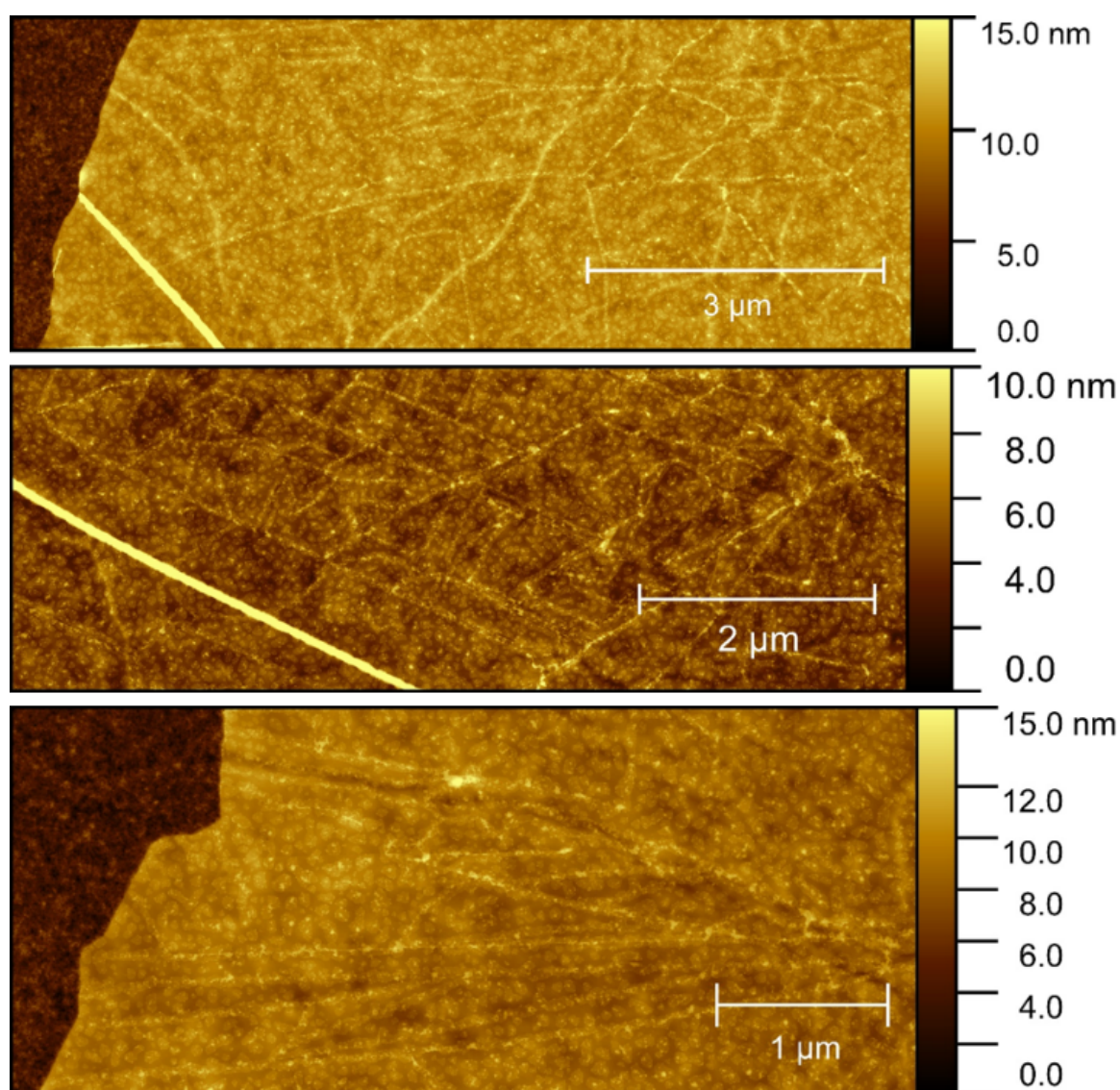


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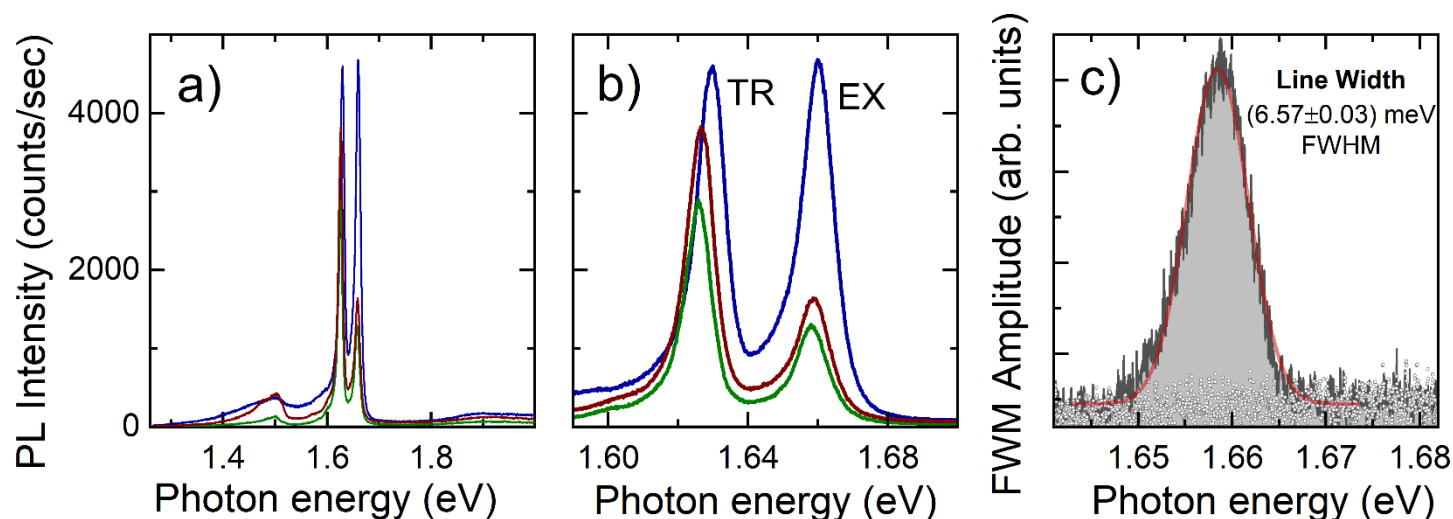
**Supplementary Information: “Coherent imaging and dynamics of exciton complexes in MoSe<sub>2</sub> monolayers epitaxially grown on a hexagonal boron nitride”**

