Supporting Information for:

Characterisation of intergrowth in metal oxide materials using structure-mining: the case of γ-MnO₂

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Figure S1: TEM images of sample 1(a-b), 2(c-d) and 3(e-f)



Figure S2: Example of fitting procedure of Bragg peaks on XRD data (a) and pair distance at 2.8 and 3.4 Å (b) and 5.3 and 5.7 Å (c) on PDF data. Here the peak fitting is performed on the calculated XRD pattern and PDF of the mined superstructure with $P_b = 0.5$ and $P_r = 0.7$. For Bragg peaks, a Pseudo-Voight function is fitted on the peak and for pair distance peak, a Gaussian function is used for the fit. For both, experimental data points are represented by blue dots, the calculated model by a solid red line and the difference curve by a solid green line.



Figure S3: Ratio between the FWHM of the 110_R and 011_R lines from the modelled XRD patterns plotted against P_r (blue dots) fitted using a quadratic trend (red solid line) using patterns for which $\%\beta$ -MnO₂ = 27 % for sample 1 (a), $\%\beta$ -MnO₂ = 40 % for sample 2 (b), $\%\beta$ -MnO₂ = 82 % for sample 3 (c).



Figure S4: Fitting of the 110_R , 110_β , 011_R and 011_β Bragg peaks on XRD data (a, c, g) and pair distance at 2.8 and 3.4 Å (b, e, h) and 5.3 and 5.7 Å (c, f, i) on PDF data, of samples 1, 2 and 3. For Bragg peaks, a Pseudo-Voight function is fitted on the peak and for pair distance peak, a Gaussian function is used for the fit. For both, experimental data points are represented by blue dots, the calculated model by a solid red line and the difference curve by a solid green line.



Figure S5: Ratio between the intensity of peaks at 5.3 and 5.8 Å from the modelled PDF patterns plotted against P_b (blue dots) fitted using a quadratic trend (red solid line) using patterns for which $\%\beta$ -MnO₂ = 27 % for sample 1 (a), $\%\beta$ -MnO₂ = 40 % for sample 2 (b), $\%\beta$ -MnO₂ = 82 % for sample 3 (c).

Refined parameter XRD	Sample 2 superstructure (0.72, 0.81)	
a (Å)	1140.855	
b (Å)	4.443	
c (Å)	2.874	
B _{iso} Mn (Å ²)	0.0843	
B _{iso} O (Å ²)	0.1683	

Refined parameter PDF	Sample 2 superstructure (0.69, 0.72)	
a (Å)	1132.151	
b (Å)	4.429	
c (Å)	2.869	
B _{iso} Mn (Å ²)	0.1594	
B _{iso} O (Å ²)	0.2921	

Table S1: Example of refined parameters during the structure-mining of sample 2 on XRD (a) and PDF (b). The refined superstructure whose results are shown here corresponds to the one for which the (Pr, Pb) couple is determined by the peak-fitting analysis, respectively (0.72, 0.81) for XRD and (0.69, 0.72) for PDF.

Sample	Pb	Pr	Phase fraction of α -MnO ₂ (%)
Sample 1	0.53	0.87	12.5
Sample 2	0.69	0.72	18.1
Sample 3	0.97	0.9	12.8

Table S2: Refined volumetric phase fraction of α -MnO₂ by Rietveld refinement of superstructures with (P_b , P_r) couple found with the peak-fitting analysis on samples 1, 2 and 3.



Figure S6: Enlargement of the low Q and low r region of the Rietveld and PDF refinements shown in Figure 6. Rietveld and PDF refinements of sample 1 are shown in (a, d), sample 2 in (b, e) and sample 3 in (c, f) respectively.



