Electronic Supplementary Information

Designable microfluidic ladder networks from backstepping microflow analysis for mass production of monodisperse microdroplets

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Note S1: Calculation of microchannel dimensions in microfluidic ladder networks for single-phase flow simulation

***Step 1**: K value*

Calculate the K value for the microfluidic ladder networks from the corresponding rule shown in **Table 1**.

***Step 2**: Relationship between K and microchannel dimensions*

The K is the flow resistance ratio of R_B/R_D. Based on Eq. (1), the relationship between the K and microchannel dimensions can be expressed as Eq. (S1):

$$K = R_B/R_D = [(w_B + h_B)^2 l_B/(w_B^3 h_B^3)]/[(w_D + h_D)^2 l_D/(w_D^3 h_D^3)] \tag{S1}$$

***Step 3**: Calculation of microchannel dimensions*

For the six microchannel sizes in Eq. (S1), five of them can be flexibly determined in a reasonable micrometer range, and then the rest one can be calculated based on the values of the five microchannel sizes and the K using Eq. (S1). The flexible choices of the five microchannel sizes are usually limited by the desired droplet sizes, and the microchannel manufacturing techniques. Here we fixed the w_D, h_D, l_D of distribution microchannel, and the w_B, h_B of branch microchannel, and then calculated the l_B of branch microchannel using Eq. (S1). The microchannel sizes of all microfluidic ladder networks for the simulation were listed in **Table S1**.

Note S2: Calculation of microchannel dimensions in microfluidic integrated devices

Step 1*: K value*

Calculate the K value for the microfluidic ladder networks from the corresponding rule shown in **Table 1**.

Step 2*: Relationship between K and microchannel dimensions*

The K is the flow resistance ratio of R_B/R_D. For the microfluidic integrated device with ladder networks containing wedge-type droplet generators, the flow resistance of branch microchannel (R_B) is the sum the flow resistances of the rectangular segment (R_{B1}) and the wedge segment (R_{B2}). Thus, the K can be expressed as Eq. (S2):

$$K = (RB1 + RB2)/RD \tag{S2}$$

R_{B2} is equivalent to the flow resistance of a rectangular microchannel with height (h_B) and length (l_{B2}) same as that of the wedge segment, but with width same as that of the middle part of the wedge segment (w_{Bm}). Thus, based on Eq. (1), the relationship between K and microchannel sizes can be expressed as Eq. (S3):

$$K = [(wB + hB)2lB1/(wB3hB3) + (wBm + hB)2lB2/(wBm3hB3)]/[(wD + hD \quad 3) \tag{S3}$$

Step 3*: Calculation of microchannel dimensions*

Similarly as mentioned in **Note S1**, for the eight microchannel sizes in Eq. (S3), seven of them can be flexibly determined in a reasonable micrometer range, and then the rest one can be calculated based on the values of the seven microchannel sizes and the K using Eq. (S3). The flexible choices of the seven microchannel sizes are usually limited by the desired droplet sizes, and the microchannel manufacturing techniques. Here we fixed w_D, h_D and l_D of the distribution microchannel, w_B, w_{Bm}, h_B and l_{B2} of branch microchannel, and then calculated the l_{B1} of the branch microchannel using Eq. (S3). Moreover, for the device designed by the existed rule, its two distribution microchannels show different widths ($W_{Din} \neq W_{Dout}$), thus we use the average value of segment flow resistances in the two distribution microchannels as the segment flow resistance R_D. The microchannel sizes of two microfluidic integrated devices were listed in **Table S2**.

Note S3: Calculation of resistance loss of path *i* in microfluidic ladder networks

For the microfluidic ladder network with *N* branch microchannels in counter-flow operation, The $h_{f,i}$ of flow path-*i* usually includes the resistance loss of inlet distribution microchannel ($h_{f,i\text{-}Din}$), branch microchannel ($h_{f,i\text{-}B}$) and inlet distribution microchannel ($h_{f,i\text{-}Dout}$), which is indicated by the blue line in Fig. S1a. The $h_{f,i\text{-}Din}$, $h_{f,i\text{-}B}$ and $h_{f,i\text{-}Dout}$ can be respectively expressed by Eq. (S4), Eq. (S5) and Eq. (S6) under the assumption of uniform flow distribution.

$$h_{f,i\text{-}Din} = [(N-1)Q_a + (N-2)Q_a + \ldots + (N-i+1)Q_a]R_D \tag{S4}$$

$$h_{f,i\text{-}B} = Q_a R_B \tag{S5}$$

$$h_{f,i\text{-}Dout} = [(N-i+1)Q_a + (N-i+2)Q_a + \ldots + (N-1)Q_a]R_D \tag{S6}$$

