Supplementary Material

Anomalous transmission and Anderson localization for alternating propagated and evanescent waves in deep-subwavelength scale

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CONTENTS

S1. Derivation of Transfer Matrix theory 3

S2. Angle dependent reflection spectra for TM waves...5

S3. Electric field modes for different angles 6

S4. Correcting the permittivity of effective medium 8

S5. Reflection spectra with corrected permittivity 9

S6. The distribution of the specific disorder 10

S7. Disorder modulated electric field modes...10

S9. Disorder modulated angles of transmission peak..12

S8. Reflectance spectra modulated by different disorder 13

References 13

S1. Derivation of Transfer Matrix theory

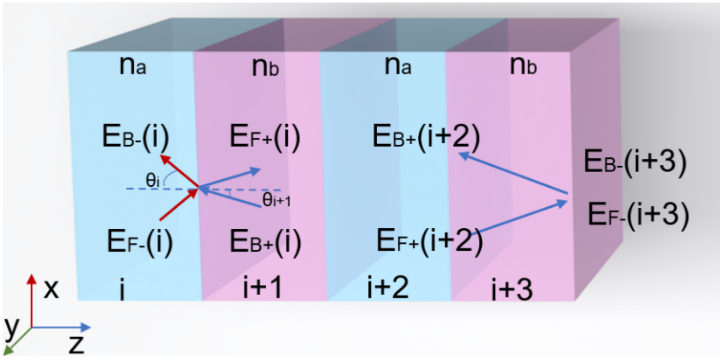

Figure S1. Schematic diagram for deriving Transfer Matrix theory

As sketched in Figure S1, the structure is composed of all dielectric multilayers, with each layer having a uniform refractive index

$$n = \begin{cases} n_{in} & -\infty < z < 0 \\ n_i & \sum_1^{2i-1} d_{i-1} < z < \sum_1^{2i} d_i \, , i = 1,2,\ldots,2N \\ n_{out} & \sum_1^{2N} d_i < z < \infty \end{cases}$$ * MERGEFORMAT (S1)

Where $n_i = n_a$ for even values of *i* and $n_i = n_b$ for odd values of *i*, d_i is the thickness of i_{th} layer and d_0 = 0. For TE waves, we separate to forward and backward components[1].

$$\begin{aligned} E(x,y,z) &= A_F e^{-j(k_{z,i}z)} e^{-j(k_{x,y}\sqrt{x^2+y^2})} + A_B e^{+j(k_{z,i}z)} e^{-j(k_{x,y}\sqrt{x^2+y^2})} \\ &= E_F(z) e^{-j(k_{x,y}\sqrt{x^2+y^2})} + E_B(z) e^{-j(k_{x,y}\sqrt{x^2+y^2})}, \end{aligned}$$ * MERGEFORMAT (S2)

where $k_{z,i} = \left[(n_i k_0)^2 - k_{x,y}{}^2\right]^{\frac{1}{2}}$, $i = a, b$, $k_0 = \frac{2\pi}{\lambda}$, we can get

$$k_{z,i} = n_i k_0 \cos\theta_i,$$ * MERGEFORMAT (S3)

where θ_i is the angle of propagation in the medium. According to Fresnel's formula, the transmission of the interface is obtained

$$\begin{bmatrix} E_{F+}(i) \\ E_{B-}(i) \end{bmatrix} = \begin{bmatrix} t_{ij} & r_{ji} \\ r_{ij} & t_{ji} \end{bmatrix} \begin{bmatrix} E_{F-}(i) \\ E_{B+}(i) \end{bmatrix},$$ * MERGEFORMAT (S4)

where t_{ij}, r_{ij} are the Fresnel transmission and reflection coefficients of the interface, and $i, j = in, a, b, out, r_{ij} = -r_{ji}, t_{ij} * t_{ji} - r_{ij} * r_{ji} = 1$.

$$\begin{bmatrix} E_{F-}(i) \\ E_{B-}(i) \end{bmatrix} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ji} \\ r_{ij} & 1 \end{bmatrix} \begin{bmatrix} E_{F+}(i) \\ E_{B+}(i) \end{bmatrix},$$ * MERGEFORMAT

(S5)

$$T_{ij} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ji} \\ r_{ij} & 1 \end{bmatrix}.$$ * MERGEFORMAT (S6)

