

Active turbulence and spontaneous phase separation in inhomogeneous extensile active gels Supplemental Material

Renato Assante,¹ Dom Corbett,¹ Davide Marenduzzo,¹ and Alexander Morozov¹

¹SUPA, School of Physics and Astronomy, The University of Edinburgh,
James Clerk Maxwell Building, Peter Guthrie Tait Road, Edinburgh, EH9 3FD, United Kingdom
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SI. CHARACTERISTIC DOMAIN SCALE IN MICROPHASE SEPARATION

To support the claim of microphase separation for the low activity and large nematic tendency, the coarsening of domains was quantified by examining the time-evolution of the characteristic domain scale, $L(t)$. This was computed as 2π times the inverse of the first moment of the static structure factor $S(k, t) = \langle \delta\phi(k, t)\delta\phi(k, t) \rangle$, where $\delta\phi = \phi - \phi_0$ and angular brackets denote here the ensemble average, so that

$$L(t) = 2\pi \left[\frac{\int_{\mathcal{K}} dk S(k, t) k}{\int_{\mathcal{K}} dk S(k, t)} \right]^{-1}, \quad (\text{S1})$$

where \mathcal{K} is the set of modes implemented within the respective run of our in-house spectral code.

As Fig. S1 shows, for a given activity the characteristic domain scale increases until reaching a plateau indicative of arrested coarsening. The plateaus for different system sizes are very similar, indicating that the late-time characteristic domain scale does not scale with system size. Fig. S2 additionally shows that the characteristic domain scale decreases with activity. Our data suggest that the functional form of the decay may differ in the microphase separated and active turbulent regime. It would be of interest to explore this issue further, for instance via finite size scaling.

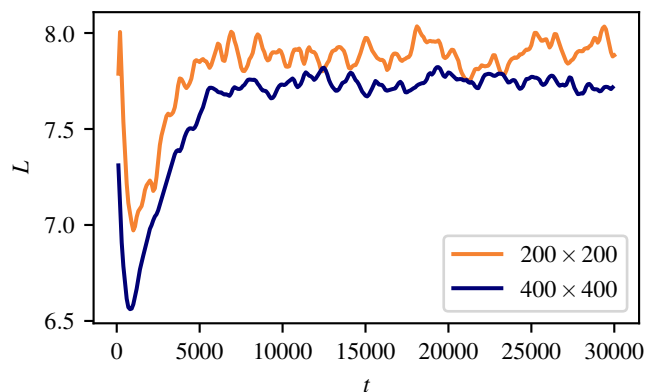


Fig. S1. Time-evolution of characteristic domain scale for microphase separated regime as described in section SI, plotted for simulated system sizes (side of simulation box, H) of 200 and 400 respectively.

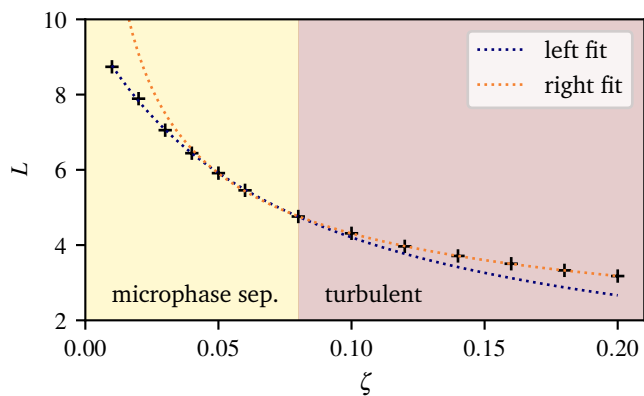


Fig. S2. Characteristic domain scale for a range of activities crossing the boundary between the microphase separated (left) and turbulent (right) regimes, for $\Delta = 1$. The different behaviour is highlighted by shading corresponding to each regime and a fit to the respective set of points, viz. $(1.4\zeta + 0.1)^{-1}$ and $1.2\zeta^{-1/2} + 0.45$.

SII. INCOMPRESSIBILITY AND NAVIER-STOKES SOLVERS

In this Section we discuss a technical point, which is of relevance to the high-activity behaviour of our system. While the concentration of active matter varies, the overall fluid is incompressible: in other words, the total density of active matter and underlying solvent is constant. Incompressibility is important especially for high activity and intermediate Δ — i.e., in the bottom right region in the phase diagram in Fig. 5, which is challenging to characterise accurately. There we typically find active turbulent patterns with vortex correlation length similar to the system size. We found that these are replaced by an apparent phase separation into high and low-concentration patches if the Navier–Stokes solver allows for fluid compressibility and the Mach number is not sufficiently small, which occurs when using, for instance, hybrid Lattice Boltzmann simulations at large activity (in our case, we found this for $\zeta > 0.1$). These states are artifacts in the current model, and using a purely incompressible and 2D fluid they are not found, in line with simulations with tracers in turbulent flows which only show aggregation in compressible fluids, or due to inertial effects [1, 2]. It would however be of interest to see whether these patterns may be recovered in thin active matter films, which can behave effectively as compressible fluids [3].

