Supplementary Information

Vibrational manipulation of dry granular materials in lab-on-a-chip devices

Timothy Hui[1*], Xiaolin Zhang[1*], Dhruva Adiga[1], Gregory H. Miller[1], and William D. Ristenpart[1†]

[1] Dept. of Chemical Engineering, University of California Davis, One Shields Ave., Davis, CA 95616 USA.

** These authors contributed equally to this article.*

† Corresponding author: William D. Ristenpart, Dept. of Chemical Engineering, University of California Davis, One Shields Ave., Davis, CA 95616 USA, (530) 752-8780, wdristenpart@ucdavis.edu.

Supplementary Video 1 | Granular motion in a T-junction channel vibrating at 30Hz.

Real-time video of blue-colored glass sand moving in response to a horizontal 30 Hz vibratory waveform (cf. Fig. 1b). Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channel appears stationary despite its motion. Rectangular channel width is 2 mm.

Supplementary Video 2 | Granular motion in a T-junction channel vibrating at 30 and 60 Hz.

Real-time video of blue-colored glass sand moving in response to a horizontal vibratory waveform with 30 and 60 Hz modes superimposed (cf. Fig. 1c). Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channel appears stationary despite its motion. Rectangular channel width is 2 mm.

Supplementary Video 3 | Granular motion in a T-junction channel vibrating at 30 and 60 Hz with negative polarity.

Real-time video of blue-colored glass sand moving in response to a horizontal vibratory waveform with 30 and 60 Hz modes superimposed, with polarity negative to that applied in Supplementary Video 2 (cf. Fig. 1d). Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channel appears stationary despite its motion. Rectangular channel width is 2 mm.

Supplementary Video 4 | Motion of various granular media in channels vibrating at 30 and 60 Hz.

Representative real-time video of yeast agglomerates, crushed ibuprofen powder, and blue-colored glass sand (top, middle, and bottom channels, respectively) moving in response to a horizontal vibratory waveform with 30 and 60 Hz modes superimposed. Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channels appear stationary despite the vibratory motion. Each rectangular channel is 2 mm wide.

Supplementary Video 5 | Mixing of granular media at a Y-junction vibrating with high induced velocity.

Real-time video of blue-colored and yellow-colored glass sand moving in response to a horizontal vibratory waveform with 30 and 60 Hz modes, negative polarity, superimposed. The vibratory amplitude ratio is $B/A = 0.23$. Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channels appear stationary despite the vibratory motion. Rectangular channel width is 2mm.

Supplementary Video 6 | Mixing of granular media at a Y-junction vibrating with low induced velocity.

Real-time video of blue-colored and yellow-colored glass sand moving in response to a horizontal vibratory waveform with 50 and 100 Hz modes, negative polarity, superimposed. The vibratory amplitude ratio is $B/A = 0.06$. Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channels appear stationary despite the vibratory motion. Rectangular channel width is 2mm.

Supplementary Video 7 | Mixing of granular media at a vibrating Y-junction with serpentine channel.

Real-time video of blue-colored and yellow-colored glass sand moving in response to a horizontal vibratory waveform with 30 and 60 Hz modes, negative polarity, superimposed. The vibratory amplitude ratio is $B/A = 0.23$. Recorded and displayed at 30 frames per second. The vibration and image capture are frequency-matched, so the channels appear stationary despite the vibratory motion. The serpentine channel cross-sectional width is 2mm.

Supplementary Video 8 | Separation of a granular mixture in a vibrating channel.

Real-time video of a mixture of glass sand (yellow) and boron carbide powder (black) moving in response to a horizontal vibratory waveform with 50 and 75 Hz modes, negative polarity, superimposed. The vibratory amplitude ratio is $B/A = 0.13$, and approximate Friction number is 1.5. Recorded and displayed at 50 frames per second. The vibration and image capture are frequency-matched, so the channels appear stationary despite the vibratory motion. The channel width is 2mm.

Supplementary Video 9 | Example of fiducial motion tracking

Top: representative high-speed video of the motion of a 10-mm wide post attached to the vibrating substrate, to serve as a fiducial for characterizing the vibratory motion. The imposed frequency modes are 30 and 60 Hz. The green box is superimposed via post facto image analysis to indicate the detected boundaries of the post. The video was captured at 1600 frames per second, and is displayed at 30 frames per second. Bottom: the corresponding centroid of the post versus time.

