# A Appendix

## A.1 Gaussian beam parameters

The plano-concave optical microcavity is characterised by the cavity length $L$ and the radius of curvature of the concave mirror $R_L$. The Rayleigh range $z_R$ of the confined mode is then

$$z_R = L\sqrt{\left(\frac{R_L}{L}-1\right)}. \tag{A.1}$$

The radius of curvature of the wavefronts $R(z)$ and the beam radius $w(z)$ are then given by

$$R(z) = z\left(1+\left(\frac{z_R}{z}\right)^2\right), \tag{A.2}$$

$$w(z) = \sqrt{\frac{2zR(z)}{kz_R}}. \tag{A.3}$$

The intensity distribution for the resonant cavity mode supported between opposing concave and planar mirrors given in equation (5) is determined as the sum of two Gaussian beams propagating in opposite directions, $I = n_m\varepsilon_0 c|E_i+E_r|^2$, where the incident $(E_i(\rho,z))$ and reflected $(E_r(\rho,z))$ waves are given by[27]:

$$E_i(\rho,z) = E_0\frac{\omega_0}{\omega}\exp\left(\frac{-\rho^2}{\omega^2}\right)\exp\left(ikz+\frac{ik\rho^2}{2R}-i\arctan\left(\frac{z}{z_R}\right)\right), \tag{A.4}$$

$$E_r(\rho,z) = E_0\frac{\omega_0}{\omega}\exp\left(\frac{-\rho^2}{\omega^2}\right)\exp\left(-ikz-\frac{ik\rho^2}{2R}+i\arctan\left(\frac{z}{z_R}\right)\right). \tag{A.5}$$

For the measurements reported here we used $R_L = 12$ $\mu$m and $L = 960$ nm.

## A.2 Balancing optical and flow forces

The maximum optical force is obtained by establishing the maximum value of $\frac{dI}{dx}$ using the mode intensity in equation (5). This maximum force opposing the drag force due to the fluid flow occurs at $x = \frac{w}{2}, y = 0, z = 0$ at which point $\frac{dI}{dx} = \frac{2P}{\pi w^3\sqrt{e}}$. Substitution of this expression into the optical force term in equation (1) and equating with the viscous drag force due to fluid flow yields equation (7). This condition corresponds approximately to the potential energy profile shown in red in figure A.1, where the potential gradient tends to zero at the downstream side of the cavity mode.

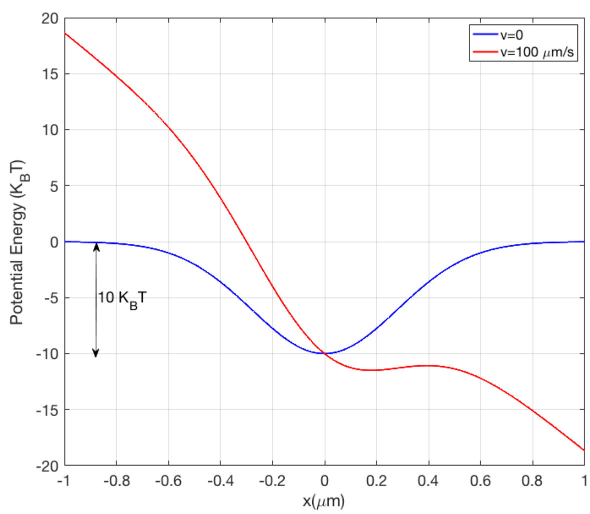


**Fig. A.1** Total potential energy along the axis $(x,0,0)$ for a PS particle diameter of 200 nm with no flow (blue curve) and with a flow speed of 100 $\mu$m s$^{-1}$ (red curve).

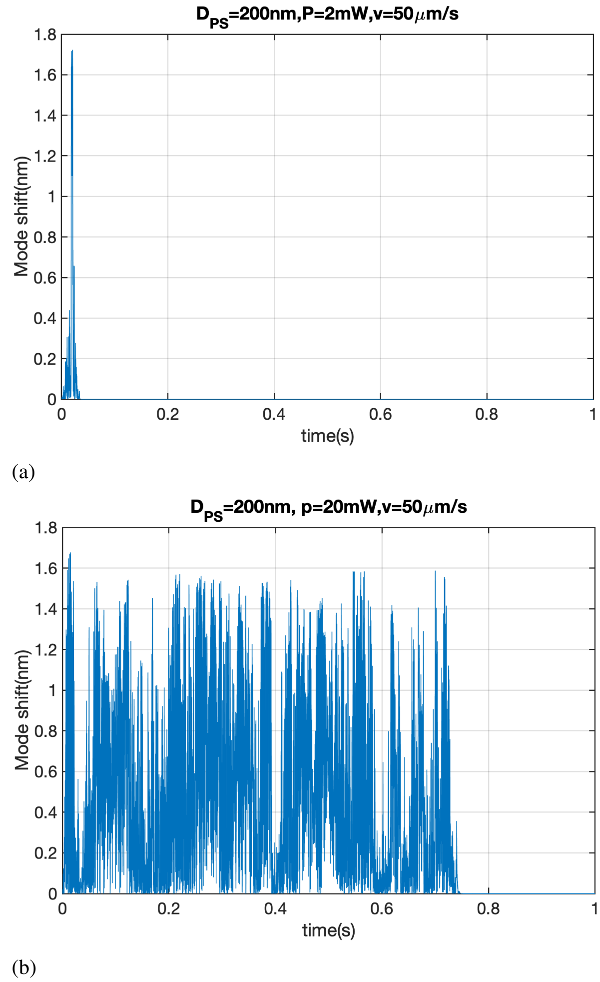


**Fig. A.2** The mode shift of a PS particle with diameter of 200 nm with a fluid flow speed of 50 $\mu$m s$^{-1}$ and intracavity powers of (a) 2 mW and (b) 20 mW.

## A.3 Monte Carlo simulation

Based on Equations (1) and (5), and the selection of the $x$ axis as the flow, the equations for incremental movements of a particle in the Cartesian coordinate system are:

$$x_i = x_{i-1} + \frac{\Delta t}{2n_m\gamma\varepsilon_0 c}\alpha\frac{dI(\underline{r})}{dx} + \sqrt{\frac{2K_BT\Delta t}{\gamma}}w_i + \Delta t v_{x0}, \tag{A.6}$$

$$y_i = y_{i-1} + \frac{\Delta t}{2n_m\gamma\varepsilon_0 c}\alpha\frac{dI(\underline{r})}{dy} + \sqrt{\frac{2K_BT\Delta t}{\gamma}}w_i, \tag{A.7}$$

$$z_i = z_{i-1} + \frac{\Delta t}{2n_m \gamma \varepsilon_0 c} \alpha \frac{dI(\underline{r})}{dz} + \sqrt{\frac{2K_B T \Delta t}{\gamma}}\, w_i. \tag{A.8}$$

Here $w_i$ is a computer generated, normally distributed random number with unity variance. The time increment $\Delta t$ is selected as 1 $\mu$s which is short enough to prevent 'tunneling' of the particle through the potential barriers. The Monte Carlo model allows simulation of the mode shift with time as the particle moves through the mode $I(r)$. Figure A.2a shows example mode shift events of a spherical PS nanoparticle diffusing through the cavity. The diameter and the velocity of the nanoparticle are 200 nm, and 50 $\mu$m s$^{-1}$, respectively. At an intracavity power of 2 mW the particle passes through the cavity mode in about 20 ms while at an intracavity power of 20 mW the particle remains in the mode for about 750 ms (figure A.2b).