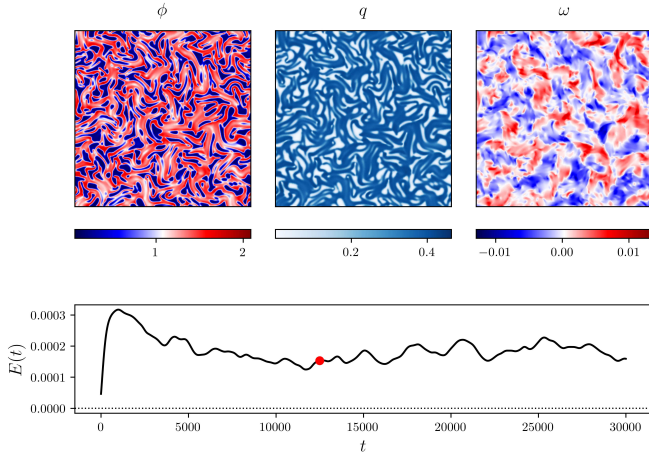


Fig. S3. Initial snapshot of Suppl. Movie 1.

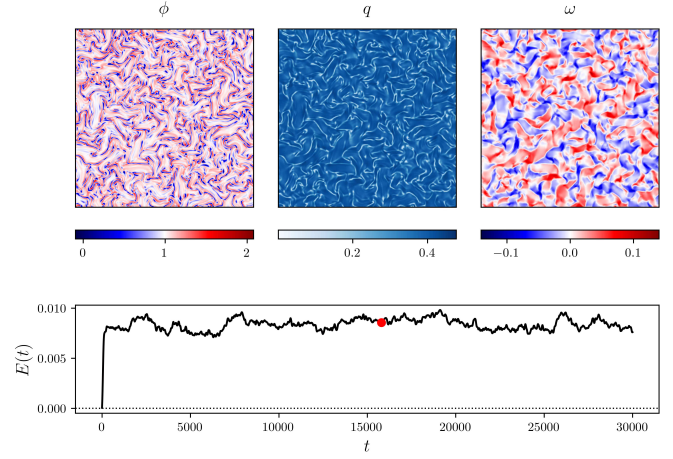


Fig. S5. Initial snapshot of Suppl. Movie 3.

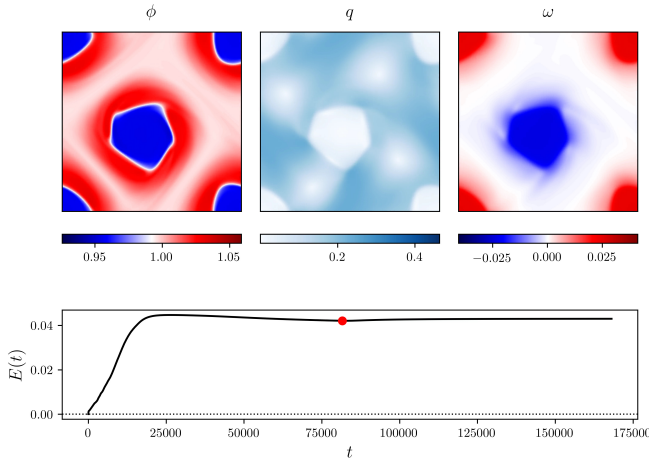


Fig. S4. Initial snapshot of Suppl. Movie 2.

SIII. SUPPLEMENTAL MOVIES

Captions for the Supplemental Movies are given below. Corresponding snapshots are shown in Figs. S1-S3.

Suppl. Movie 1: this movie shows the dynamics corresponding to the microphase separated regime. The graph at the bottom shows the time evolution of the kinetic energy.

Suppl. Movie 2: same as Suppl. Movie 1, but for the regimes where patterns are observed.

Suppl. Movie 3: same as Suppl. Movie 1, but for the active turbulence regimes.

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 [3] R. Voituriez, J. Joanny, and J. Prost, Generic phase diagram of active polar films, *Phys. Rev. Lett.* **96**, 028102 (2006).