Numerical methods at low-moderate Fr range

When Fr is small or $\hat{v}_{osc}$ is not negligible, we solve Eq. (4) as a shooting method boundary value problem over one period T of the motion: the initial value $\hat{v}(0)$ is adjusted with Newton's method until the boundary condition $\hat{v}(0) = \hat{v}(T)$ is satisfied. If Newton's method fails to converge bisection is used as a fallback. The velocity $\hat{v}(T)$ is calculated from $\hat{v}(0)$ by numerical integration of Eq. (4) over the period. The $\hat{v}$ trajectory either traces $\hat{x}''$ until $|\hat{x}''(\hat{t})| \leq \mathrm{Fr}^{-1}$, or it follows a linear path with slope $\pm \mu_k/(\mu_s \mathrm{Fr})$ until the condition $\hat{v}(\hat{t}) = \hat{x}'(\hat{t})$ is satisfied. The trajectory is thus determined by finding the special points where $|\hat{x}''(\hat{t})| \leq \mathrm{Fr}^{-1}$ or where a line intersects the $\hat{x}(\hat{t})$ curve. The special points are determined by first bracketing then refinement. The velocity $\hat{v}_{st}$ is computed as the mean of the converged trajectory over the period. In the large-Fr, negligible $\hat{v}_{osc}$ limit we use Newton's method, with bisection as a fallback, to find the zero of $f(\hat{v}_{st}) = \int_0^T \mathrm{sign}[\hat{x}'(\hat{t}) - \hat{v}_{st}] d\hat{t}$. For each iteration, the intersections of the horizontal line $\hat{v}_{st}$ with curve $\hat{x}'(\hat{t})$ are determined by bracketing and refinement. These points of intersection partition the period into positive and negative values of the sign function, which are readily summed to compute f.

Table S1 Properties of the granular media. Individual particle sizes were estimated from optical microscopy as the mean and standard deviation of 10 particles, except for the manufacturer-provided mean size of the boron carbide. The particle-particle static friction coefficient was measured via the angle of repose method, and is reported as the mean and standard deviation of 6 trials. The particle-substrate static friction coefficient was measured via the sliding tilt angle method, and is reported as the mean and standard deviation of 5 trials.

	Particle Size (µm)	**Static Friction Coefficient, Particle-Particle**	**Static Friction Coefficient, Particle-Substrate**
Glass Sand (Yellow)	86 ± 30	0.84 ± 0.05	1.8 ± 0.4
Glass Sand (Blue)	88 ± 26	0.83 ± 0.06	2.1 ± 0.3
Boron Carbide	60 ± 18	0.82 ± 0.09	2.0 ± 0.5
Yeast	370 ± 140	0.79 ± 0.04	0.55 ± 0.06
Ibuprofen	66 ± 33	1.31 ± 0.14	2.3 ± 0.8

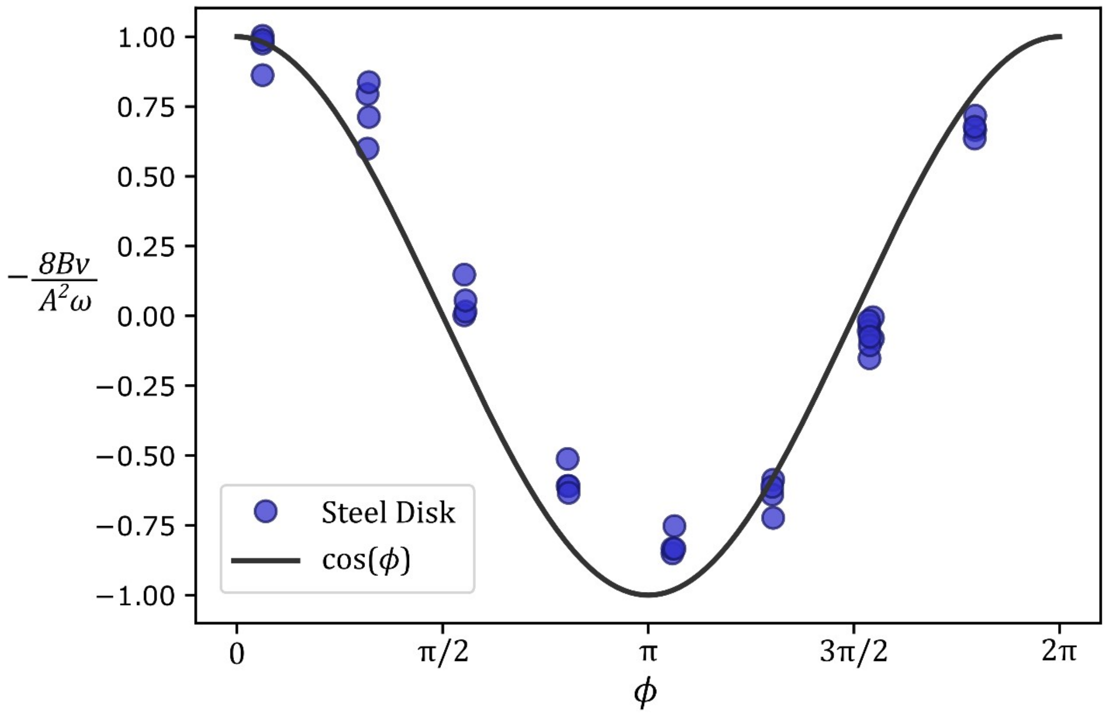

Figure S1. Velocity versus imposed phase lag. The dimensionless velocity of steel disks moving on a vibrating aluminum surface, normalized on the predicted velocity magnitude (cf. Eq. 3 in the main manuscript for $|B/A| > \frac{1}{4}$), versus the imposed phase lag ϕ. Markers are individual experimental trials, and the black line is the theoretical prediction $\cos\phi$. The vibratory waveform is 30 and 60 Hz, with average vibratory amplitudes of $A = 0.31$ mm, $B = 0.14$ mm, and average Friction number $Fr = 4.7$.

