Electronic Supplementary Information: Dynamic ordering caused by a source-sink relation between two droplets

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S1. Description of movies

- **Movie 1: d2sm00497f2.mp4**
  A real-time movie of decanol and paraffin droplets put on the surface of water saturated by 1-decanol. No self-propulsion took place.
- **Movie 2: d2sm00497f3.mp4**
  Initial motion of droplets, accelerated 20 times. $t \in [0\text{ h}, 1\text{ h}]$ and $c_{\mathrm{SB}} = 0.005$ wt%.
- **Movie 3: d2sm00497f4.mp4**
  Initial motion of droplets, accelerated 20 times. $t \in [0\text{ h}, 1\text{ h}]$ and $c_{\mathrm{SB}} = 0.05$ wt%.
- **Movie 4: d2sm00497f5.mp4**
  Initial motion of droplets, accelerated 20 times. $t \in [0\text{ h}, 1\text{ h}]$ and $c_{\mathrm{SB}} = 0.2$ wt%.
- **Movie 5: d2sm00497f6.mp4**
  Binary-star like motion of the droplets, accelerated 20 times. $t \in [4\text{ h}, 5\text{ h}]$ and $c_{\mathrm{SB}} = 0.005$ wt%.
- **Movie 6: d2sm00497f7.mp4**
  Rectilinear oscillation of the decanol droplet, accelerated 20 times. $t \in [4\text{ h}, 5\text{ h}]$ and $c_{\mathrm{SB}} = 0.05$ wt%.
- **Movie 7: d2sm00497f8.mp4**
  Constant distance state of the two droplets, accelerated 20 times. $t \in [4\text{ h}, 5\text{ h}]$ and $c_{\mathrm{SB}} = 0.2$ wt%.
- **Movie 8: d2sm00497f9.mp4**
  A real time movie of approaching motion between a 10 μL paraffin droplet colored by Sudan black B (0.05 wt%) and a 200 μL paraffin droplet without colorant.

S2. Diameter change of the droplets

Figure S1 shows the changes in diameter of two droplets. The diameter of decanol droplets decreased with time. The lower the SB concentration, the quicker the decrease in diameter. On the other hand, the diameter of paraffin droplets changed little over the course of the experiment.

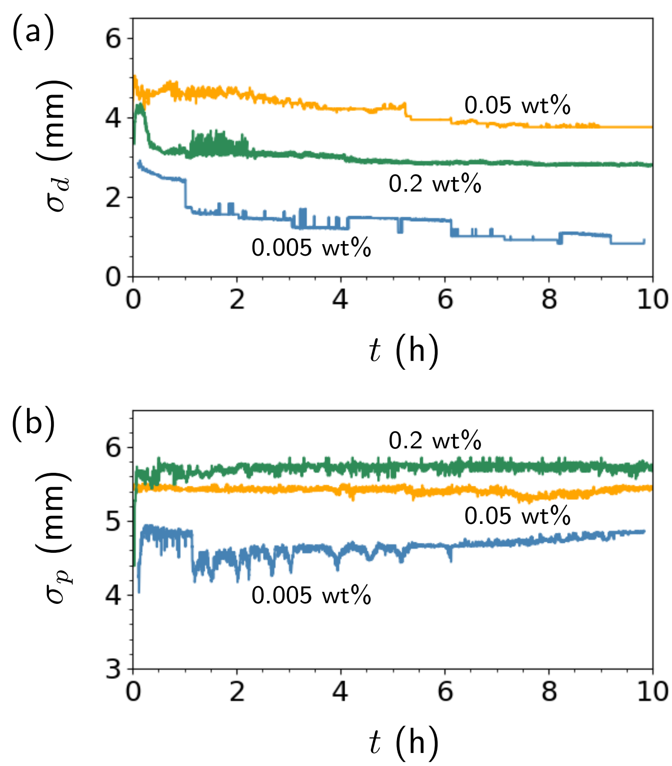

Figure S1: The diameter of (a) a decanol droplet, σ_d and (b) a paraffin droplet, σ_p, as a function of time, t. Each curve represents a result at different SB concentrations, c_{SB}.

S3. Deeper water

Figure S2 shows the distance between two droplets as a function of time t, when the water volume was 50 ml (the depth was about 8 mm) instead of the standard values: volume 30 ml and the depth of about 5 mm. Their qualitative behavior did not change, that is, we observed a constant distance when c_{SB} was low or high and distance oscillations for the intermediate values. There are, however, qualitative differences. For example, it took longer before the trajectories were stabilized.