Fig S7: Summary of the ML-MotEx analysis of the XRD and PDF data of sample 1 (a-d), 2 (e, f) and 3 (g-j) respectively. Violin plot of the SHAP values obtained in the analysis of the different datasets, showing

preferable % β -MnO₂, P_b and P_r values and if intergrowth blocks in the starting model are favorable rather as a β - or R-MnO₂ block for the fit quality. (a, c, e, g, i) Each block in the supercell are colored with respect to the results from the ML-MotEx analysis. (b, d, f, h, j) If ML-MotEx prefer the block to be *R*- *MnO*₂, *it is colored blue, if ML-MotEx does not differentiate between the blocks, it is colored black and if ML-MotEx prefers* β -*MnO*₂ *it is colored red.*

Supplementary Notes 1: Whole Powder Pattern Modelling (WPPM)

Whole Powder Pattern Modelling is an approach for analysis XRD data based on modelling the experimental data bin one step developed by Scardi et al.¹ Instead of refining structural parameters by the mean of analytical functions such as pseudo-Voigt or Voigt, they model the XRD experimental pattern by convoluting the diffraction pattern of a crystalline phase by broadening models related to different effects, from the instrumental broadening of a peak to nanostructural effects such as finite size, strains or dislocations. Indeed, the diffracted intensity expressed as a function of the scattering vector shkl is a Fourier transform of the electronic density in a crystal. This analysis thus bases on the assumption that the contribution of each of the broadening effects listed before can be expressed as a sum of Fourier components that are added to the expression of the diffracted intensity. The analysis of Fourier components of these convolution functions allows a quantification of these different effects.

Following that type of methodology, it is possible to convolute the effect of a crystallite size distribution on the model. This size distribution must be modelled by a mathematical function and here we chose a lognormal distribution g(D) expressed as follow in Equation 6:

$$g(D) = \frac{1}{D\sigma\sqrt{2\pi}} e^{-\frac{(\ln D - \mu)^2}{2\sigma^2}}$$
 (6)

Where μ corresponds to the lognormal mean and σ corresponds to the lognormal variance. From these parameters μ and σ it is then possible to calculated the average size D_{average} of the distribution and its standard deviation s.d. as shown on Equation 4 and 5:

$$D_{average} = e^{(\mu + \frac{\sigma^2}{2})}$$
 and $s. d. = (e^{2\mu + \sigma^2} (e^{\sigma^2} - 1))$ (4)-(5)

Some softwares are implemented with the WPPM method where the parameters related to the different contributions to the experimental XRD pattern are refined alongside unit cell parameters using a least-square method. Here we used PM2K, developed by Leoni et al.² In our case, we use WPPM to model the crystallite size distribution along the [100] direction of β - and R-MnO₂ domains in our samples by modelling the peak shape of the (110) peak of each parent structure. The fit of the peak performed on PM2K as well as the refined μ and σ values are shown in Fig S7. This yields a size distribution along the [100] direction, i.e. the intergrowth direction, we then weight to yielded size distribution by a factor (2)^{1/2}/2 for β -MnO₂ domains as this corresponds to the ratio between the length of the [110] and [100] vectors in this structure, and by (a²+b²)^{1/2}/a for R-MnO₂ as it corresponds to the same ratio in the R-MnO₂ structure. This allows to determine the Domains Size Distribution of R- and β -MnO₂ domains in our samples as shown in Fig 8.



Figure S8: Fitting of the 110 reflection of R- and β -MnO₂ on XRD patterns of samples 1 (a-b), 2 (c-d) and 3 (e-f) by Whole Powder Pattern Modelling. The experimental data are represented by black dots, the calculated model in solid red line and difference curve in solid gray line.



Figure S9: Calculated Domain Size Distribution of R- and β -MnO2 domains in supercells for which structure refinements are shown on Figure 6, i.e. supercells determined by peak-fitting method, for sample 1 (a-b), sample 2 (c-d) and sample 3 (e-f)

References:

1. Scardi, P.; Leoni, M., Whole powder pattern modelling. Acta Crystallogr. A 2002, 58, 190-200.

2. Leoni, M.; Confente, T.; Scardi, P., PM2K: a flexible program implementing Whole Powder Pattern Modelling. *Z. Kristallogr.* **2006**, 249-254.