Electronic Supplementary Material (ESI) for Lab on a Chip.

Supporting Information

Oscillating high aspect ratio micro-channels can effectively atomize liquids into uniform aerosol droplets and dial their size on-demand

Hoai Nguyen An Le[1*], Jason Brenker[1*], Abanoub Shenoda[1], Zara Sheikh[2], Jackson Gum[1], Hui Xin Ong[2,3], Daniela Traini[2,3], and Tuncay Alan[1**]

S1. Stability of capillary waves for a rigid channel

Traditionally, the interfacial dynamics of unconstrained liquids supported by an oscillating substrate can be described by a nonlinear differential equation.

$$\ddot{A}(t) + 2\alpha\dot{A}(t) + \left(\frac{\sigma k^3}{\rho} + \alpha^2 + ak(2\pi F)^2 \cos(2\pi F t)\right)A(t) = 0 \quad (1)$$

where *A(t)* is the unsteady capillary wave amplitude, *F* is the forcing frequency, *k=2π/λ* is the wave number, a is the vertical vibration amplitude of the rectangular channel (Figure S1a), $\alpha = 2\nu k^2$ is the damping factor, where ν is the kinematic viscosity of the liquid. By substituting *τ=πFt*, Equation (3) can be transformed into the well-known damped Mathieu equation:

$$\ddot{A}(\tau) + 2\gamma\dot{A}(\tau) + (p + 2q\cos(2\tau))A(\tau) = 0 \quad (2)$$

where, $\gamma = \alpha/\pi F$, $p = (\sigma k^3/\rho + \alpha^2)/(\pi F)^2$, and $q = 2ak$.

Equation (2) can be solved by the harmonic balance technique resulting in two regions (stable and unstable) as shown in Figure 6b which represents the stability of the liquid interface during vibration. [1-4]

Theoretically, the critical threshold amplitude after which the capillary waves break up into droplets is given by $a_c = 2(\mu/\rho)(\rho/\pi\sigma F)^{\wedge}(1/3)$. However, the critical amplitude for spray formation is expected to be 3-6 times larger than this theoretical value [5]. Increasing the actuation amplitude beyond the actual critical value (a_c) results in spray formation (Figure S1b).

S2. Periodic channel deformation from structural dynamics simulation

To study the deformation of the silicon substrate under a simple harmonic forcing, finite element analysis was conducted using the Harmonic Response module using ANSYS 2021 R2. The chip vibrates at 1 MHz with a 1 μm applied vertical displacement at the bottom of the silicon substrate. The substrate is free to move in the horizontal direction. From the result file, we obtained the periodic mode shape of the substrate at the applied harmonic forcing.