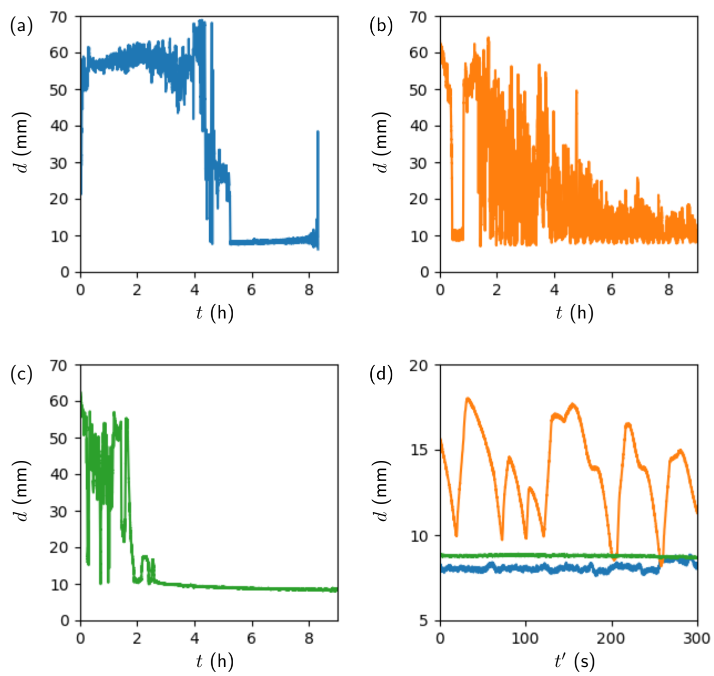

Figure S2: Distance between the two droplets as a function of time when the volume of water was 50 ml and its depth was about 8 mm. (a) $c_{\mathrm{SB}} = 0.005$ wt%, (b) $c_{0.05}$ wt%, and (c) $c_{\mathrm{SB}} = 0.2$ wt%. (d) d in shorter time scale starting at $t = 6$ h. Blue: $c_{\mathrm{SB}} = 0.005$ wt%, orange: $c_{\mathrm{SB}} = 0.05$ wt%, and green: $c_{\mathrm{SB}} = 0.2$ wt%.

S4. Attraction between two paraffin droplets

Figure S3 shows the motion of two paraffin droplets of different sizes (10 μl and 200 μl). The smaller one was colored by 0.05 wt% SB. They are attracted by each other and eventually merged. Their relative speed was almost constant, about 1 mm/s, except when their distance was less than 20 mm. Beyond this distance, they accelerated significantly.

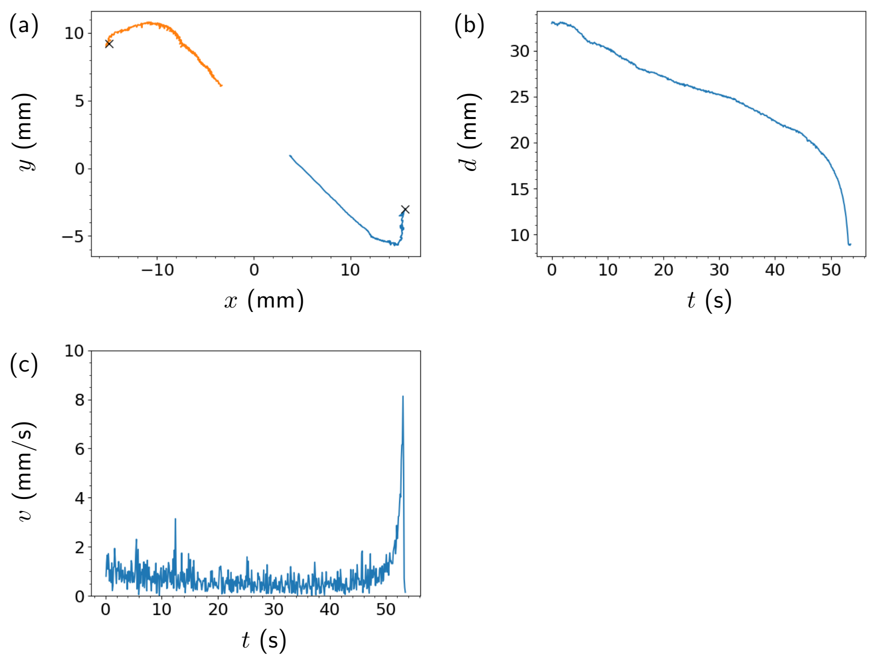

Figure S3: The attraction between a paraffin droplet of 10 μl and a paraffin droplet of 200 μl. (a) Their trajectories. The initial locations are marked by crosses. (b) The center-to-center distance between the droplets as a function of time. The droplets touched at $d \simeq 9$ mm. (c) The relative speed of the smaller droplet to the larger one.

S5. Period of simulated trajectories

Figure S4 shows the period of limit cycles and stable spirals obtained by simulations. The period changes smoothly across the boundary. Moreover, it becomes minimum at the boundary.

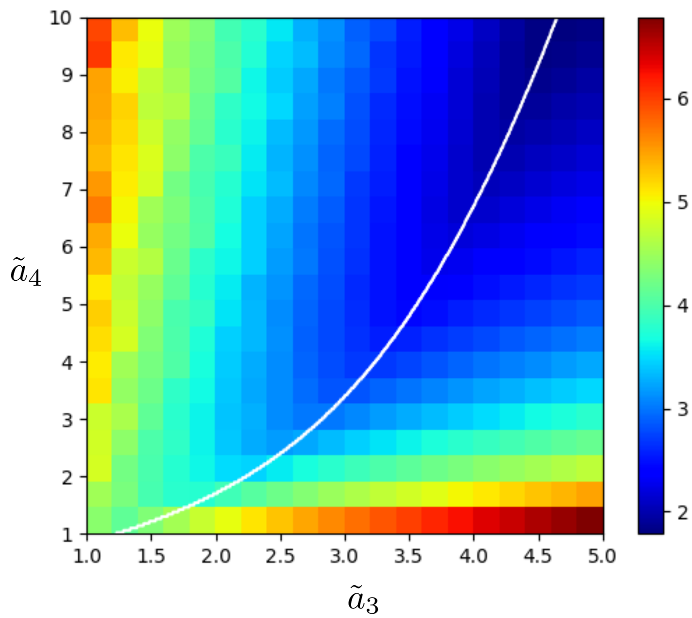

Figure S4: The period of limit cycles and stable spirals obtained by simulations, as a function of $\tilde{a}_3$ and $\tilde{a}_4$. $\tilde{a}_1 = \tilde{a}_2 = 1$. The period was estimated as twice the average time between two consecutive time intervals at which $\tilde{v}$ becomes 0. The white curve represents the stability boundary shown in Fig. 10 in the main text.