The $h_{f,i}$ of flow path-*i* is the sum of $h_{f,i\text{-}Din}$, $h_{f,i\text{-}B}$ and $h_{f,i\text{-}Dout}$, and can be expressed as Eq. (S7) (namely the Eq. (6)):

$$h_{f,i} = [-i^2 + (2N+1)i - 2N]Q_a R_D + Q_a R_B \tag{S7}$$

Similarly, for the microfluidic ladder network with *N* branch microchannels in co-flow operation, the $h_{f,i}$ of flow path-*i* usually includes the resistance loss of inlet distribution microchannel ($h_{f,i\text{-}Din}$), branch microchannel ($h_{f,i\text{-}B}$) and inlet distribution microchannel ($h_{f,i\text{-}Dout}$), which is indicated by the blue line in Fig. S1b. The $h_{f,i\text{-}Din}$, $h_{f,i\text{-}B}$ and $h_{f,i\text{-}Dout}$ can be respectively expressed by Eq. (S8), Eq. (S9) and Eq. (S10) under the assumption of uniform flow distribution.

$$h_{f,i\text{-}Din} = [(N-1)Q_a + (N-2)Q_a + \ldots + (N-i+1)Q_a]R_D \tag{S8}$$

$$h_{f,i\text{-}B} = Q_a R_B \tag{S9}$$

$$h_{f,i\text{-}Dout} = [iQ_a + (i+1)Q_a + \ldots + (N-1)Q_a]R_D \tag{S10}$$

The $h_{f,i}$ of flow path-*i* is the sum of $h_{f,i\text{-}Din}$, $h_{f,i\text{-}B}$ and $h_{f,i\text{-}Dout}$, and can be expressed as Eq. (S11) (namely the Eq. (12) in the main text):