By Karolina Ewa Polczyńska *et al.*

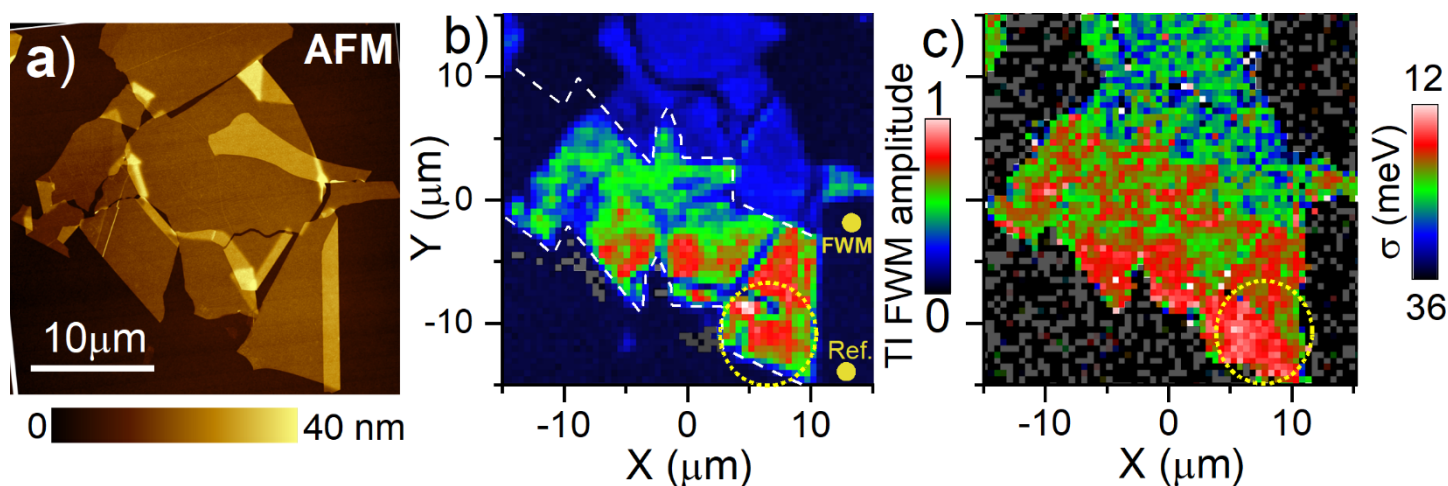
The document contains three supplementary figures, complementing the main text, showing: the AFM Imaging, photoluminescence and four-wave mixing spectra, and four-wave mixing imaging.



**Figure S1 – AFM imaging of the investigated sample.** The scale is given by horizontal bars, respectively. The MoSe<sub>2</sub> grows on the surface of the hBN: one can distinguish monolayer flakes, bilayers, and areas not yet covered by the MoSe<sub>2</sub>. The characteristic stripes, which are building up on a few micron scale, could be due to the surface reconstruction mechanism.



**Figure S2 – Complementary spectra.** a) Typical micro-photoluminescence (PL) spectra of the investigated sample excited by a CW laser at 532nm at  $T=10\text{K}$ . The spectra are dominated by narrow exciton and trion lines with their relative intensities depending on the underlying residual doping. Note a quasi-constant transition energy indicating lack of strain. A defect band is also visible at the low energy side, as in bare TMD monolayers obtained via mechanical exfoliation. b) as in a) but zoomed into the spectral region of exciton and trion transitions. A typical line width of 10meV FWHM is measured. c) Spectrally-resolved FWM amplitude measured at  $\tau_{12}=0.5\text{ps}$  and  $\tau_{23}=0$ . Locally the FWM line width reads around 6.6meV, indicating comparable homogeneous and inhomogeneous contributions to the spectral line shape. Open circles indicate the noise level.



**Figure S3 - FWM imaging and correlations between FWM amplitude and inhomogeneous broadening.** A complementary set of data as shown in Fig. 4 of the main text but obtained on a different hBN flake, as shown in the AFM image in a). The corresponding imaging of the FWM amplitude is shown in b). It is anti-correlated with the inhomogeneous broadening  $\sigma$  shown in c): the strongest FWM signal corresponds to the lowest  $\sigma$ , as indicated with the dotted yellow circle. In b), we observe a significantly weaker FWM signal at the upper part of the flake compared to the lower area. We notice that the form of this boundary mimics the shape of hBN edge at the bottom of the flake, as indicated by white dashed lines. Let us note that the reference's intensity drops significantly when reflecting from the hBN, compared to the  $\text{SiO}_2$  substrate. Hence, the generated amplitude of the FWM spectral interference is lowered when the reference pulse impinges the hBN flake. This effect is thus due to the signal detection arrangements, and not due to intrinsic sample properties.