

Supplementary Information for Photonic Band Structure Calculation of 3D- Finite Nanostructured Supercrystals

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Table S1: High symmetry (critical) points and symmetry lines in the first Brillouin zone of a simple cubic lattice.

Symmetry points (u, v, w)	$[k_x, k_y, k_z]$
Γ : (0,0,0)	[0,0,0]
X : ($\frac{1}{2}, 0, 0$)	$[\pi/a, 0, 0]$
M : ($\frac{1}{2}, \frac{1}{2}, 0$)	$[\pi/a, \pi/a, 0]$
R : ($\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$)	$[\pi/a, \pi/a, \pi/a]$

Symmetry lines	Length
Δ : (0, v ,0), $0 < v < \frac{1}{2}$	$\overline{\Gamma X} = \pi/a$
T : ($\frac{1}{2}, \frac{1}{2}, w$), $0 < w < \frac{1}{2}$	$\overline{MR} = \pi/a$
Λ : (w, w, w), $0 < w < \frac{1}{2}$	$\overline{\Gamma R} = \sqrt{3}\pi/a$
Σ : ($u, u, 0$), $0 < u < \frac{1}{2}$	$\overline{\Gamma M} = \sqrt{2}\pi/a$
S : ($u, \frac{1}{2}, u$), $0 < u < \frac{1}{2}$	$\overline{XR} = \sqrt{2}\pi/a$
Z : ($u, \frac{1}{2}, 0$), $0 < u < \frac{1}{2}$	$\overline{XM} = \pi/a$

Table S2: Key physical parameters for each system: supercrystal overall size (L), unit cell volume (V_{cell}), nanoparticle volume (V_{NP}), and filling fraction ($F = V_{\text{NP}}/V_{\text{cell}}$).

Supercrystals	L [μm]	V_{cell} [nm^3]	V_{NP} [nm^3]	F
Spherical NPs, $a=100$ nm	2.0	10^6	2.68×10^5	0.268
Spherical NPs, $a=130$ nm	3.5	2.20×10^6	2.68×10^5	0.122
Nanorods, $a=100$ nm	2.0	10^6	8.38×10^4	0.084

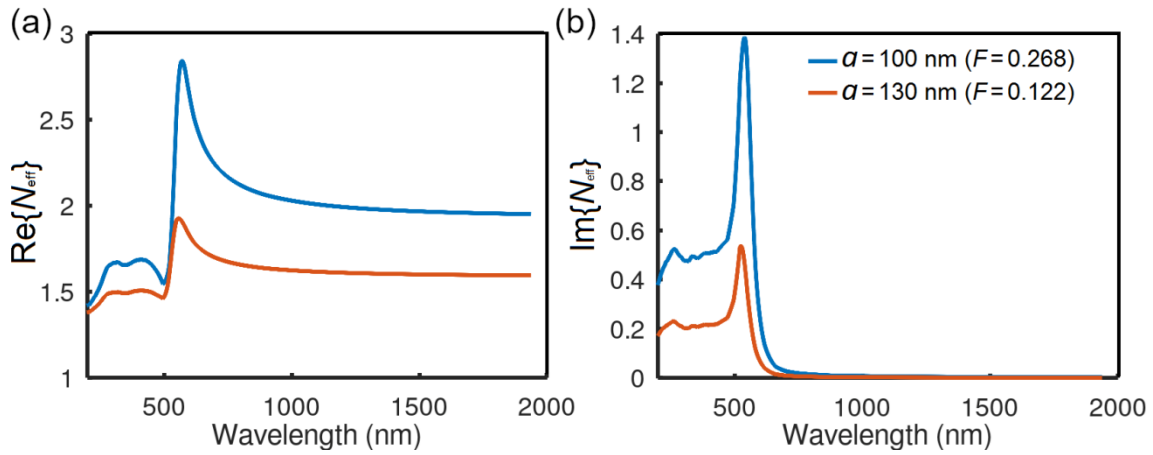


Figure S1: (a) Real and (b) imaginary parts of the effective refractive index of superlattices composed by Au NPs of 80 nm in diameter arranged in a simple cubic lattice and with lattice parameters of $a=100$ nm (blue) and $a=130$ nm (red). The refractive index of the Au NPs is taken from Johnson and Christy's experimentally tabulated data and the refractive index of the medium is $n_m=1.33$ (water). F is the filling factor.

