

## **Supporting Information**

# **On the nature of the two-positron bond: Evidence for a novel bond type**

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**Table S1** Total energy of  $(PsH)_2$  system as a function of the inter-proton distance computed at the MC-HF/[aug-cc-pVTZ:aug-cc-pVTZ], MC-HF/[aug-cc-pVQZ:aug-cc-pVQZ], VMC and DMC levels of theory for the singlet positronic and electronic spin states. The statistical error of the last digits of the QMC results is given in parentheses. All values are in atomic units.

| $R_{HH}$ | MC-HF       | MC-HF       | VMC         | DMC        |
|----------|-------------|-------------|-------------|------------|
|          | aug-cc-pVTZ | aug-cc-pVQZ |             |            |
| 3.2      | -1.273450   | -1.275147   |             |            |
| 3.6      | -1.287555   | -1.289003   |             |            |
| 4.0      | -1.299544   | -1.300855   |             |            |
| 4.4      | -1.309134   | -1.310377   |             |            |
| 4.8      | -1.316450   | -1.317666   |             |            |
| 5.2      | -1.321790   | -1.323002   |             |            |
| 5.6      | -1.325519   | -1.326738   | -1.58090(2) | -1.5882(1) |
| 5.7      |             |             | -1.58120(2) | -1.5884(1) |
| 5.8      |             |             | -1.58149(2) | -1.5886(1) |
| 5.9      |             |             | -1.58165(2) | -1.5887(1) |
| 6.0      | -1.328008   | -1.329237   | -1.58178(2) | -1.5887(1) |
| 6.1      |             |             | -1.58180(2) | -1.5887(1) |
| 6.2      |             |             | -1.58181(2) | -1.5887(1) |
| 6.3      |             |             | -1.58178(2) | -1.5886(1) |
| 6.4      | -1.329587   | -1.330830   | -1.58166(2) | -1.5884(1) |
| 6.8      | -1.330528   | -1.331792   |             |            |
| 7.2      | -1.331043   | -1.332332   |             |            |
| 7.6      | -1.331292   | -1.332603   |             |            |
| 8.0      | -1.331389   | -1.332710   |             |            |
| 8.1      | -1.331398   | -1.332720   |             |            |
| 8.2      | -1.331404   | -1.332725   |             |            |
| 8.3      | -1.331407   | -1.332727   |             |            |
| 8.4      | -1.331408   | -1.332725   |             |            |
| 8.5      | -1.331406   | -1.332720   |             |            |
| 8.6      | -1.331403   | -1.332713   |             |            |
| 8.7      | -1.331398   | -1.332705   |             |            |
| 8.8      | -1.331393   | -1.332696   |             |            |
| 9.2      | -1.331367   | -1.332651   |             |            |
| 9.6      | -1.331337   | -1.332607   |             |            |
| 10.0     | -1.331308   | -1.332568   |             |            |
| 10.4     | -1.331277   | -1.332537   |             |            |
| 10.8     | -1.331247   | -1.332512   |             |            |
| 11.2     | -1.331218   | -1.332490   |             |            |
| 11.6     | -1.331190   | -1.332470   |             |            |
| 12.0     | -1.331164   | -1.332453   |             |            |

**Table S2** Total energy of the singlet electronic spin state of  $[H^-, e^+, H^-]$  system as a function of the inter-proton distance computed at the MC-HF/[auc-cc-pVTZ:auc-cc-pVTZ], VMC and DMC levels of theory. The statistical error of the last digit of the QMC results is given in parentheses. All values are given in atomic units.