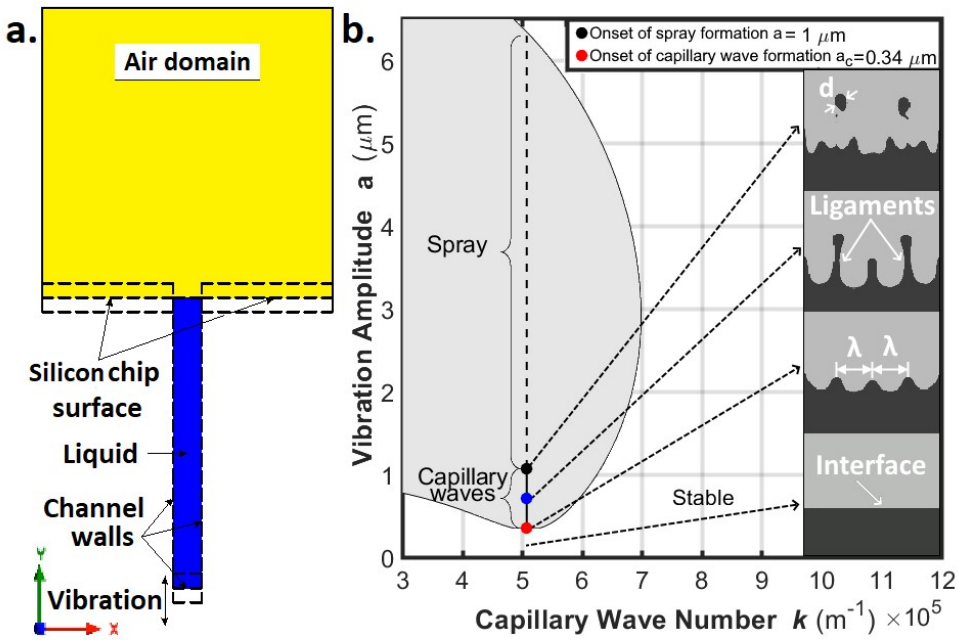

Figure S1. (a) 2D CFD model of the interaction of the vibrating channel with the liquid sample (b) The damped Mathieu stability chart for liquid water actuated at 1 MHz. Grey regions are unstable, the white area is stable, and the minimum (red) point indicates the theoretical critical threshold for droplet ejection. Insets are snapshots from a CFD simulation of a 2D microchannel (of width 35 μm and depth of 200 μm) 80% filled with liquid showing the stages of droplet ejection starting from a flat interface followed by the generation of capillary waves.

The dimensions of the silicon chip is 1 cm long and 0.5 mm thick, while the channel dimensions are 20 μm wide and 250 μm deep as shown in Fig S2 (a). Moreover, Fig S2 (b) shows a schematic diagram for one cycle of the periodic channel deformation with respect to a datum, in the beginning the channel has its original shape (straight perpendicular walls), and after 0.25 cycle the channel moves upward and the side walls flap and open up outwardly. At 0.5 cycle the channel goes back to its original shape and at 0.75 cycle the channel moves downward and the side walls close and move inwardly. After completing one cycle the channel returns to its primary shape. We call the outward and inward movement of the channel “the pinching mode”.

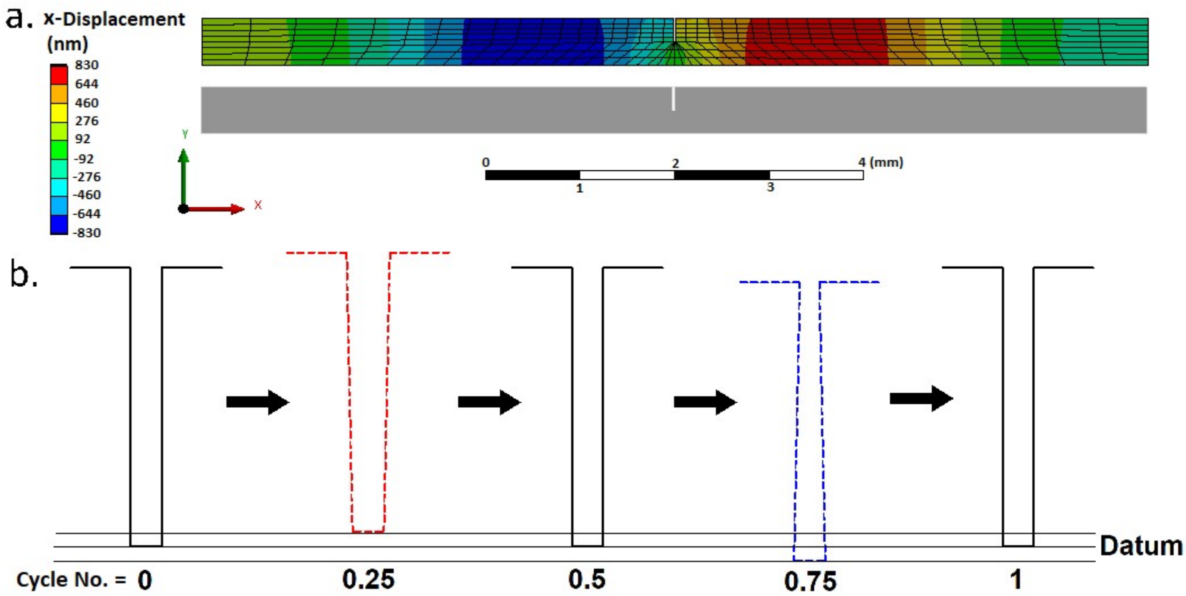

Figure S2. (a) 2D harmonic response simulation using ANSYS for the silicon substrate containing a single microchannel. The applied forcing is a 1 μm vertical displacement at the bottom of the substrate at 1 MHz. The legend bar shows the deformation in the x direction in nm. (b) A schematic diagram shows one complete cycle of the periodic channel deformation with respect to a datum.