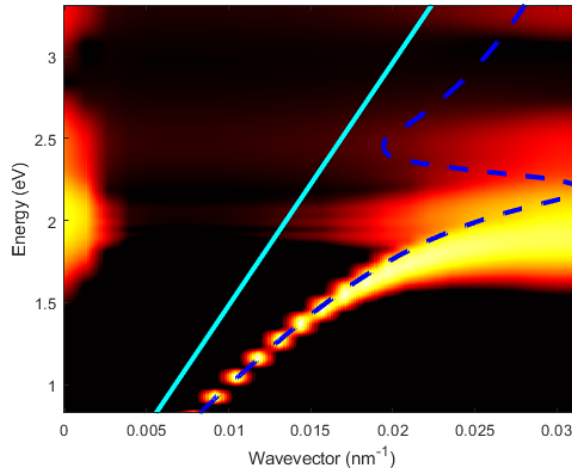


Figure S2: Dispersion relation of an infinite Au-NPs supercrystal with a lattice parameter of 100 nm, light dispersion in water (cyan solid line), and dispersion relation of the light in the effective medium (blue dashed line).

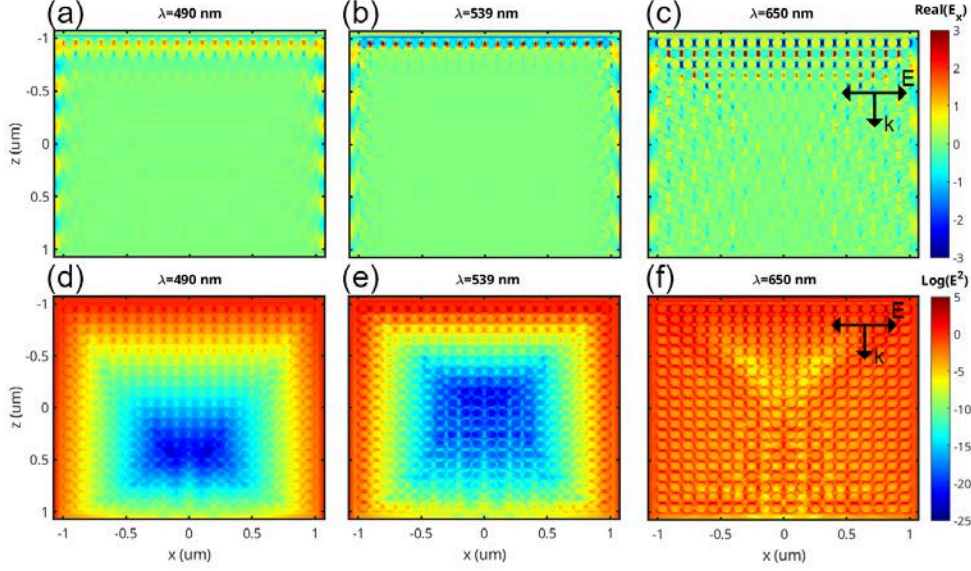


Figure S3: Real part of the x -component of the electric field (upper panels) and electric field intensity enhancement in log-scale (lower panels) of a cubic supercrystal composed of 80-nm Au NPs in a cubic array of lattice parameter $a=100$ nm. The cubic supercrystal is excited with a plane wave of $\mathbf{k} = \mathbf{k}_X$ (X point of the Brillouin zone) and at wavelengths corresponding to (a, d) the upper branch, (b, e) the polaritonic band gap, and (c, f) the lower branch of the photonic band structure (Figure 4a).