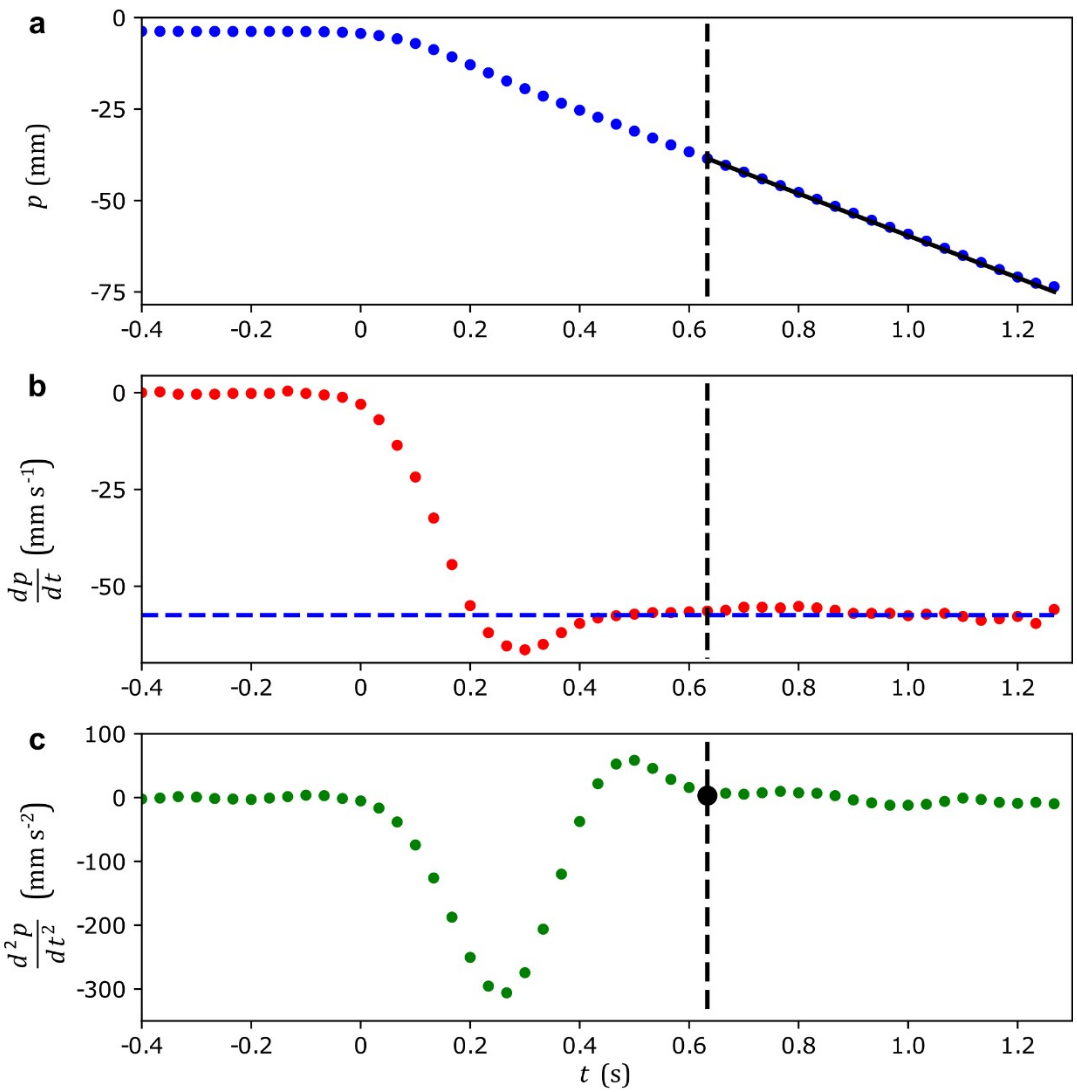

Figure S2. Representative tracking of a granular front. The position (a), velocity (b), and acceleration (c) of the leading front of blue-colored glass sand moving in a straight channel (cf. Fig. 2a). The vibratory waveform is 30 and 60 Hz applied at $t = 0$, then ramped up to maximum applied amplitude at $t = 0.25$ s. The maximum vibratory amplitude was $A = 0.83$ mm, $B = 0.32$ mm. The vertical dashed line denotes the time at which the acceleration (c) returned to zero and denotes the beginning of the range over which linear regression was used to determine the steady velocity, shown as a solid black line in (a) and horizontal blue dashed line in (b).