Electronic Supplementary Material (ESI) for Materials Advances.

Supporting information

Birefringence in anodic aluminum oxide: an optical method for measuring porosity

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Appendix 1

$$\frac{1}{n_2^2} = \frac{\cos^2\beta_2}{n_{or}^2} + \frac{\sin^2\beta_2}{n_{ex}^2} \tag{S1.1}$$

$$\frac{1}{n_1^2} = \frac{1}{n_{or}^2} \tag{S1.2}$$

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{n_{or}^2} - \frac{\cos^2\beta_2}{n_{or}^2} - \frac{\sin^2\beta_2}{n_{ex}^2} \tag{S1.2}$$

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{sin^2\beta_2}{n_{or}^2} - \frac{\sin^2\beta_2}{n_{ex}^2} \tag{S1.3}$$

$$\frac{n_2^2 - n_1^2}{n_1^2 n_2^2} = \sin^2\beta_2 \frac{n_{ex}^2 - n_{or}^2}{n_{or}^2 n_{ex}^2} \tag{S1.4}$$

$$\frac{n_1 + n_2}{n_1^2 n_2^2}(n_2 - n_1) = \frac{n_{or} + n_{ex}}{n_{or}^2 n_{ex}^2}\sin^2\beta_2\,(n_{\mathrm{ex}} - n_{or}) \tag{S1.5}$$

Because $\Delta n << n$, (S1.5) can be simplified to:

$$n_2 - n_1 = \sin^2\beta\,(n_{\mathrm{ex}} - n_{or}) \tag{S1.6}$$

$$n_2 - n_1 = \sin^2\beta\,\Delta n$$
(S1.7)

Appendix 2

Let us introduce the following parameters:

$$\Delta n^* \equiv n_2 - n_1$$
(S2.1)

$$\Delta\beta \equiv \beta_1 - \beta_2$$
(S2.2)

$$n = \frac{n_1 + n_2}{2}$$
(S2.3)

Since $\Delta n << n$, it can be assumed that

$$\Delta\beta \ll \beta$$
(S2.4)

According to Snell's law of refraction:

$$n_1 \sin\beta_1 = n_2 \sin\beta_2$$
(S2.5)

The optical path difference between the paths of two rays is:

$$\delta = h(n_1\cos\beta_1 - n_2\cos\beta_2) = h\left(n_1\cos\beta_1 - n_1\frac{\sin\beta_1}{\sin\beta_2}\cos\beta_2\right) =$$
$$= n_1 h \sin\beta_1\left(\frac{\cos\beta_1}{\sin\beta_1} - \frac{\cos\beta_2}{\sin\beta_2}\right) = n_1 h \sin\beta_1\,(\cot\beta_1 - \cot\beta_2)$$
(S2.6)

Taking (S2.4) and (S2.5) into account:

$$\frac{\cot\beta_1 - \cot\beta_2}{\beta_1 - \beta_2} = \frac{\cot\beta_1 - \cot\beta_2}{\Delta\beta} = (\cot\beta)' = -\frac{1}{\sin^2\beta}$$
(S2.7)

$$\cot\beta_1 - \cot\beta_2 = -\frac{\Delta\beta}{\sin^2\beta}$$
(S2.8)

From (S2.6), (S2.8), and (S2.4) the optical path difference is determined by the equation:

$$\delta = -\frac{nh}{\sin\beta}\Delta\beta$$
(S2.9)

From (S2.5), one can derive $\Delta\beta$ in terms of Δn^*:

$$1 - \frac{\Delta n^*}{n_2} = \frac{n_2 - \Delta n^*}{n_2} = \frac{\sin(\beta_2 + \Delta\beta)}{\sin\beta_2} = \frac{\sin\beta_2 + \cos\beta_2\,\Delta\beta}{\sin\beta_2} = 1 + \frac{\cos\beta_2\,\Delta\beta}{\sin\beta_2}$$

(S2.10)

Since of $\Delta\beta << \beta$, one can obtain:

$$\Delta\beta = -\frac{\Delta n^*}{n_2}\frac{\sin\beta_2}{\cos\beta_2} = -\frac{\Delta n^*}{n}\frac{\sin\beta}{\cos\beta}$$

(S2.11)

Combining (S2.9) and (S2.11), one can get:

$$\delta = \frac{h}{\cos\beta}\Delta n^* = \frac{h}{\cos\beta}(n_2 - n_1)$$

(S2.12)