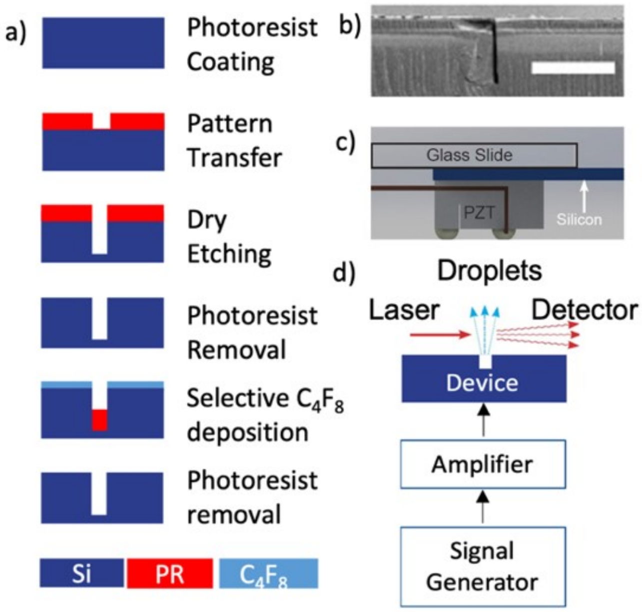

Figure S3. a) Schematic drawings depicting the device fabrication and surface coating steps, (b) scanning Electron Micrograph of a channel cross-section, (c) schematic of the device assembly (d) actuation and characterization steps.

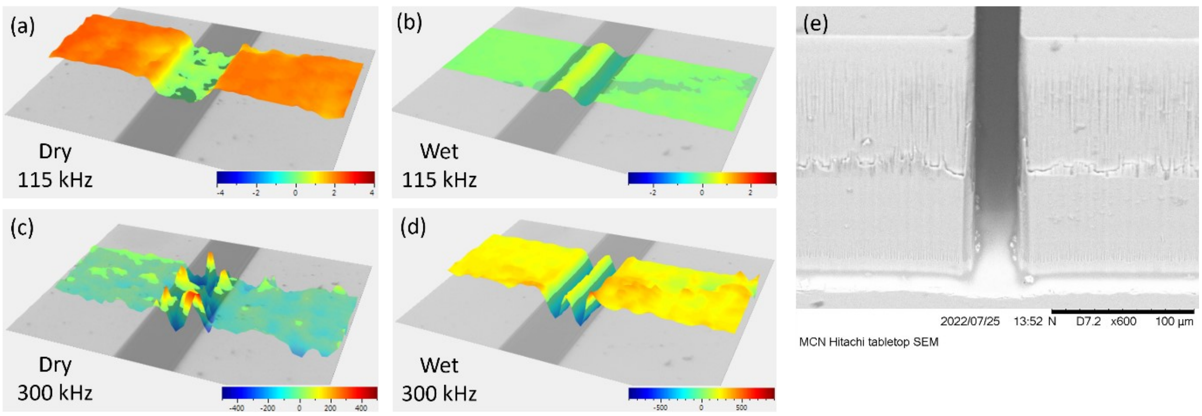

Figure S4. LDV scans of a 35 µm wide channel (a) before and (b) after being filled with water (115kHz actuation frequency, scale bar shows instantaneous displacements in nm) (c) LDV scan of a 35 µm wide channel (c) before, and (d) after being filled with water under (300kHz actuation frequency, scale bar shows instantaneous displacements in pm) The scans of the channel when dry produce only noise as the bottom of the channel is not within the focal depth of field of the LDV. (e) SEM scan of a 30 µm wide channel highlighting high aspect ratio channel geometry and low surface roughness and lack of features of comparable size to wavelengths observed.

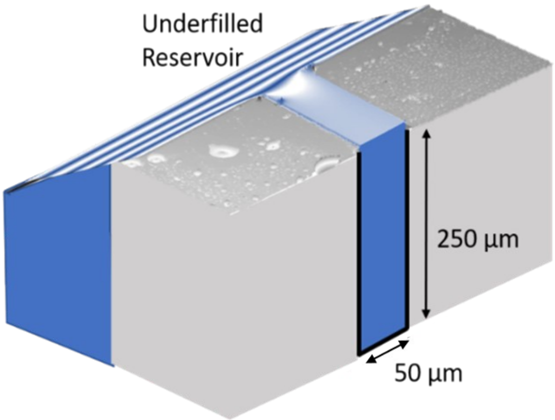

Figure S5. Optical profilometer scan of the inlet of a 50 µm wide, 250µm deep channel connected to a large fluid reservoir to the left of the image which is underfilled. The scan represents a 0.46 mm by 0.62 mm region with the vertical axis exaggerated for visual clarity. The grey and blue colours represent the Si surface and water respectively.

