

**Electronic Supplementary Information**

**A Comprehensive diffusion-induced stress coupled multiscale modeling and analysis in hard-carbon electrode of Li-ion batteries**

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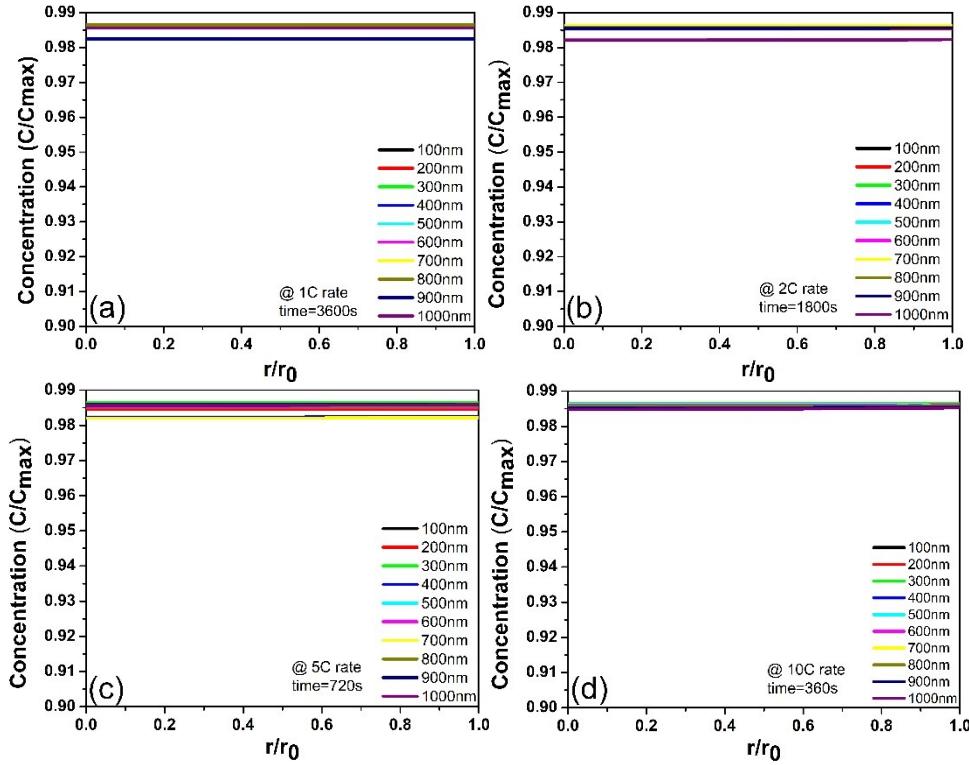
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**Table S1.** The current densities ( $\text{mA}/\text{cm}^2$ ) applied to particles of various radius at different C-rates.

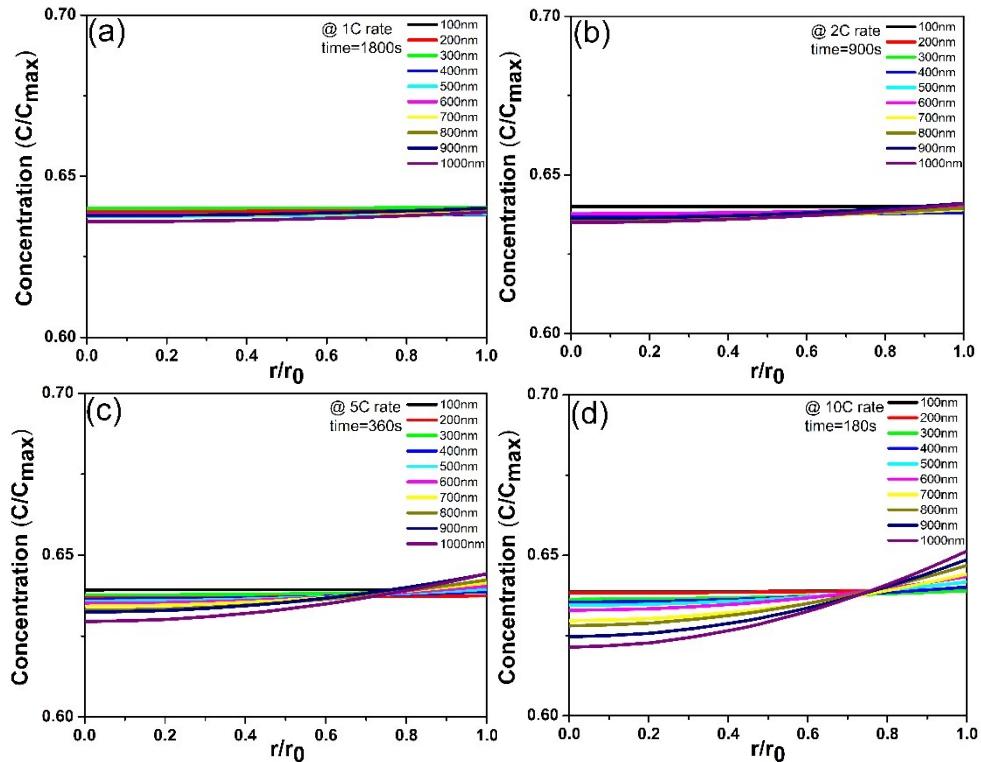
S. No.	Radius of the particle (nm)	Applied current density ( $\text{mA}/\text{cm}^2$ )			
		@ 1 C rate	@ 2 C rate	@ 5 C rate	@ 10 C rate
1	100	0.004167	0.008333	0.020833	0.041667
2	200	0.008333	0.016667	0.041667	0.083333
3	300	0.012500	0.025000	0.062500	0.125000
4	400	0.016667	0.033333	0.083333	0.166667
5	500	0.020833	0.041667	0.104167	0.208333
6	600	0.025000	0.050000	0.125000	0.250000
7	700	0.029167	0.058333	0.145833	0.291667
8	800	0.033333	0.066667	0.166667	0.333333
9	900	0.037500	0.075000	0.187500	0.375000
10	1000	0.041667	0.083333	0.208333	0.416667

**Table S2.** Diffusivity (D) of  $\text{Li}^+$  in  $\text{Li}_x\text{C}$  at room temperature (300 K) and Elastic modulus (E) of  $\text{Li}_x\text{C}$  calculated using the MD simulation, with ReaxFF potential.