Appendix 3

To measure the effective refractive index of AAO films, SEM and optical spectroscopy techniques were applied. According to the cross-sectional SEM images, the thickness (*h*) of the AAO film was 52.5 ± 0.5 μm. Transmittance spectra were collected from both as-prepared AAO film and the membrane after barrier layer etching. Fabry–Pérot optical interference fringes are seen in the optical spectra (Fig. S3a). The positions of the transmittance maxima λ_m are determined by the Bragg–Snell law:

$$2hn = m\lambda_m, \quad (S3.1)$$

where m is an integer. Hence, the dependence of *m* on $1/\lambda_m$ (Fig. S3b) should be linear with a slope of 2*hn*. The effective refractive indices of AAO films before and after barrier layer etching, calculated from the slope of *m* versus $1/\lambda_m$ dependence are given in Table S3.1.

Table S3.1. Optical characteristics of AAO films.

	Slope $\Delta(m')/\Delta(1/\lambda)$	n
As-prepared AAO film	169.7 ± 0.2	1.616 ± 0.015
AAO film after barrier layer etching	165.6 ± 0.4	1.577 ± 0.015

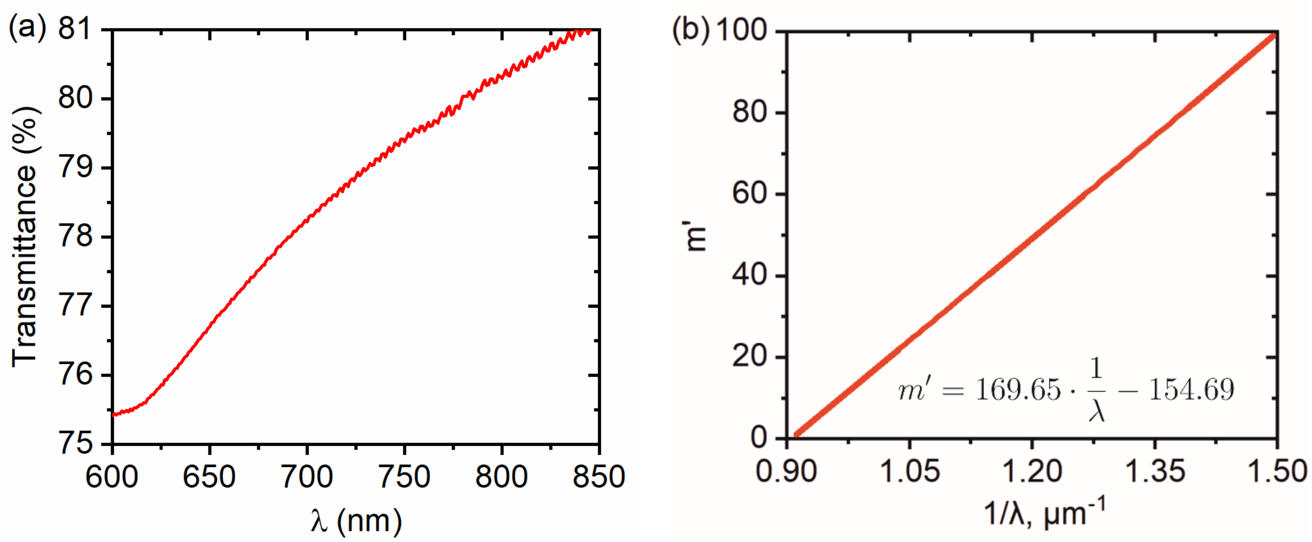


Fig. S3. The transmittance spectrum of an as-prepared AAO film (a). Dependence of *m* on $1/\lambda_m$, where λ_m is the wavelength of the *m*-th maximum of the transmittance, numbered from longer to shorter wavelengths (b).

Appendix 4

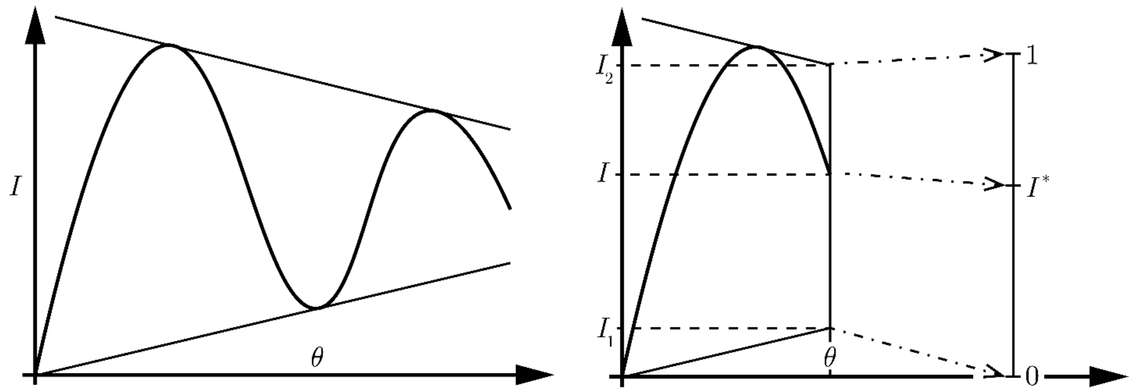


Fig. S4. The schematic of the algorithm of dragging data to fit 0-1 range.