| $R_{HH}$ | MC-HF       | VMC         | DMC        |
|----------|-------------|-------------|------------|
|          | auc-cc-pVTZ |             |            |
| 3.2      | -1.111312   |             |            |
| 3.6      | -1.123523   |             |            |
| 4.0      | -1.135653   |             |            |
| 4.4      | -1.146367   |             |            |
| 4.8      | -1.155217   |             |            |
| 5.2      | -1.162160   |             |            |
| 5.6      | -1.167326   |             |            |
| 5.7      | -1.168361   |             |            |
| 5.8      | -1.169300   |             |            |
| 5.9      | -1.170146   | -1.33533(2) | -1.3399(1) |
| 6.0      | -1.170902   | -1.33559(2) | -1.3401(1) |
| 6.1      | -1.171570   | -1.33577(2) | -1.3403(1) |
| 6.2      | -1.172153   | -1.33591(2) | -1.3404(1) |
| 6.3      | -1.172654   | -1.33600(2) | -1.3405(1) |
| 6.4      | -1.173075   | -1.33600(2) | -1.3404(1) |
| 6.5      | -1.173418   | -1.33600(2) | -1.3404(1) |
| 6.6      | -1.173687   | -1.33595(2) | -1.3404(1) |
| 6.7      | -1.173884   | -1.33586(2) | -1.3402(1) |
| 6.8      | -1.174011   | -1.33567(2) | -1.3401(1) |
| 6.9      | -1.174071   | -1.33551(2) | -1.3398(1) |
| 7.0      | -1.174068   |             |            |
| 7.1      | -1.174003   |             |            |
| 7.2      | -1.173881   |             |            |
| 7.6      | -1.172887   |             |            |
| 8.0      | -1.171278   |             |            |
| 8.4      | -1.169314   |             |            |
| 8.8      | -1.167221   |             |            |
| 9.2      | -1.165157   |             |            |
| 9.6      | -1.163210   |             |            |
| 10.0     | -1.161418   |             |            |
| 10.4     | -1.159786   |             |            |
| 10.8     | -1.158307   |             |            |
| 11.2     | -1.156969   |             |            |
| 11.6     | -1.155761   |             |            |
| 12.0     | -1.154671   |             |            |

**Table S3** Vibrational energy levels of  $[H^-, e^+, H^-]$  and  $(PsH)_2$  based on their respective ground-state potential energy curves obtained at the DMC level of theory given in atomic units.

| Level | $[H^-, e^+, H^-]$ | $(PsH)_2$ |
|-------|-------------------|-----------|
| 0     | -1.3394           | -1.5876   |
| 1     | -1.3373           | -1.5855   |
| 2     | -1.3354           | -1.5836   |
| 3     | -1.3335           | -1.5818   |
| 4     | -1.3316           | -1.5805   |
| 5     | -1.3299           | -1.5793   |
| 6     | -1.3282           |           |
| 7     | -1.3267           |           |
| 8     | -1.3252           |           |
| 9     | -1.3237           |           |

**Table S4** Total energy of PsH system computed at the MC-HF/[auc-cc-pVTZ:auc-cc-pVTZ] and MC-HF/[auc-cc-pVQZ:auc-cc-pVQZ] levels of theory while the exact value is from [G. Ryzhikh, J. Mitroy *J. Phys. B* **1999**, 32, 4051]. All values are in atomic units.

|     | MC-HF       | MC-HF       | Exact     |
|-----|-------------|-------------|-----------|
|     | auc-cc-pVTZ | auc-cc-pVQZ |           |
| $E$ | -0.665519   | -0.666175   | -0.789196 |