The transmission inside each layer can be expressed as

$$\begin{aligned} E_{\text{F-}}(i) &= E_{\text{F+}}(i-1) e^{-j(k_{z,i} d_i)} \\ E_{\text{B+}}(i-1) &= E_{\text{B-}}(i) e^{-j(k_{z,i} d_i)}, \end{aligned}$$ * MERGEFORMAT (S7)

$$T_i = \begin{bmatrix} e^{j\varphi_i} & 0 \\ 0 & e^{-j\varphi_i} \end{bmatrix},$$ * MERGEFORMAT (S8)

where $\varphi_i = k_{z,i} d_i = k_0 n_i d_i \cos\theta_i$. According to the Eq. (S7) and (S8), we can get

$$\begin{bmatrix} E_{F-}(0) \\ E_{B-}(0) \end{bmatrix} = T_{0N} \begin{bmatrix} E_{F+}(N-1) \\ E_{B+}(N-1) \end{bmatrix},$$ * MERGEFORMAT (S9)

$$T_{0N} = T_{01} T_1 T_{12} T_2 ... T_{N-1} T_{(N-1)N} = \begin{bmatrix} A_{0N} & B_{0N} \\ C_{0N} & D_{0N} \end{bmatrix}.$$ *

MERGEFORMAT (S10)

After calculating the relevant transfer matrices, the transmission coefficient for a wave incident on the *N*-th layer is $\frac{1}{A_{0N}}$ and the reflection coefficient is $\frac{B_{0N}}{A_{0N}}$. For the internal electric field at left incidence($E_{F-}(0) = 1, E_{B+}(N) = 0$), we can get

$$\begin{bmatrix} E_{F-}(n) \\ E_{B-}(n) \end{bmatrix} = \frac{1}{A_{0,n} A_{n,N} + B_{0,n} C_{n,N}} \begin{bmatrix} A_{n,N} \\ C_{n,N} \end{bmatrix}.$$ * MERGEFORMAT

(S11)

For the case of the ordered medium at the total reflection angle,

$d_a = d_b, \varphi_a = j*\varphi_b,$, $r_{ab} = -r_{ba} = j, t_{ab}*t_{ba} = 2$, we can get

$$n_a cos\theta_a = n_a \sqrt{1 - \frac{{n_{in}}^2}{n_a^2} sin\theta_{in}} = \sqrt{n_a^2 - {n_{eff}}^2} = \sqrt{\frac{n_a^2 - n_b^2}{2}}.$$ * MERGEFORMAT (S12)

Clearly, there exists only one variable $n_a^2 - n_b^2$ for all products of T_a, T_{ab}, T_{ba}, T_b. Similarly, substituting $n_a \cos\theta_a$ into T_{ina}, T_{bout}, there exist only two variables n_{out}^2-n_{eff}^2 and $n_{in}^2 - n_{eff}^2$, except $n_a^2 - n_b^2$. With the same and certain background refractive index, the reflectivity calculated from Transfer Matrix theory is only determined by $n_a^2 - n_b^2$, $n_a^2 + n_b^2$.

S2. Angle dependent reflection spectra for TM waves

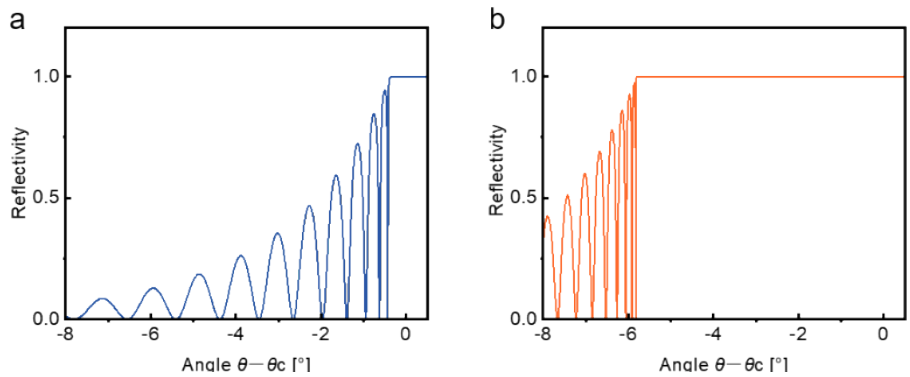

Figure S2. Angle dependent reflection spectra for TM waves. (a) nin = nout = 2.6, na = 2.3, nb = 2.1; (b) nin = nout = 3.4, na = 3.2, nb = 2.1. For TM waves, the incident angle corresponding to the last peak of the reflection spectra calculated by the transfer matrix theory is smaller than θc, illustrating the permittivity correction for effective medium is negative. Similarly, the larger refractive index difference carries a larger correction. Actually there exist two different definitions of TE polarization floating around literatures. The TE wave we use here is defined as an electromagnetic wave whose electric vector is perpendicular to the incident surface, and correspondingly, the TM wave is an electromagnetic wave whose electric vector is parallel to the incident surface.

