Supplementary materials

Dynamics study of the post-transition-state-bifurcation process of the (HCOOH)H⁺ \rightarrow CO + H₃O⁺ / HCO⁺ + H₂O dissociation: Application of machine-learning techniques

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S1 Figures and Tables for the supporting information



Figure S1: The fitted potential energy as a function of the correction energy



Mode 1, $\tilde{\nu_1} = 1870i$ cm⁻¹

Figure S2: Normal mode vector with the vibrational frequency $\tilde{\nu}_1$ for the imaginary frequency mode 1 at the transition state TS_{1-2}



Figure S3: Temperature-dependent populations of vibrational states with normal mode frequencies. The blue, green, orange and red bars represent the populations at 298, 1000, 2000 and 4000 K for (a) modes 2-6 and (b) modes 7-12.



Figure S4: The distribution of the expectation values of the configuration spaces (a) for Q_2 and (b) Q_{12} modes by PIMD sampling, and (c) for Q_2 and (d) Q_{12} modes by classical sampling. The blue, green, orange and red dotted lines represent the coordinate expectation values for 298, 1000, 2000 and 4000 K. The black sold line present the magnitude of the zero-point vibrational wave function $|\Psi_0|^2$



Figure S5: (a) Accuracy scores and importance of the 54 features for (b) RF and (c) LGB models using 120 data in the binary classification in terms of the classical MD simulations at 298 K



Figure S6: (a) Accuracy scores and importance of the 54 features for (b) RF and (c) LGB models using 80 data in the binary classification in terms of the classical MD simulations at 298 K



Figure S7: (a) Accuracy scores and importance of the six features for (b) RF and (c) LGB models using 166 data in the binary classification in terms of the classical MD simulations at 298 K



Figure S8: (a) Accuracy scores and importance of 54 transition-state features for (b) RF and (c) LGB models in the binary classification with respect to the 4000 K classical simulation

Coordinate	PES	MP2	Ref. [1]
EQ ₁	0.0	0.0	0.0
TS_{1-2}	49.0	52.2	47.8
INT ₂	14.1	17.7	15.1
TS_{2-3}	19.8	20.7	16.5
INT ₃	0.9	3.7	0.7
TS_{3-4}	17.4	18.4	-
INT_4	8.5	11.5	-
$HCO^+ + H_2O$	40.8	46.5	39.7
$CO + H_3O^{\tilde{+}}$	19.2	20.9	16.3

Table S1: The potential energies at key geometries calculated by our PES (in kcal mol^{-1}) and MP2/6-31G(d), and the comparison with the previous study (MP2/6-31G(d))

Table S2: The zero-po PIMD simulations $< E$ different frequencies \tilde{v}_c	int vib > and «	rationa the ana	l energies lytical va	s (ZPE), the second state of the second state	he expect > (in kca	ation valu l mol ⁻¹) f	es of internal energy by or each temperature with
			29	98 K	1000 K		
	\tilde{v}_{lpha}	ZPE	< E >	$< U_q >$	< E >	$< U_q >$	
	\tilde{v}_2	0.61	0.78	0.79	2.04	2.05	
	\tilde{v}_3	1.20	1.24	1.24	2.22	2.22	
	~.	1 / 1	1 4 2	1 4 2	0.01	0.01	

\tilde{v}_2	0.61	0.78	0.79	2.04	2.05
\tilde{v}_3	1.20	1.24	1.24	2.22	2.22
$ ilde{v}_4$	1.41	1.43	1.43	2.31	2.31
\tilde{v}_5	1.52	1.54	1.54	2.35	2.36
\tilde{v}_6	1.53	1.54	1.55	2.35	2.36
\tilde{v}_7	1.79	1.80	1.80	2.49	2.50
$ ilde{ u}_8$	1.84	1.84	1.85	2.51	2.53
\tilde{v}_9	2.51	2.49	2.51	2.92	2.95
\tilde{v}_{10}	2.88	2.84	2.88	3.18	3.22
\tilde{v}_{11}	4.70	4.57	4.70	4.68	4.79
\tilde{v}_{12}	5.05	4.88	5.05	4.97	5.11
		2000 K		4000 K	
\tilde{v}_{lpha}		< E >	$< U_q >$	< E >	$< U_q >$
\tilde{v}_2		3.99	4.00	7.95	7.96
\tilde{v}_3		4.09	4.09	8.00	8.01
\tilde{v}_4		4.13	4.14	8.02	8.03
\tilde{v}_5		4.16	4.17	8.04	8.04
\tilde{v}_6		4.16	4.17	8.03	8.04
\tilde{v}_7		4.23	4.24	8.08	8.08
$ ilde{v}_8$		4.25	4.25	8.09	8.09
\tilde{v}_9		4.48	4.49	8.18	8.21
$ ilde{v}_9 \ ilde{v}_{10}$		4.48 4.63	4.49 4.65	8.18 8.28	8.21 8.29
$egin{array}{l} ilde{v}_9 \ ilde{v}_{10} \ ilde{v}_{11} \end{array}$		4.48 4.63 5.60	4.49 4.65 5.68	8.18 8.28 8.80	8.21 8.29 8.85

	298 K	1000 K	2000 K	4000 K			
	$< U_q >$	$< U_q >$	$< U_q >$	$< U_q >$			
	0.59	1.99	3.97	7.95			
\tilde{v}_{lpha}	< E >	< E >	< E >	< E >			
	Classical						
\tilde{v}_2	0.59	1.99	3.99	7.94			
\tilde{v}_3	0.59	1.98	3.97	7.94			
\tilde{v}_4	0.59	1.98	3.97	7.98			
\tilde{v}_5	0.59	1.99	3.98	7.93			
\tilde{v}_6	0.59	1.99	3.96	7.95			
$ ilde{ u}_7$	0.59	1.99	3.98	7.93			
$ ilde{ u}_8$	0.59	1.99	3.97	7.91			
\tilde{v}_9	0.59	1.99	3.98	7.95			
\tilde{v}_{10}	0.59	1.99	3.98	7.95			
\tilde{v}_{11}	0.59	1.99	3.98	7.96			
\tilde{v}_{12}	0.59	1.99	3.98	7.96			

Table S3: Temperature-dependent internal energy of the classical Hamiltonian for harmonic oscillator $\langle U_c \rangle$ and the energy expectation values by classical sampling $\langle E \rangle$ (in kcal mol⁻¹)

References

 [1] Osamu Sekiguchi, Vebjørn Bakken, and Einar Uggerud. Decomposition of protonated formic acid: One transition state - Two product channels. J. Am. Soc. Mass Spectrom., 15(7):982–988, 2004.