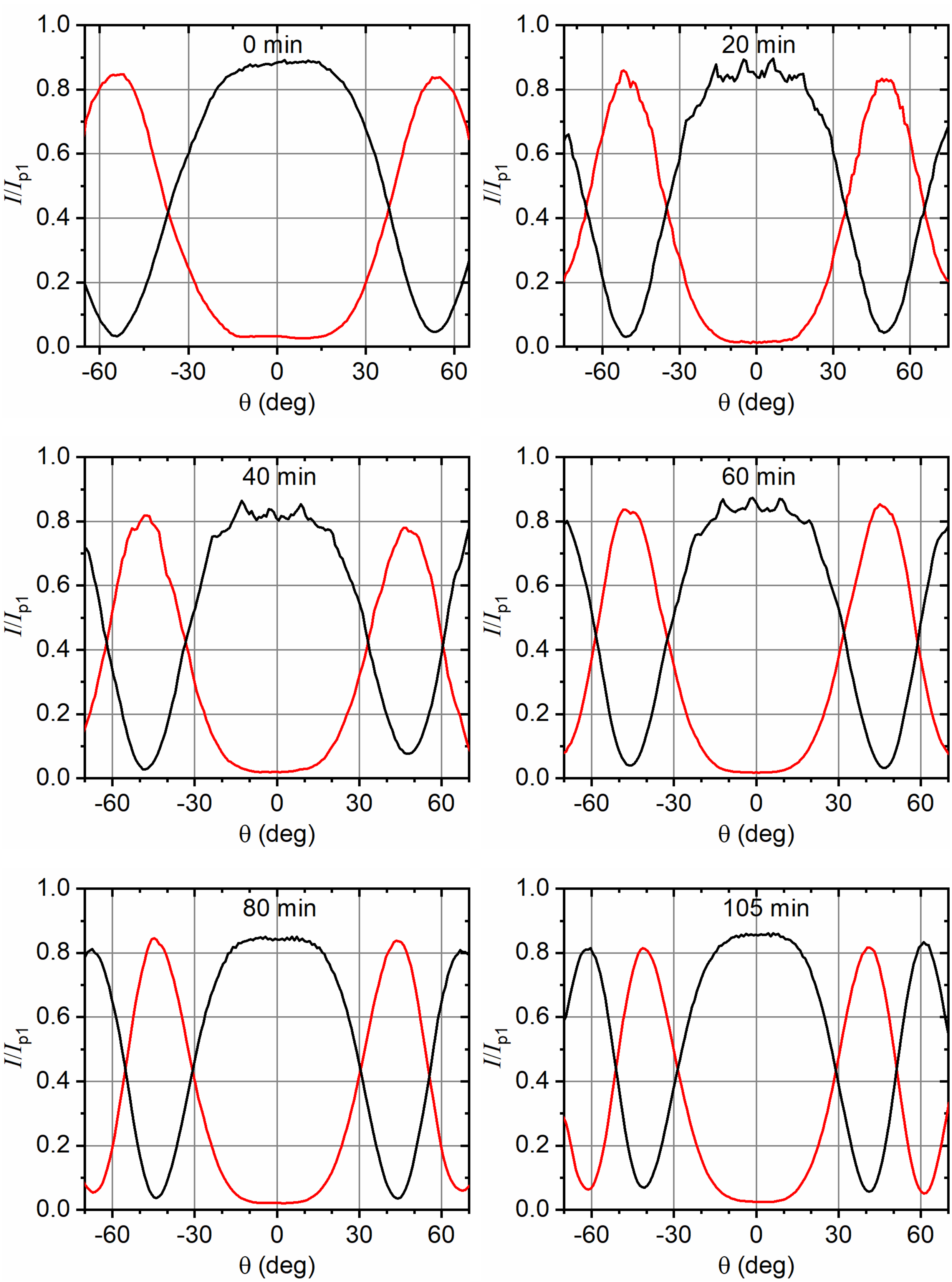
$$I_1 \to 0$$

$$I_2 \to 1$$

$$I \to I^*$$

$$I^* = \frac{I - I_1}{I_2 - I_1}$$

Appendix 5


0 min
20 min
40 min
60 min
80 min
105 min
I/I_{p1}
θ (deg)
1.0
0.8
0.6
0.4
0.2
0.0
-60
-30
0
30
60