Phase/SOC (%)	Diffusivity (D in $\text{cm}^2/\text{s}$ )	Elastic modulus (E in GPa)
$\text{Li}_{0.0224}\text{C} / \text{SOC (10\%)}$	$1.96 \times 10^{-11}$	53.99
$\text{Li}_{0.0448}\text{C} / \text{SOC (20\%)}$	$5.84 \times 10^{-11}$	52.91
$\text{Li}_{0.0672}\text{C} / \text{SOC (30\%)}$	$2.08 \times 10^{-11}$	49.41
$\text{Li}_{0.0896}\text{C} / \text{SOC (40\%)}$	$5.52 \times 10^{-11}$	41.58
$\text{Li}_{0.112}\text{C} / \text{SOC (50\%)}$	$6.76 \times 10^{-11}$	31.73
$\text{Li}_{0.1344}\text{C} / \text{SOC (60\%)}$	$2.15 \times 10^{-10}$	35.02
$\text{Li}_{0.1568}\text{C} / \text{SOC (70\%)}$	$3.01 \times 10^{-10}$	30.49
$\text{Li}_{0.1792}\text{C} / \text{SOC (80\%)}$	$4.82 \times 10^{-10}$	29.47
$\text{Li}_{0.2016}\text{C} / \text{SOC (90\%)}$	$1.99 \times 10^{-11}$	20.22

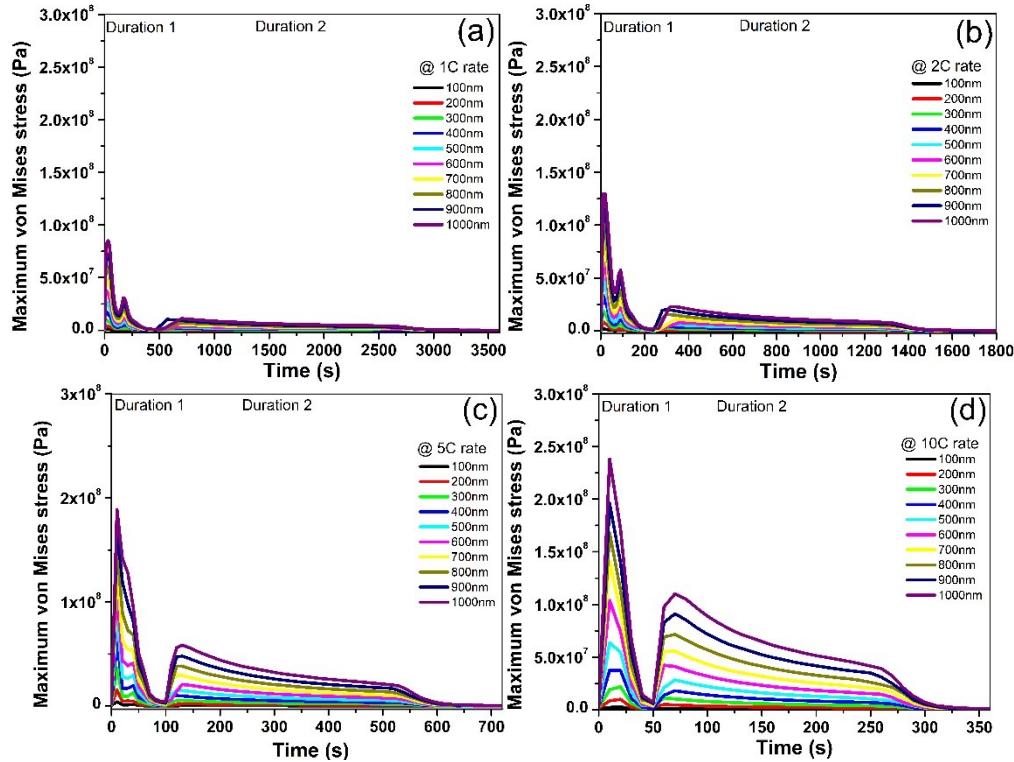


**Fig. S1** Evolution of the particle size-dependent dimensionless concentration field for the two-way coupled approach over the dimensionless radius at (a) 1C, (b) 2C, (c) 5C, and (d) 10C-rates. All the profiles were shown for the corresponding last times steps (SOC 100) for different particle size (100-1000 nm).



**Fig. S2** Evolution of the particle size-dependent dimensionless concentration field for the two-way coupled approach over the dimensionless radius at (a) 1C, (b) 2C, (c) 5C, and (d) 10 C-rates. All the profiles were shown for the corresponding half of the lithiation time. (e) values of the Li<sup>+</sup> concentration at half of the particle radius for all the particles with respect to C-rates and (f) schematic of the spherical particle showing the location of  $r/r_0$  where the values of concentration are obtained.

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**Fig. S3** Maximum von Mises stress over time, for different spherical particles of radius 100-1000 nm, for the two-way coupled approach over different lithiation rates (C-rates), (a) 1C, (b) 2C, (c) 5C, and (d) 10C, peak values of maximum von Mises stresses with respect to C-rate for various particle sizes at (e) duration 1 (up to SOC 15), and (f) duration 2 (SOC range 15-100). (One-way coupled)

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