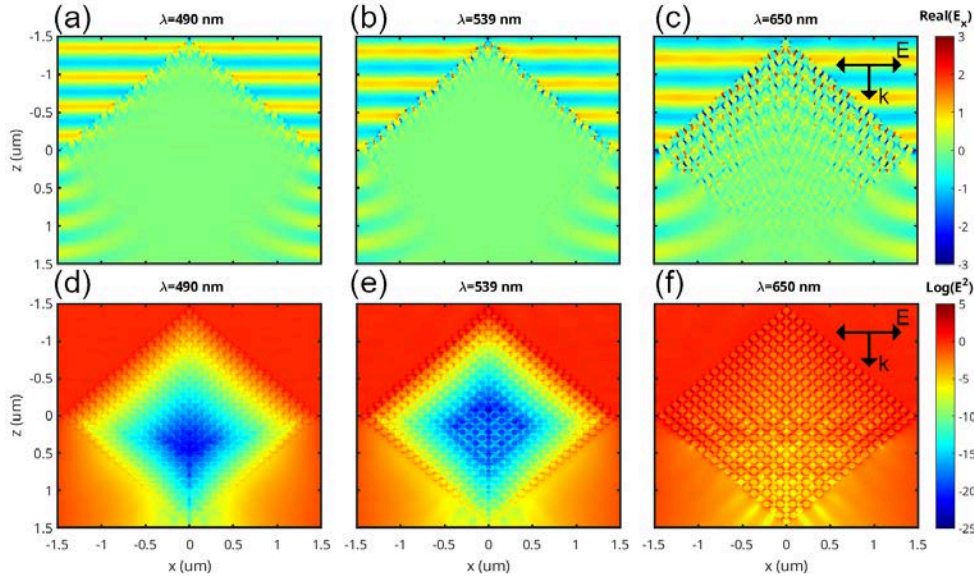


Figure S4: Real part of the x -component of the electric field (upper panels) and the electric field intensity enhancement in log-scale (lower panels) of a rhombic dodecahedral supercrystal composed of 80-nm Au NPs in a cubic array of lattice parameter $a=100$ nm. The cubic supercrystal is excited with a plane wave of $\mathbf{k} = \mathbf{k}_M$ (M point of the Brillouin zone) and at wavelengths corresponding to (a, d) the upper branch, (b, e) the polaritonic band gap, and (c, f) the lower branch of the photonic band structure (Figure 4c).

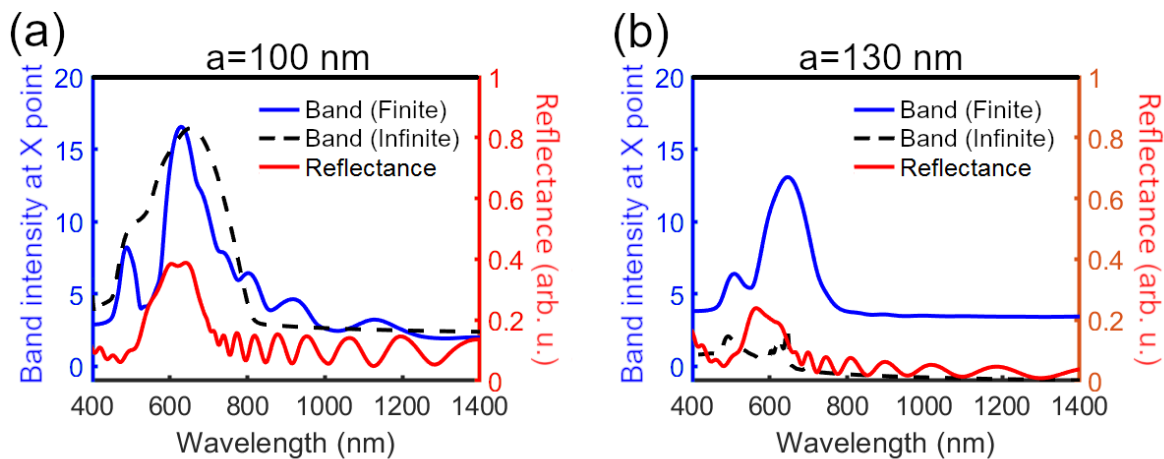


Figure S5: Band structure at \mathbf{X} point ($\mathbf{k} = \mathbf{k}_X$), blue line, and reflectance, red line, of a cubic Au NPs supercrystal with a lattice parameter of (a) 100 nm and (b) 130 nm. The band structure for the infinite case is also plotted, dashed black line.

