I. The effect of the depolarizer metasurface on phase.

As shown in Fig.1(a), we simulated the transmitted E-field of linearly polarized light under different rotation angles of the nanopillar (0°, 10°, 20°). For one certain polarization direction such as x-axis, the single unit of the depolarizer metasurface, i.e. a half-wave plate unit, will not change the phase of linearly polarized light, but the projected amplitude along the given polarization direction because of the rotation of polarization. Therefore, through randomly arranging the directions of the half-wave plate units, the output light is plane wave with no intrinsic polarization and the power in x-axis should be 50% due to the nature of unpolarized light.

![Fig.1 Electric field of LP and CP with the change of rotation angles.](image)

Since the spatial distribution of transmitted amplitude in each unit pixel, which is related to the rotation angle of the unit, is random, the wavefront of linearly polarized light will show changes in the aspect of the sidelobes of the beam. The directivity of array with both uniform phase and amplitude is shown in Fig.2(a). By setting uniform phase but random amplitude for each units, the far field pattern exhibits a little changes in the directivity of sidelobes (as shown in Fig.2(b)-2(d)). However, the main beam direction remains stable so the transmitted light is less affected by the depolarizer
metasurface.

Fig.2 Directivity of the linearly polarized light passing through the depolarizer MS.

For circularly polarized light, the unit will not affect the amplitude but the phase. According to the PB phase method, the circularly polarized light passing through a half-wave plate with rotation angle of $\theta$ will convert to its handedness-flipped polarization state with the phase change of $2\theta$, as shown in Fig.1(b). It can be seen that the phase proportionally varies with the increase of rotation angle while the amplitude keeps unchanged. Based on the phase array theory which is widely used in antenna applications, the discrete distribution of the element’s phase would modulate the wavefront and thus manipulate the beam of light. Therefore, the beam will diverge when circularly polarized waves pass through the depolarizer metasurface, though the total transmitted power is still 50% in the given polarization direction. Fig.R6 display the 3D directivity of circularly polarized light passing through the polarizer MS with $200 \times 200$ units. It can be seen that the beam diverges almost uniformly within a certain spatial domain and thus the energy spreads in discrete directions. Therefore, for circularly polarized light, the proposed design may suits for the special applications where the beam quality and focusing condition are not strictly required.
To prove these conclusions in theory, we derived the E-field expression of light that passed through the half-wave plate unit with rotation angle of $\theta$. The $x$-direction E-field ($E_x$) was considered first. As shown in Fig.4, the $E_x$ can be resolved into the superposition of two orthogonal components whose vectors are overlapped with the half-wave plate unit:

$$
E_x = E_0 \cos \theta \hat{u} - E_0 \sin \theta \hat{v}
$$

(1) 

After passing through the phase unit, it becomes:

$$
\hat{E}_x = E_0 \cos \theta \cdot e^{j \varphi_u} \hat{u} - E_0 \sin \theta \cdot e^{j \varphi_u} \hat{v}
$$

(2) 

where $\varphi_v = \varphi_u + \pi$. Based on the Cartesian transformation, the normalized vectors have the relationship as follow:

$$
\begin{align*}
\hat{u} &= \hat{x} \cos \theta + \hat{y} \sin \theta \\
\hat{v} &= \hat{y} \cos \theta - \hat{x} \sin \theta
\end{align*}
$$

(3) 

thus the E-field can be expressed as:

$$
\hat{E}_x = (E_0 \cos^2 \theta \hat{x} + E_0 \cos \theta \sin \theta \hat{y} - E_0 \sin^2 \theta \hat{x} + E_0 \cos \theta \sin \theta \hat{y}) \cdot e^{j \varphi_u}
$$

$$
= (E_0 \cos 2\theta \cdot \hat{x} + E_0 \sin 2\theta \cdot \hat{y}) \cdot e^{j \varphi_u}
$$

(4)
For y-polarized light, the transmitted E-field \((E_y)\) can be obtained using the same derivation:

\[
E_y = (E_0 \sin 2\theta \cdot \hat{x} - E_0 \cos 2\theta \cdot \hat{y}) \cdot e^{i\phi_y} \tag{5}
\]

It is clear that the phase of E-field has nothing to do with the rotation angle \(\theta\) of the nanopillar, and the amplitude varies with the change of \(\theta\). Note that \(\phi_y\) is a constant value when the structure of nanopillar is determined.

Further, for circularly polarized light, the transmitted E-field should be:

\[
E = E_x + jE_y = (E_0 e^{i2\theta} \hat{x} - jE_0 e^{i2\theta} \hat{y}) \cdot e^{i\phi_y} \tag{6}
\]

which is the handedness-flipped polarization state with the phase change of 2\(\theta\).

### II. The influence of dimensional size of polarizer.

The linewidth and period of the Al grating are set to be 80 nm and 160 nm, and the thickness is 40 nm. Fig.5(a) illustrates that the simulated extinction ratio (ER) of the linear polarizer is about 36 at 770 nm, and 27 at 850 nm. As shown in Fig.5(b), with the increase of Al thickness, the ER gets larger but the transmission of the co-polarized light decreases at the same time. Consider that the transmission of the cross-polarization is low enough (~3% at the center wavelength 850 nm), the thickness is finally chosen to be 40 nm by comprehensively making compromise among the transmission efficiency and ER. Limited by the fabrication conditions, the dimensional size of the Al grating might not exactly match the settings. By adding machining error of ±10 nm in the simulations, it can be seen from Fig.5(c) and 5(d) that the ER gets worse, so we think the machining error might be one of the reasons for the poor performance.

Besides, we believe that the parameters of the LP are not the optimal, higher ER and transmission efficiency can be realized simultaneously through further optimizing the structure.
Fig. 5 R7 Influence of dimensional size of polarizer.