble) aerators. A long residence times is equating a longer interaction between the gaseous and the aqueous phase. This leads to an increase in mass transport. A comparison of fine and macro bubbles results in a ~ 280 times longer residence time. As a result of continuous aeration, the theoretically introduced surface per time $\dot{a}$ (m²/min) is calculated based on the volumetric gassing rate $\dot{V}$ (m³/min) as shown in equation (12).

$$\dot{a} = SV \cdot \dot{V}_{gas} \qquad\qquad (\ 12\ )$$

To enable a more detailed comparison of the introduced surfaces, the product of the surface stream $\dot{a}$ and the residence time τ is resulting in an expression of the operative introduced surface ($a_{operative}$ in m²) using equation (13).

$$a_{eff} = \dot{a} \cdot \tau \qquad\qquad (\ 13\ )$$