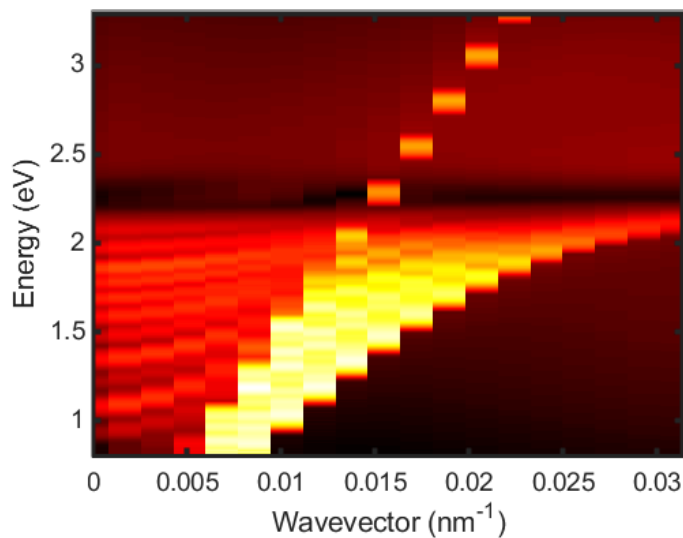


Figure S6: Dispersion relation of slab with effective refractive index (N_{eff}) representing a Au-NPs supercrystal with a lattice parameter of 100 nm in water.

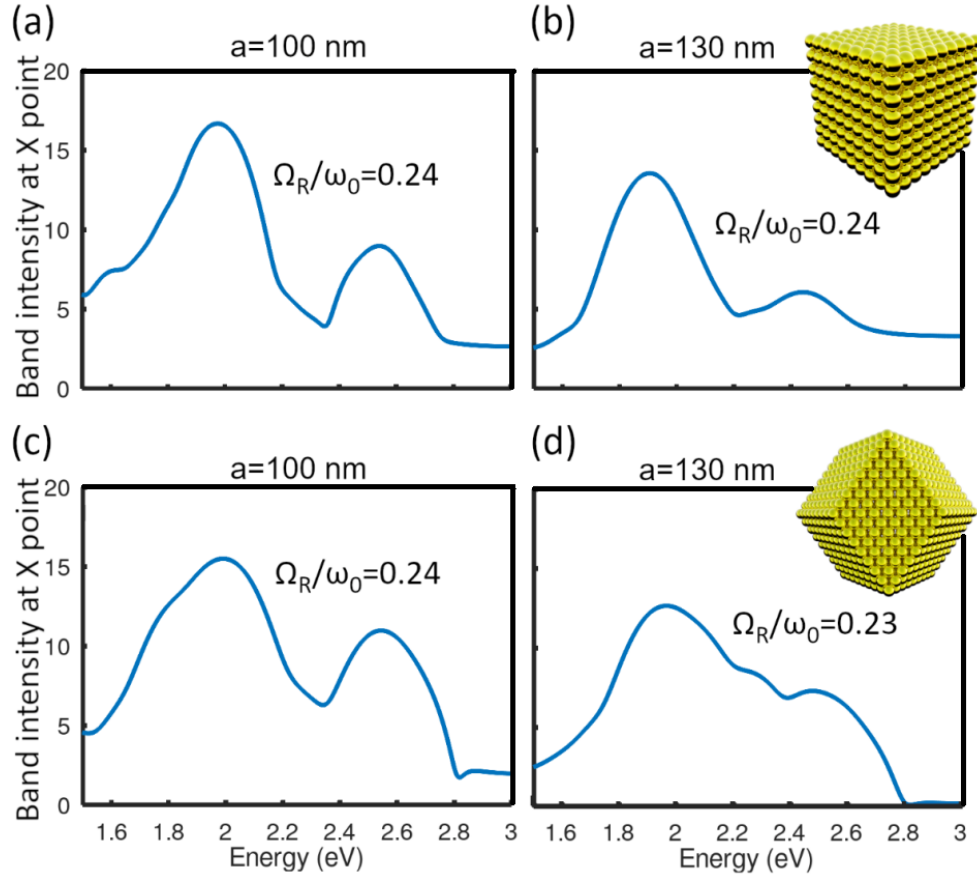


Figure S7: Photonic bands and mode splitting to plasmonic mode energy ratios (Ω_R/ω_0) at the **X** point for the finite supercrystals with cubic (**a**, **b**) and rhombic dodecahedral (**c**, **d**) habits. The LSPR energy is $\hbar\omega_0 \sim 2.3$ eV.