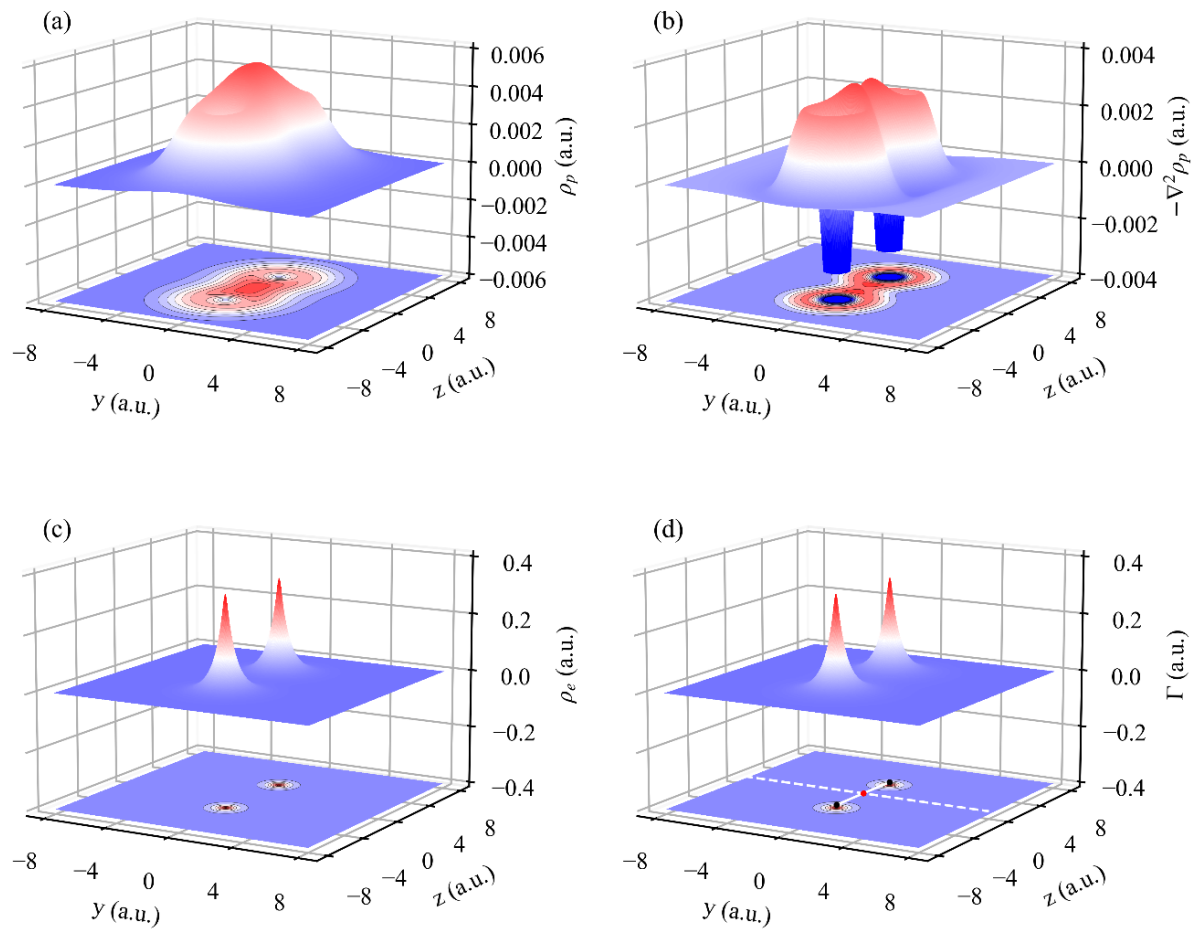
**Table S5** TC-IQA terms computed at the MC-HF/[aug-cc-pVTZ:aug-cc-pVTZ] level of theory at 6.0 Bohr inter-nuclear distance for the singlet positronic and electronic spin states of  $(PsH)_2$  system. All values are given in atomic units.

| $E_{\text{intra}}$                       |                                          |                                          |                                          |                                          |                                          |
|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| $T_e(\Omega_{k=1,2})$                    | $V_{e\text{-nuc}}(\Omega_{k=1,2})$       | $V_{ee}^{\text{cl}}(\Omega_{k=1,2})$     | $V_{ee}^{\text{xc}}(\Omega_{k=1,2})$     |                                          |                                          |
| 0.6142                                   | -1.5609                                  | 0.9515                                   | -0.4683                                  |                                          |                                          |
| $T_p(\Omega_{k=1,2})$                    | $V_{p\text{-nuc}}(\Omega_{k=1,2})$       | $V_{ep}^{\text{cl}}(\Omega_{k=1,2})$     | $V_{pp}^{\text{cl}}(\Omega_{k=1,2})$     | $V_{pp}^{\text{xc}}(\Omega_{k=1,2})$     |                                          |
| 0.0655                                   | 0.3399                                   | -0.6081                                  | 0.1236                                   | -0.1040                                  |                                          |
| $E_{\text{inter}}$                       |                                          |                                          |                                          |                                          |                                          |
| $V_{e\text{-nuc}}(\Omega_1, \Omega_2)$   | $V_{e\text{-nuc}}(\Omega_2, \Omega_1)$   | $V_{\text{nuc-nuc}}(\Omega_1, \Omega_2)$ | $V_{ee}^{\text{cl}}(\Omega_1, \Omega_2)$ | $V_{ee}^{\text{xc}}(\Omega_1, \Omega_2)$ |                                          |
| -0.3273                                  | -0.3273                                  | 0.1667                                   | 0.6432                                   | -0.0123                                  |                                          |
| $V_{ep}^{\text{cl}}(\Omega_1, \Omega_2)$ | $V_{ep}^{\text{cl}}(\Omega_2, \Omega_1)$ | $V_{p\text{-nuc}}(\Omega_1, \Omega_2)$   | $V_{p\text{-nuc}}(\Omega_2, \Omega_1)$   | $V_{pp}^{\text{cl}}(\Omega_1, \Omega_2)$ | $V_{pp}^{\text{xc}}(\Omega_1, \Omega_2)$ |
| -0.3050                                  | -0.3050                                  | 0.1549                                   | 0.1549                                   | 0.1457                                   | -0.0232                                  |