$$h_{f,i} = Q_a[-i^2 (N + 1)i + 0.5N^2 - 1.5N]R_D + Q_a R_B \tag{S11}$$

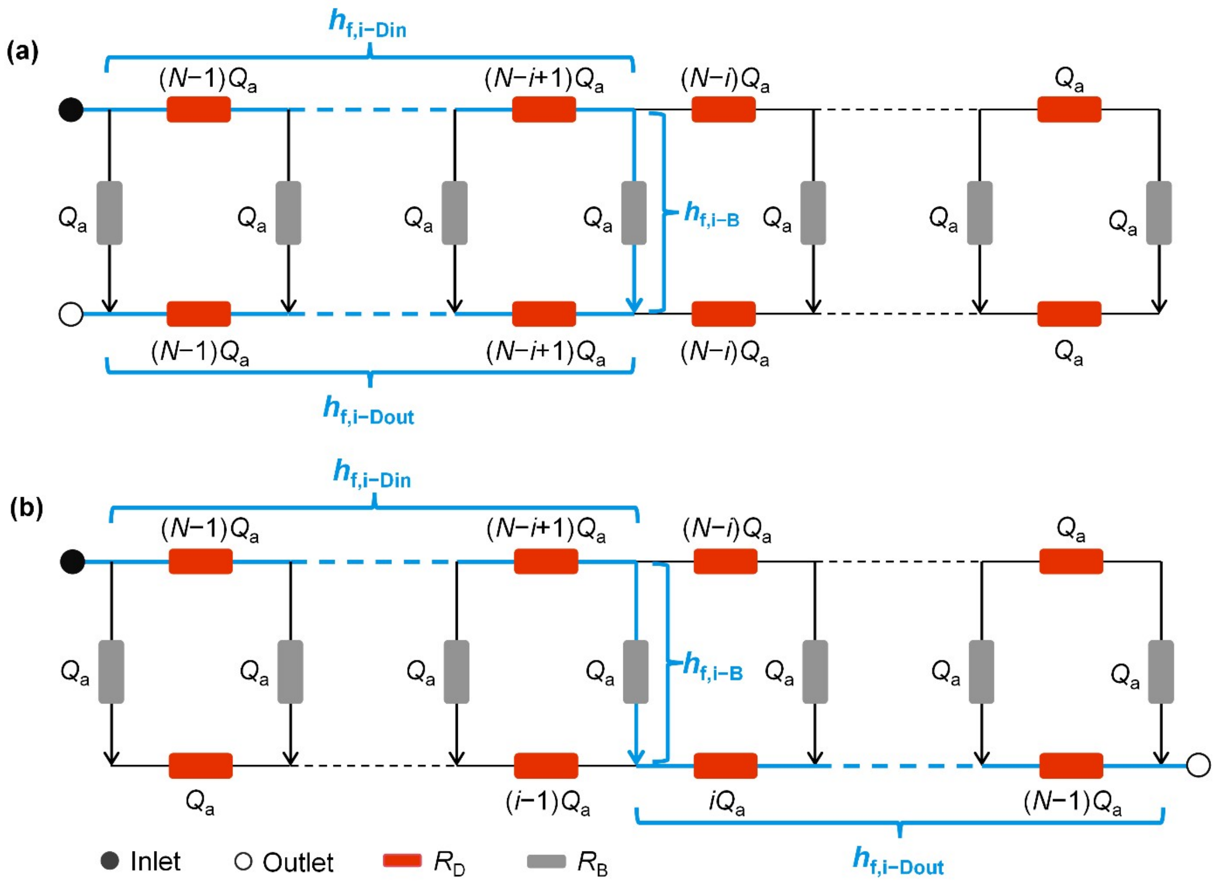

Figure S1. Schematic illustration of flow path-*i* for the microfluidic ladder network with *N* branch microchannels in counter-flow operation (a) and co-flow operation (b).

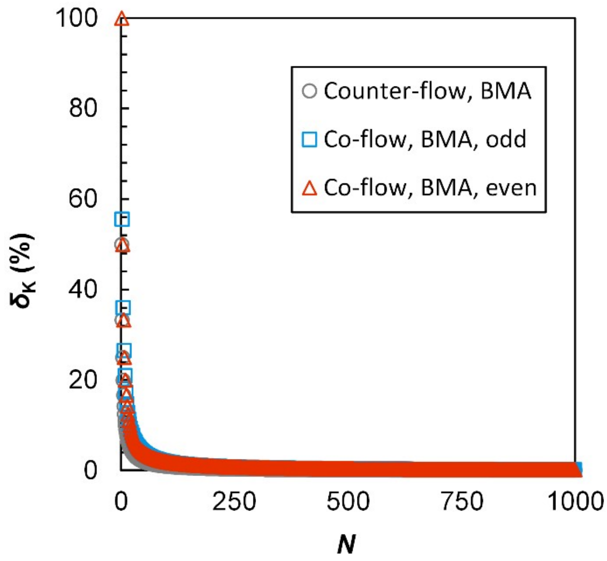

Figure S2. Relative deviation (δ_K) of K value after approximation in the equation of BMA rule. δ_K is defined as the ratio of (K–K_R) to K, where K and K_R are respectively calculated by the equation of BMA rule with and without the approximation.

Table S1. Designed parameters of microfluidic ladder networks for numerical simulation

Rule	N (-)	K_{min} (-)	w_B (μm)	h_B (μm)	l_B (μm)	w_D (μm)	h_D (μm)	l_D (μm)
Counter-flow, existed	20	4000	100	40	1500	3000	400	17167
	80	16000	100	40	1500	3000	400	4292
	100	20000	100	40	1500	3000	400	3433
	150	30000	100	40	1500	3000	400	2289
	200	40000	100	40	1500	3000	400	1717
Counter-flow, BMA	20	40000	100	40	18000	3000	400	20600
	80	640000	100	40	18000	3000	400	1288
	100	1000000	100	40	18000	3000	400	824
	150	2250000	100	40	18000	3000	400	366
	200	4000000	100	40	18000	3000	400	206
Co-flow, BMA	20	10000	100	40	5000	3000	400	22889
	80	160000	100	40	5000	3000	400	1431
	100	250000	100	40	5000	3000	400	916
	150	562500	100	40	5000	3000	400	407
	200	1000000	100	40	5000	3000	400	229

Table S2. Microchannel size of microfluidic integrated devices

Device	w_B (μm)	h_B (μm)	l_{B1} (μm)	l_{B2} (μm)	w_{Din} (μm)	w_{Dout} (μm)	h_D (μm)	l_D (μm)
E-200 (designed)	80	40	932	450	1800	4000	240	320
E-200 (experimental)	80.07	39.77	798.17	421.41	1619.57	3619.57	264.80	306.27
B-200 (designed)	80	40	14534	450	4000	4000	240	320
B-200 (experimental)	82.66	37.92	12956.29	417.34	3600.99	3600.99	265.68	309.22

Table S3. Designed parameters of microfluidic ladder networks with different deviation parameters

Device	Deviation parameter	K	w_B (μm)	h_B (μm)	l_B (μm)	w_D (μm)	h_D (μm)	l_D (μm)
1	0.01	10^6	80	40	14800	4000	240	320
2	0.001	10^7	80	40	20000	6500	400	320
3	0.0001	10^8	80	40	30000	10000	650	320