Supplementary

Single wall and graphene-modified multiwall wasp nest shape Bi$_2$Mo$_2$O$_9$ self-assembly for performance-enhanced asymmetric supercapacitor

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Figure S1. Reflectance spectra of Reduced Graphene Oxide (a), annealed S-Bi$_2$Mo$_2$O$_9$ (b), and M-Bi$_2$Mo$_2$O$_9$ (c).
Figure S2. Growth mechanism traced from SEM images of (a) S-Bi$_2$Mo$_2$O$_9$ and (b) M-Bi$_2$Mo$_2$O$_9$. 
Table S1. Specific Capacitance, Energy Density and Power Density Values from GCD curves of asymmetric device

<table>
<thead>
<tr>
<th>Potential Window (V)</th>
<th>Current Density (A g⁻¹)</th>
<th>Discharge Time (s)</th>
<th>Specific Capacitance (F g⁻¹)</th>
<th>Energy Density (Wh Kg⁻¹)</th>
<th>Power Density (W Kg⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1</td>
<td>260</td>
<td>173.333</td>
<td>54.167</td>
<td>750</td>
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<tr>
<td></td>
<td>2</td>
<td>115</td>
<td>153.333</td>
<td>47.916</td>
<td>1499</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>53</td>
<td>106.000</td>
<td>33.125</td>
<td>2250</td>
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<tr>
<td></td>
<td>4</td>
<td>25</td>
<td>66.666</td>
<td>20.833</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>11</td>
<td>36.666</td>
<td>11.458</td>
<td>3750</td>
</tr>
</tbody>
</table>
2.3.1. Trasatti Method Exploration:

\[ q_T = q_i + q_o \]  
\[
C_T = C_i + C_o
\]

\( q_T \) is a total voltametric charge, \( q_i \) is an inner surface charge, and \( q_o \) is an outer surface charge. \( C_T \) is the total capacitance which is the sum of PC and EDLC.

As \( V \rightarrow \infty \) in the outer surface, the more approachable area is possible in the electrode-electrolyte interface due to the surface process. Likewise, \( V \rightarrow 0 \) in the inner surface, there is less access area because of near surfaces and loose grain boundaries that give ions enough time to diffuse and react.

The main procedures involved in the analysis are calculating specific capacitance, total capacitance, calculating charge at the outer surface \( (C_o) \), the inner surface \( (C_i) \), and capacitance contribution percentage. Various scan rates are used for measuring cyclic voltammograms, and respective specific capacitance is calculated based on the following equation,

\[
C_q = \frac{S}{2v \times \Delta U} \]  

Where \( C_q \) is Specific Capacity \( (\text{C/g}) \), \( S \) represents the Area enclosed, Scan rate is represented by \( v \) \( (\text{mV/s}) \), and \( m \) is active mass deposited in the electrode surface \( (\text{mg}) \).

For calculation of total capacitance, consider a semi-infinite linear diffusion pattern \( (\text{viz}, \text{ions arbitrarily diffused between the electrolyte and electrode, when } V \rightarrow 0, q \rightarrow q_T) \). Linear association between the square root of scan rates \( (v^{1/2}) \) and the reciprocal of the observed specific capacitance \( (1/C_q) \) yields the maximum capacitance.

\[ \frac{1}{q(v)} = \text{Const. } v^{1/2} + \frac{1}{q_T} \]  

Multiplying \( dU \) on both sides,

\[ \frac{dU}{q(v)} = \text{Const. } v^{1/2} + \frac{dU}{q_T} \]  

\[ \frac{1}{C(v)} = \text{Const. } v^{1/2} + \frac{1}{C_T} \]  

Likewise, the calculation of charge at the outer surface \( (C_o) \) and the inner surface \( (C_i) \) considers a semi-infinite ion diffusion pattern \( (\text{viz}, \text{ions arbitrarily diffuse from electrolyte to}) \)
electrode, when $V \rightarrow \infty$, $q \rightarrow q_0)$. According to the Cottrell equation, the estimated specific capacitance and the reciprocal root of scan rates give a linear correlation.

$$C(v) = \text{Const.} \cdot v^{1/2} + C_T$$

The maximum capacitance and the outer surface charge ($C_o$) are estimated by,

$$C_t = C_T - C_o$$

The Trasatti method is the easiest to explain, whether it is a surface-controlled capacitive electrode or a diffusion-controlled faradic electrode. The linear fit of $C$ vs. $v^{-1/2}$ and $C^{-1}$ vs. $v^{-1/2}$ for S-Bi$_2$Mo$_2$O$_9$ and M-Bi$_2$Mo$_2$O$_9$ are shown in Figure S3 (a, c) and (b, d).

Figure S3. Linear fit ($C$ vs $v^{-1/2}$) of annealed S-Bi$_2$Mo$_2$O$_9$ and M-Bi$_2$Mo$_2$O$_9$ (a, c), Linear fit ($C^{-1}$ vs $v^{1/2}$) of annealed S-Bi$_2$Mo$_2$O$_9$ and M-Bi$_2$Mo$_2$O$_9$ (b, d).
Figure S4. Comparative CV profile of activated carbon and active material (M-Bi$_2$Mo$_2$O$_9$) at 30 mVs$^{-1}$ in three electrode system