Supplemental Material S1

Variable bond design

The variable bonds are made out of Field's metal and silicone. Each variable bond consists of a block of Field's metal and a copper heating element encapsulated in a thin layer of soft silicone. The dimensions of the Field's metal block are $15\text{mm} \times 3\text{mm} \times 1.5\text{mm}$. The copper heater is laser-cut out of a thin sheet of copper with thickness 0.1mm. The width of the copper channel along the serpentine feature of the heater is 0.2mm. We use Smooth-on $\text{Ecoflex}^{\mathbb{TM}}$ 20 as our silicone, which has a modulus $\approx 50\text{kPa}$. The variable bond is assembled upside down in a rectangular mold with dimensions $15\text{mm} \times 4\text{mm} \times 2\text{mm}$. We start with a pre-cured strip of $\text{Ecoflex}^{\mathbb{TM}}$ 20 with thickness 200µm, and lay the copper heater on top of it. A very thin layer of spray-on silicone is put on the copper element to ensure that when an electric voltage is applied across the heater, the current does not find a lower resistance path through the Field's metal core. Once the spray-on silicone cures, the Field's metal block is placed over the heater and the mold is filled with $\text{Ecoflex}^{\mathbb{TM}}$ 20. Once cured, the variable bond is taken out of the mold and is ready to be used.

The copper heater is embedded inside the bond in a way that prevents any extension or compression of the silicone at the edges of the bond. This configuration ensures that in the cold state the variable bond is resilient to any applied longitudinal strain, giving it a high spring constant. The soft state of the bond is achieved by applying an electric voltage across the heater, causing the Field's metal core to melt due to Joule heating. With the Field's metal in the liquid state, the variable bond becomes extremely soft with its stiffness close to that of $\mathrm{Ecoflex}^{\mathbb{M}} 20$.

Regular bond design

Each regular bond is lasercut out of silicone rubber sheets with thickness 1.6mm and a shore hardness of A90. The bonds have a non-uniform width throughout their length with a long wide part in the middle and short thin parts on each end (as seen in the schematic in the main text). The thin parts ensure that the bonds are easy to rotate about their ends. The middle part of each bond is $3.8 \text{mm} \times 15 \text{mm}$ with a thin part measuring $1.8 \text{mm} \times 6 \text{mm}$ on each end. This is the same geometry as the bonds in the experimental networks. The disordered structure of our experimental networks causes the length of each bond to be slightly different. We make our individual bonds the same length as the average bond length in the networks. The silicone encapsulated part of the variable bonds are all the same dimensions: 4mm wide, 15mm long, and 2mm thick. These have 1.8mm wide silicone rubber strips connected on their ends.

Fabricating allosteric networks

The allosteric networks are made in experiments by laser-cutting silicone rubber sheets with shore hardness A90 and thickness 1.6mm. The same material is used to make individual bonds in the previous section. Each bond is 3.8mm wide in the middle 1.8mm wide near the nodes with an average bond length of 24mm. For each allosteric response that we want to incorporate, we have a set of bonds that need to be softened. We find this set of bonds using the design protocol described earlier for a stiffness ratio of 0.01. Once we have identified the set of soft bonds, we simply cut out the middle portion of these bonds from the network and attach a variable bond in their place using Loctite[®] instant adhesive. Once all the bonds corresponding to a particular response are replaced, we connect them in a single circuit by soldering copper wires to the ends of the copper heaters on each bond. Variable bonds are then heated by running an electric current of 1.5A through them. This rapidly switches the variable bonds to their soft state via Joule heating, with the transition time being under a minute. When the current is turned off, the variable bonds cool down completely to reverse back to their stiff state within a few minutes.

Supplemental Material S2



Fig. S1: Strain distribution in allosteric networks. Heat maps showing simulated strain magnitudes in an allosteric network due to an applied strain at the input site (black nodes). Output 1 is labeled with blue nodes and output 2 is labeled with green nodes. Bonds in the stiff state are shown in dotted lines, and bonds in the soft state are shown in dashed lines. (a) Variable bonds corresponding to output 1 are in the soft state. (b) Variable bonds corresponding to output 2 are in the soft state. (c) Both sets of variable bonds are in their soft state. Scale bar shows the magnitude of strain and is normalized by the largest strain in each configuration. Note that the strains are localized and the regions with maximum strain include the input and output sites, as well as the variable bonds that have been softened.