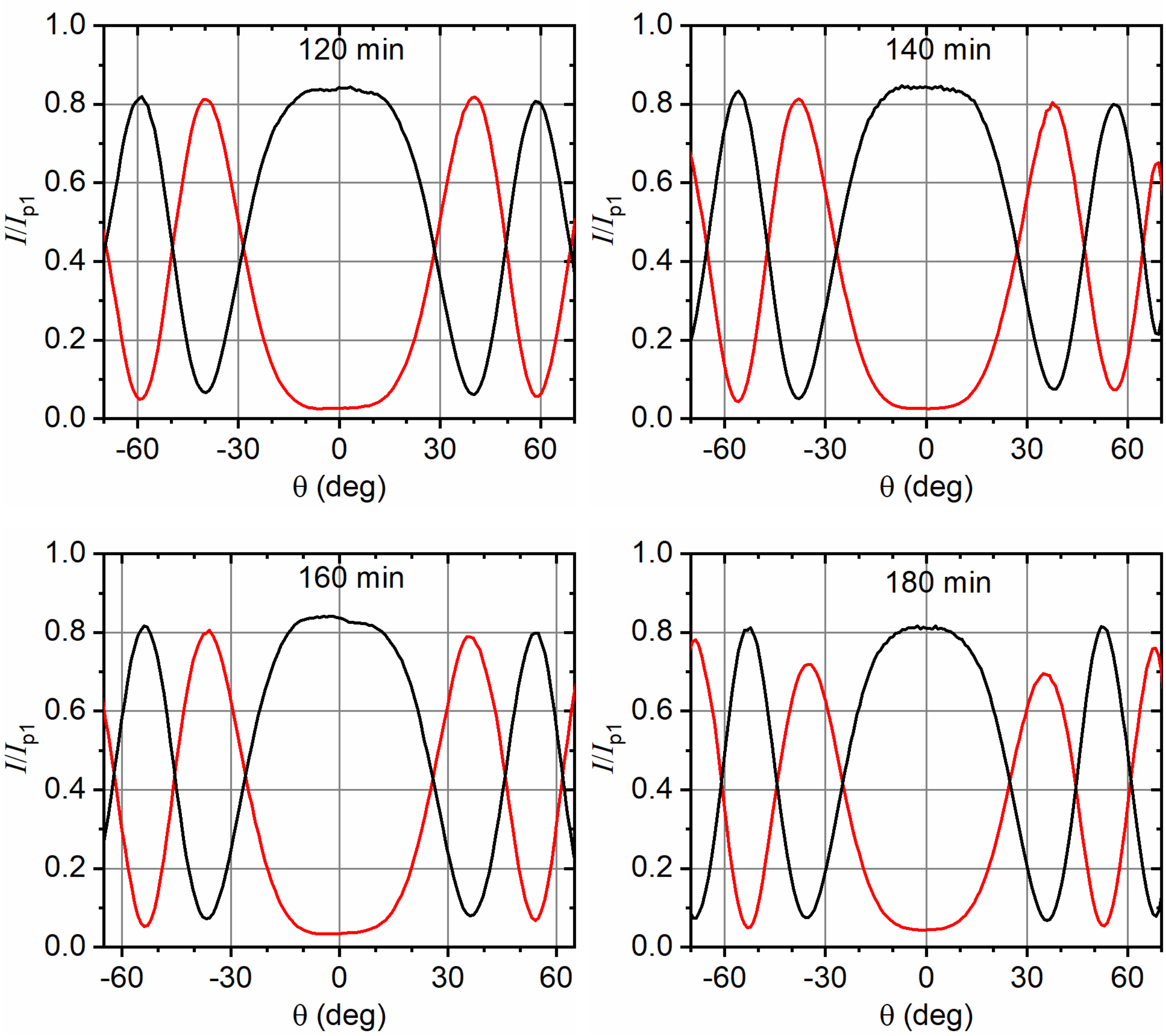


Fig. S5. The intensity of the transmitted laser beam depending on the rotation angle of AAO film placed between two polarizers whose vibration directions are oriented perpendicular (red line) and parallel (black line) to each other. Etching durations in 2 M H_2SO_4 solution at 20 °C are given in panels.