Channel width (µm)	Frequency (MHz)	Dv10 (µm)	Dv50 (µm)	Dv90 (µm)	Span Dv90-DV10 / Dv50	Dn10 (µm)	Dn50 (µm)	Dn90 (µm)
3	1	4.44	5.09	5.66	0.24	4.33	4.89	5.52
3	2.5	4.18	4.53	5.15	0.21	4.03	4.47	4.96
3	4.1	3.67	4.27	4.76	0.26	3.58	4.08	4.64
3	4.6	3.24	3.76	4.09	0.23	3.21	3.61	4.06
5	1	4.59	5.14	5.66	0.21	4.49	4.99	5.54
5	2.4	4.24	4.8	5.51	0.26	4.07	4.64	5.29
5	4.1	3.88	4.38	4.71	0.19	3.84	4.23	4.66
10	0.9	4.93	5.58	6.4	0.26	4.74	5.41	6.16
10	1	4.92	5.46	6.31	0.25	4.75	5.38	6.10
10	2.6	4.54	5.13	5.66	0.22	4.44	4.96	5.53
10	4	4.22	4.73	5.46	0.26	4.05	4.61	5.24
20	1	6.25	7	7.66	0.20	6.14	6.80	7.53
20	1.2	5.67	6.05	6.54	0.14	5.59	6.01	6.46
20	1.6	5.04	5.84	6.39	0.23	4.97	5.59	6.30

Table S1. Volume and number based aerosol statistics presented in Figure 4 (a-d)

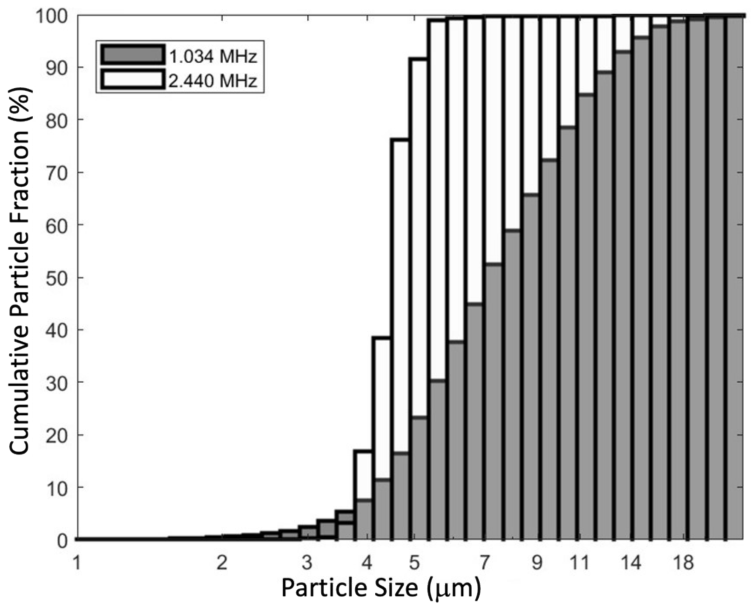

Figure S6. Cumulative droplet size distributions generated by a device containing sixteen 20 *μm-wide channels.* Increasing the frequency reduces the mean droplet diameter from 7.5 *μm* and 4.6 *μm.* a Kolmogorov-Smirnov test of equality of distributions, confirms that the differences between the two distributions are large enough to be attributed to the change in frequency rather than noise (p=0.045)

Supplementary Video 1. Operation of the device. Fluid contained in a sessile droplet placed at the end of a single channel is rapidly drawn into the channel and atomized.

Supplementary Video 2. Demonstration of the on-demand actuation capability, and use of different device/inlet configurations. The device can atomize fluids from different inlets: a 0.1 mL sessile droplet, 20 mL beaker, and a mL beaker, a freshly cut orange. A close-up, high-speed recording of the fluid interface indicates controlled droplet production.

References

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