Electronic Supplementary Material (ESI) for Physical Chemistry Chemical Physics.

Supplementary Information for "Velocity Map Imaging with No Spherical Aberrations"

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I. FULL DERIVATIONS

A. Expression for time-of-flight

The time-of-flight of particles to the detectors is determined by plugging $z(t_f) = L$ into Eq. 4 from the main manuscript:

$$L = z(t_t) = \left(z_0 + \frac{U_z}{4U_\rho}\right) cosh\left(\sqrt{2}\omega_\rho t_f\right) + \frac{v_z^0}{\sqrt{2}\omega_\rho} sinh\left(\sqrt{2}\omega_\rho t_f\right) - \frac{U_z}{4U_\rho} \tag{1}$$

This equation is of the form:

$$C = Acosh(x) + Bsinh(x) \tag{2}$$

where:

$$A = z_0 + \frac{U_z}{4U_\rho} \tag{3}$$

$$B = \frac{v_z^0}{\sqrt{2}\omega_\rho} \tag{4}$$

and

$$C = L + \frac{U_z}{4U_\rho}. \tag{5}$$

If $|A| > |B|$ we can define:

$$A = \alpha cosh(\beta), \quad B = \alpha sinh(\beta) \tag{6}$$

With

$$\alpha = \sqrt{A^2 - B^2} = \sqrt{\left(z_0 + \frac{U_z}{4U_\rho}\right)^2 - \left(\frac{v_z^0}{\sqrt{2}\omega_\rho}\right)^2} \tag{7}$$

and

$$\beta = tanh^{-1}\left(\frac{A}{B}\right) = tanh^{-1}\left(\frac{\sqrt{2}\omega_\rho}{v_z^0}(z_0 + \frac{U_z}{4U_\rho})\right) \tag{8}$$

, so that

$$C = \alpha(cosh(\beta)cosh(x) + sinh(\beta)sinh(x) = \alpha cosh(\beta + x). \tag{9}$$

Consequently:

$$\sqrt{2}\omega_\rho t_f = x = cosh^{-1}\left(\frac{C}{\alpha}\right) - \beta. \tag{10}$$

Plugging in the values for $C,\ \alpha$ and β leads to:

$$t_f = \frac{1}{\sqrt{2}\omega_\rho}\left[cosh^{-1}\left(\frac{L+\frac{U_z}{4U_\rho}}{\sqrt{\left(z_0+\frac{U_z}{4U_\rho}\right)^2-\left(\frac{v_z^0}{\sqrt{2}\omega_\rho}\right)^2}}\right) - tanh^{-1}\left(\frac{v_z^0}{\sqrt{2}\omega_\rho}\cdot\frac{1}{z_0+\frac{U_z}{4U_\rho}}\right)\right] \tag{11}$$

In the case where $|A| < |B|$ we use a similar derivation with:

$$C = cosh(x) + Bsinh(x) = \alpha(sinh(\beta)cosh(x) + cosh(\beta)sinh(x)) = \alpha sinh(\beta + x) \tag{12}$$

where

$$A = \alpha sinh(\beta) = z_0 + \frac{U_z}{4U_\rho} \quad B = \alpha cosh(\beta) = \frac{v_z^0}{\sqrt{2}\omega_\rho} \tag{13}$$

$$C = L + \frac{U_z}{4U_\rho} \tag{14}$$

$$\alpha = \sqrt{B^2 - A^2} = \sqrt{\left(\frac{v_z^0}{\sqrt{2}\omega_\rho}\right)^2 - \left(z_0 + \frac{U_z}{4U_\rho}\right)^2} \tag{15}$$

and

$$\beta = tanh^{-1}\left(\frac{A}{B}\right) = tanh^{-1}\left(\frac{\sqrt{2}\omega_\rho}{v_z^0}(z_0 + \frac{U_z}{4U_\rho})\right). \tag{16}$$

leading to:

$$\sqrt{2}\omega_\rho t_f = x = sinh^{-1}\left(\frac{C}{\alpha}\right) - \beta \tag{17}$$

$$t_f = \frac{1}{\sqrt{2}\omega_\rho}\left[sinh^{-1}\left(\frac{L+\frac{U_z}{4U_\rho}}{\sqrt{\left(z_0+\frac{U_z}{4U_\rho}\right)^2-\left(\frac{v_z^0}{\sqrt{2}\omega_\rho}\right)^2}}\right) - tanh^{-1}\left(\frac{v_z^0}{\sqrt{2}\omega_\rho}\cdot\frac{1}{z_0+\frac{U_z}{4U_\rho}}\right)\right] \tag{18}$$

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B. Second order expansion of impact position

We perform expansion of Eq. 11 around $v_z^0 = z_0 = 0$. Noting that:

$$\left(\frac{dt_f}{dz_0}\right)_{v_z^0=z_0=0} = \frac{1}{\sqrt{2}\omega_\rho}\left[\frac{\frac{dx}{dz_0}}{\sqrt{x^2-1}} + \frac{\frac{dy}{dz_0}}{1-y^2}\right]_{v_z^0=z_0=0} = -\frac{2\beta+1}{\sqrt{2+2/\beta}}\frac{1}{\omega_\rho L} \tag{19}$$

and:

$$\left(\frac{dt_f}{dv_z^0}\right)_{v_z^0=z_0=0} = \frac{1}{\sqrt{2}\omega_\rho}\left[\frac{\frac{dx}{dv_z^0}}{\sqrt{x^2-1}} + \frac{\frac{dy}{dv_z^0}}{1-y^2}\right]_{v_z^0=z_0=0} = \frac{\beta}{\omega_\rho^2 L} \tag{20}$$

Leads to:

$$t_f = \frac{cosh^{-1}(2\beta+1)}{\sqrt{2}\omega_\rho} - \frac{2\beta+1}{\sqrt{2+2/\beta}}\frac{1}{\omega_\rho L}z_0 + \frac{\beta}{\omega_\rho^2 L}v_z^0 + ... \tag{21}$$

Expanding the expression for ρ_f around $\omega_\rho t_f = \pi/2$ leads to:

$$\rho_f = \rho_0 cos(\omega_\rho t_f) + \frac{v_\rho^0}{\omega_\rho} sin(\omega_\rho t_f) \simeq -\rho_0\omega_\rho\Delta t_f + \frac{v_\rho^0}{\omega_\rho} \tag{22}$$

$$\rho_f = \frac{v_\rho^0}{\omega_\rho} + \frac{1}{2^{3/2}}\left(\frac{4\beta^2+2\beta}{\sqrt{\beta^2+\beta}}\right)\frac{1}{L}\rho_0 z_0 - \frac{\beta}{\omega_\rho L}\rho_0 v_z^0 = \frac{v_\rho^0}{\omega_\rho} + 1.38\frac{\rho_0 z_0}{L} - \frac{0.9161}{\omega_\rho L}\rho_0 v_z^0 \tag{23}$$