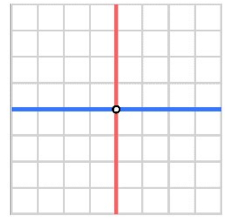
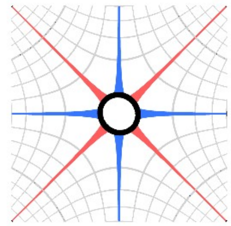
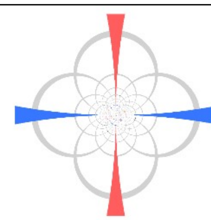
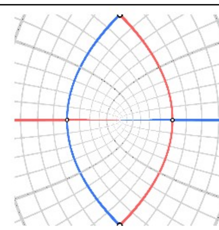
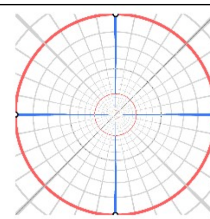
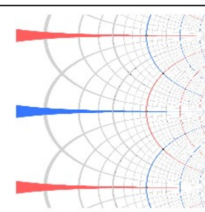
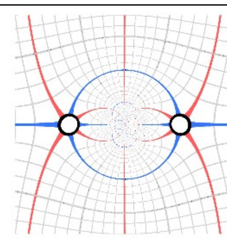
Electronic Supplementary Material (ESI) for Lab on a Chip.

Supplementary Table 1 – Catalog of simple conformal maps

Transform	Description	Illustration
$f(z) = z$	Identity transform. As a source term, represents a straight flow of uniform velocity.	
$f(z) = z^n$	Maps the upper half plane to a corner of angle pi/n. Useful for generating radially symmetrical geometries.	
$f(z) = \frac{1}{z}$	Inversion transform: Maps the interior of the unit circle to its exterior and its exterior to its interior. As a source term, represents a dipole flow.	
$f(z) = \sqrt{z}$	Maps streamlines to parabolas. Useful for modelling flow around rounded finger-like obstacles.	
$f(z) = log(z)$	Changes from cartesian to polar coordinates. As a source term, represents a simple point source.	
$f(z) = e^z$	Inverse of the log transform. Maps radial geometries to channel flow.	
$f(z) = z + \frac{1}{z}$	Superposition of straight flow and dipole flow. Maps straight flow to straight flow around a unit cylinder.	

In addition to these, if k is a real number, $f(z) = k * z$ resizes the domain by a factor of k, and $f(z) = z + k$ effects a translation of the origin by k units. $f(z) = e^{ik} * z$ rotates the domain by k radians. Many more transformations and descriptions of their applications, as well as numerical methods for computing maps that are not represented by simple functions can be found in Schinzinger & Laura: “Conformal Mapping: Methods and Applications”, 2012. A large family of conformal maps which map the domain to the interior of polygons are called Schwarz-Christoffel transforms. Schwarz-Christoffel transforms are of special use when modelling flow in channels, flow inside polygonal chambers, or flow in periodic arrangements of sources. In-depth discussion of conformal maps can be found in Tobin Driscoll’s “Schwarz-Christoffel Mapping”, 2002. Classic textbooks on special functions such as Abramowitz & Stegun also contain detailed discussion of special functions and their applications as conformal maps, complete with illustrations. More complex source terms and procedures for generating complex arrangements of sources and obstacles can be found in ODL Strack: “Analytical Groundwater Mechanics”, 2017.