**Table S6** TC-IQA terms computed at the MC-HF/[aug-cc-pVTZ:aug-cc-pVTZ] level of theory at 8.4 Bohr inter-nuclear distance for the singlet positronic and electronic spin states of  $(PsH)_2$  system. All values are given in atomic units.

| $E_{\text{intra}}$                       |                                          |                                          |                                          |                                          |                                          |
|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| $T_e(\Omega_{k=1,2})$                    | $V_{e\text{-nuc}}(\Omega_{k=1,2})$       | $V_{ee}^{\text{cl}}(\Omega_{k=1,2})$     | $V_{ee}^{\text{xc}}(\Omega_{k=1,2})$     |                                          |                                          |
| 0.6011                                   | -1.5448                                  | 0.9368                                   | -0.4667                                  |                                          |                                          |
| $T_p(\Omega_{k=1,2})$                    | $V_{p\text{-nuc}}(\Omega_{k=1,2})$       | $V_{ep}^{\text{cl}}(\Omega_{k=1,2})$     | $V_{pp}^{\text{cl}}(\Omega_{k=1,2})$     | $V_{pp}^{\text{xc}}(\Omega_{k=1,2})$     |                                          |
| 0.0652                                   | 0.3285                                   | -0.5872                                  | 0.1182                                   | -0.1129                                  |                                          |
| $E_{\text{inter}}$                       |                                          |                                          |                                          |                                          |                                          |
| $V_{e\text{-nuc}}(\Omega_1, \Omega_2)$   | $V_{e\text{-nuc}}(\Omega_2, \Omega_1)$   | $V_{\text{nuc-nuc}}(\Omega_1, \Omega_2)$ | $V_{ee}^{\text{cl}}(\Omega_1, \Omega_2)$ | $V_{ee}^{\text{xc}}(\Omega_1, \Omega_2)$ |                                          |
| -0.2373                                  | -0.2373                                  | 0.1190                                   | 0.4730                                   | -0.0028                                  |                                          |
| $V_{ep}^{\text{cl}}(\Omega_1, \Omega_2)$ | $V_{ep}^{\text{cl}}(\Omega_2, \Omega_1)$ | $V_{p\text{-nuc}}(\Omega_1, \Omega_2)$   | $V_{p\text{-nuc}}(\Omega_2, \Omega_1)$   | $V_{pp}^{\text{cl}}(\Omega_1, \Omega_2)$ | $V_{pp}^{\text{xc}}(\Omega_1, \Omega_2)$ |
| -0.2309                                  | -0.2309                                  | 0.1158                                   | 0.1158                                   | 0.1129                                   | -0.0052                                  |





**Fig. S1** Relief maps of (a) the one-positron density, (b) the Laplacian of the one-positron density, (c) the one-electron density, and (d) the Gamma density of  $(PsH)_2$ . The protons are 6.0 Bohr apart, placed at (0.0, 0.0, 3.0) and (0.0, 0.0, -3.0) in the coordinate system. The black and red spheres in panel (d) are the (3, -3) and (3, -1) CPs of the Gamma density, respectively, while the solid white lines are the gradient paths connecting the (3, -3) and (3, -1) CPs. The dashed white line is the intersection of the zero-flux surface and the yz plane, which acts as the boundary of the AIM.