

Supplementary information for “A theory of skyrmion crystal formation”

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Details of supporting movies

We have recorded the evolution process described in Fig. 6 of the main text. In all videos below, magnetization distribution is the average of 8 layers. The color [color bar shown in Fig. 6(a)] encodes the azimuthal angle of skyrmion magnetization and the black denotes the background.

Movie-a shows stability of metastable helical state at 20 K and at the optimal field of 0.3 T. The initial state has 10 stripe skyrmions. The skyrmion number has only increased by 1 to 11 skyrmions after 30 ns although the thermal equilibrium state is an SkX of about 140 skyrmions.

Movie-b shows how a helical state reaches the thermal equilibrium state of an SkX at 29 K and at the optimal field of 0.3 T. There are 10 stripe skyrmions in the initial state. It transforms into the thermal equilibrium state of an SkX of 132 skyrmions after 30 ns.

Movie-c shows how an SkX with too many (150) skyrmions, metastable and out of equilibrium, reaches the thermal equilibrium state of a different SkX of 136 skyrmions at 29 K and at the optimal field of 0.3 T.

Movie-d shows how an SkX of 150 skyrmions reaches the thermal equilibrium state of a helical

state during the zero-field cooling process from 30 K to 0 K at a rate of 1 K/ns. The SkX becomes helical states within 1 ns at 30 K. The system has 7 skyrmions with mixture of circular skyrmions, stripe skyrmions and ramified stripe skyrmions.

Movie-e shows how an SkX of 150 skyrmions reaches the thermal equilibrium phase of a helical state during the zero-field warming from 0 K to 30 K at a rate of 1 K/ns. The SkX does not change much at relative low temperature. It starts to transform into helical state above 22 K. The final state after 30 ns has 4 skyrmions of mixture of one circular skyrmion and 3 ramified stripe skyrmions.

Movie-f shows how an initial ferromagnetic states, far from its equilibrium phase at 29 K and at the optimal field of 0.3 T, transforms into the thermal equilibrium state of an SkX of 133 skyrmions. To visualise how skyrmions develop from the thermally generated nucleation centres one by one. The video contains two sections. The first 0.5 ns is a faster process and the first group of skyrmions are generated within order of 10 ps. The second section of about 9.5 ns is a slower process in which skyrmions are developed within nanosecond time scale. The final state after 10 ns is a SkX of 133 skyrmions.