Supplementary information: Curvature-Controlled Geometrical Lensing Behavior in Self-Propelled Colloidal Particle Systems

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- Movie 1: Video of cyclic phase separation of active particles confined to a spherocylinder with radius $R = 10\sigma$, length $L = 2\pi R$, persistence length $l = 100\sigma$ and packing fraction $\phi = 0.24$.
- Movie 2: Video of cyclic cluster waves of active particles confined to a spherocone with radii $R = 10\sigma$, $r = 4\sigma$, length $L = 2\pi R$, persistence length $l = 100\sigma$ and packing fraction $\phi = 0.24$.
- Movie 3: Video of positional cluster fluctuations of active particles confined to a metaball with radius $R = 9.5\sigma$, distance $L = 24\sigma$, persistence length $l = 100\sigma$ and packing fraction $\phi = 0.28$.

Persistence length

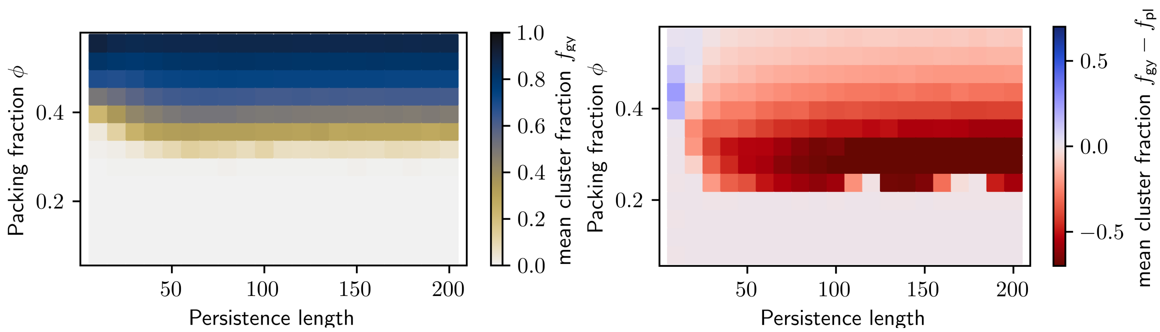

Figure 1. Left: Color map of the mean cluster fraction $f = \frac{1}{N}\sum_i^N \frac{m_i}{N}$ for SPPs on a $2 \times 2 \times 2$ periodic gyroid with mean curvature $K = 0.015\sigma^{-2}$. The phase space is sampled based on the packing fraction ϕ and the persistence length $l = \frac{v_0}{D_r} = 100$ with a constant active particle velocity $v_0 = 1$ and changing rotational diffusion constant D_r. Right: Comparison color map between the weighted cluster fraction of the phase diagram on the left and of a corresponding system, where the SPPs are confined to a flat plane with equal surface area as gyroid surface, respectively. Here, blue regions indicate curvature regions with higher mean cluster fraction than on their flat counterpart whereas red domains correspond to lower clustering.

High curvature regime on the gyroid surface

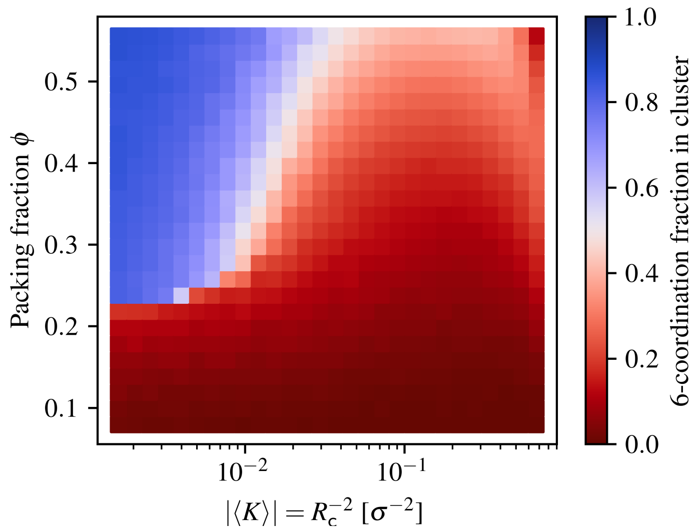

Figure 2. Color map of the weighted mean coordination number for SPPs within the largest cluster on a $2 \times 2 \times 2$ periodic gyroid surface. The phase space is sampled based on the packing fraction ϕ and the mean curvature $|\langle K \rangle|$ at persistence length $l = 100$. In the high curvature regime the SPPs fail to form tightly packed hexagonal configurations, but rather under-coordinated, percolating networks. The arrangements are highly disordered and reminiscent of geometrically frustrated amorphous glasses.

Cluster size fluctuations on curved surfaces

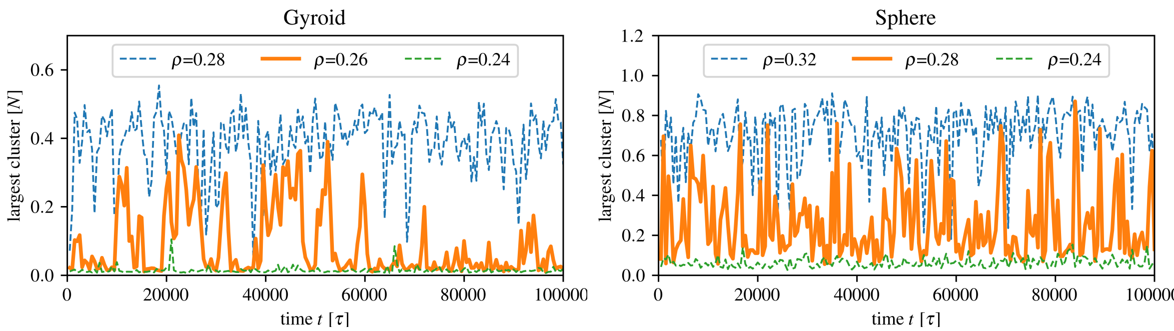

Figure 3. Number of particles within the largest cluster during representative simulations on a $2 \times 2 \times 2$ periodic gyroid surface (left) and a sphere (right) at $|K| = 0.0076\sigma^{-2}$ and persistence length $l = 100$. In between the phase-separated steady state and the gas phase we observe large cluster size fluctuations. On the gyroid these fluctuations are caused by the variance in Gaussian curvature and the fluidization of the dense domains. On the sphere the finite nature of the system combined with the fluidization leads to metastable clusters.