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Asymmetric edge supercurrents in $MoTe_2$ Josephson junctions: Supporting Information

Pingbo Chen^{1,2,†}, Jinhua Wang^{2,†}, Gongqi Wang², Bicong Ye^{2,3}, Liang Zhou², Le Wang⁴, Jiannong Wang³, Wenqing Zhang^{5,6}, Weiqiang Chen^{2,6}, Jiawei Mei^{2,6} and Hongtao He^{2,6*}

¹Department of physics, Harbin Institute of Technology, Harbin, 150001, China ²Department of Physics, Southern University of Science and Technology, Shenzhen 518055, China ³Department of Physics, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, 999077, China

⁴Shenzhen Institute for Quantum Science and Engineering, Southern University of Science and Technology, Shenzhen, 518055, China

⁵Department of Materials Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, China

⁶Shenzhen Key Laboratory for Advanced Quantum Functional Materials and Devices, Southern University of Science and Technology, Shenzhen 518055, China

*Corresponding author email address: heht@sustech.edu.cn
†These authors contributed equally to this work

Extraction of the supercurrent distribution with the Dynes-Fulton method

For a Josephson junction exposed to a perpendicular magnetic field B , the magnitude of the critical current $^{I_c(B)}$ is strongly correlated with the supercurrent distribution $^{J_c(x)}$. To understand this relationship, we first consider the complex Fourier transform of $^{J_c(x)}$, which yields a complex critical current function,

$$I_c(B) = \left| I_c(B) \right| = \left| \int_{-\infty}^{+\infty} J_c(x) exp(\frac{i2\pi L_{eff}Bx}{\Phi_0}) dx \right|$$
 (1)

where x represents the dimension along the width of the junction, and $L_{eff} = L + 2\lambda$ stands for the effective length of the junction. Additionally, Φ_0 is the magnetic flux quantum with a value of $\frac{h}{2e}$. In the following, we will derive $J_c(x)$ from the observed $I_c(B)$ pattern using the method proposed by Dynes and Fulton [1].

When the current distribution is symmetric, *i.e.*, the odd part of $exp(\frac{i2\pi L_{eff}Bx}{\Phi_0})$ vanishes in the

 $I_E(B) = \int_{-\infty}^{+\infty} J_E(x) cos \left(\frac{2\pi L_{eff} Bx}{\Phi_0}\right) dx$ integral, Eq. (1) becomes . However, if the current distribution

has a small odd component, we also need to consider the Fourier transform of this odd component,

$$I_0(B) = \int_{-\infty}^{+\infty} J_0(x) sin\left(\frac{2\pi L_{eff}Bx}{\Phi_0}\right) dx$$
. Thus, the overall complex critical current is
$$I_c(B) = I_E(B) + iI_0(B)$$

The observed critical current $I_c(B)$ is given by $I_c(B) = \sqrt{I_E^2(B) + I_0^2(B)}$. Through appropriate mathematical manipulation, we can recover $J_c(x)$ from the observed $I_c(B)$. The even component, $I_E(B)$, is obtained by flipping the sign of alternate lobes in the $I_c(B)$ pattern, as shown in Supplementary Fig. 1 (b). The odd component, $I_0(B)$, is derived by interpolating between the minima of $I_c(B)$ and flipping sign between lobes, as shown in Supplementary Fig. 1 (c). Finally, by performing an inverse Fourier transform of $I_c(B)$, we can obtain the supercurrent density profile,

$$J_c(x) = \frac{1}{\Delta BW} \left| \int_{B_{min}}^{B_{max}} I_c(B) exp(\frac{-i2\pi L_{eff}Bx}{\Phi_0}) dB \right|.$$
 (3)

(2)

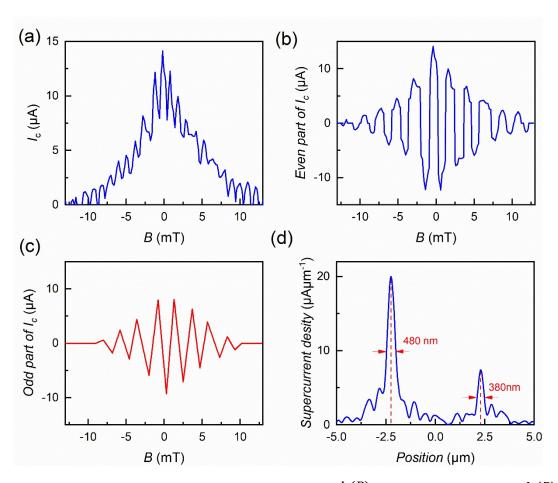
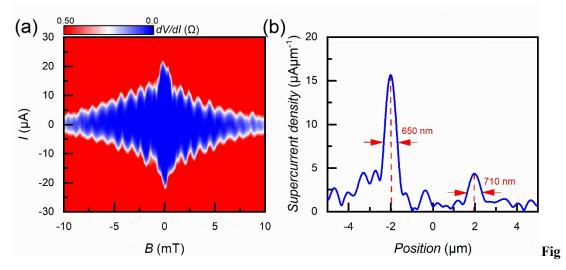


Figure S1. (a) Field dependence of the critical current $I_c(B)$. (b) The critical current $I_E(B)$ corresponding to the even component of the current distribution $J_E(x)$. (c) The critical current $I_O(B)$ corresponding to the odd component of the current distribution $J_O(x)$. (d) The derived current density distribution $J_c(x)$.



ure S2. Asymmetric edge supercurrents were also observed in another Nb/MoTe₂/Nb Josephson junction with L=400 nm, W=3.94 μ m, and t=50 nm. (a) Color map of the differential resistance

versus the current bias and magnetic field at T=0.9 K. (b) The supercurrent density distribution derived with the Dynes-Fulton method, clearly showing the presence of edge states with characteristic widths of 650 nm and 710 nm, respectively.

Supporting reference

[1] R. C. Dynes and T. A. Fulton, *Phys. Rev. B*, 1971, **3**, 3015-3023.