Electronic Supplementary Information for Reshaping Sub-millimetre Bubbles from Spheres to Tori

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Other Supplementary Materials for this manuscript include the following:

Movies S1 to S4



Fig. S1. Schematic of apparatus used to generate positive or negative pressure in the headspace of the microscopy cell containing *cerato-ulmin* dispersions.



Fig. S2. The toroidal bubbles are stable for at least five days at ambient storage pressure.



Fig. S3. Symmetric eighth geometric model and thickness region partition for finite element analysis.



Fig. S4. Characterization of axially asymmetric genus 1 objects following reference (57). (A) Cross section of a Dupin cyclide in symmetry planes (vertical and horizontal slices through the middle).



Fig. S5. CU-coated air bubbles fall apart into small debris (arrow indicated) as overpressure is supplied. Inset images show zoom-in view of two bubbles after falling apart.





Fig. S6. DDM results from bubbles debris (red arrow) in Fig. S4. The DDM matrix $|F_D(q,\Delta t)|^2$ and normalized auto-correlation function $f(q,\Delta t)$ projected on Δt for different q indicate exponential growth or decay behavior, respectively. (A) Growth of $|F_D(q,\Delta t)|^2$ with delay time Δt for six values of q (µm⁻¹). The continuous lines are fits of the data. (B) Normalized autocorrelation function $f(q,\Delta t)$ extracted from $|F_D(q,\Delta t)|^2$ at various q versus Δt . Lines are exponential fits to the data. (C) $f(q,\Delta t)$ collapses when plotted as a function of $\Delta t q^2$. This scaling is compatible with a Brownian diffusive process. (D & E) Γ and τ are plotted against q^2 . (F) The bubble debris indicates characteristic Brownian diffusion, yielding a diffusion coefficient of $D_t =$ 0.61 µm²/s and a corresponding uniform radius of 400 nm.

Energy of detachment from surface

We use the model of Prabhudesai et al.(Ref 44 of the main document) to estimate the free energy of CU molecule detachment from an air-water interface, assuming the CU single molecule is a solid, spherical particle with a diameter of D and contact angle of θ . The energy required to remove one CU molecule from the interface is expressed by ΔE_{da} ,

 $\Delta E_{dw} \text{ and } \Delta E_{d}, \text{ where } \Delta E_{da} = \frac{1}{4} \pi D^2 \gamma_{aw} (1 - \cos \theta)^2 \text{ is the free energy of particle detachment}$ into air, $\Delta E_{dw} = \frac{1}{4} \pi D^2 \gamma_{aw} (1 + \cos \theta)^2 \text{ is the free energy of particle detachment into water and}$ $\Delta E_d = \frac{1}{4} \pi D^2 \gamma_{aw} (1 - |\cos \theta|)^2 \text{ is the minimum energy for detachment into the bulk phase.}$

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Movie S1.

Cerato-ulmin rodlike bubbles transition to spherical shapes upon pressure changes above the *cerato-ulmin* dispersion. Images were collected at 137 frames/second, then complied into videos.

Movie S2.

Cerato-ulmin spherical bubbles transition to toroidal shapes following a prescribed pressure manipulation. Images were collected at 120 frames/second, then complied into videos and played at a speed of 4 times faster than recorded.

Movie S3.

Cerato-ulmin spherical bubbles transition to toroidal shapes following a prescribed pressure manipulation. Images were collected at 120 frames/second, then complied into videos and played at a speed of 8 times faster than recorded.

Movie S4.

Repeated shape transition from sphere to tori while applying pressure oscillation. Images were collected at 120 frames/second, then complied into videos and played at a speed of 4 times faster than recorded.