S3. Electric field modes for different angles

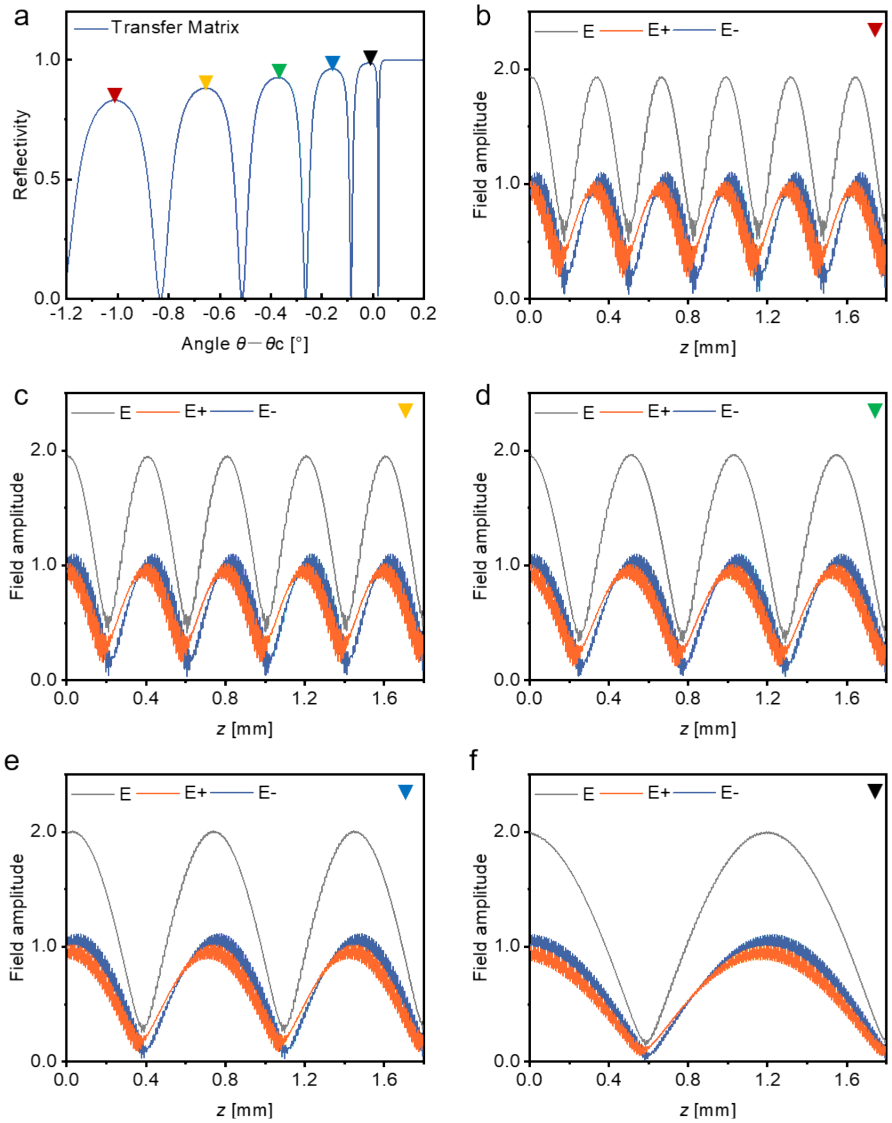

Figure S3. Electric field modes for the incidence angles corresponding to the maximum of reflectivity in ordered case. (a) Same with Figure 1(c) in main text, the incidence angles corresponding to the maximum of reflectivity are marked by arrows with different colors. (b) - (f) Electric field modes for the marked incidence angles in (a). Clearly, as the incidence angle approach to θ_c, the oscillating spatial frequencies of EM modes decrease. The coherent superposition of the forward and backward wave components inside the multilayer medium forms a standing wave. At the reflection maximum, the oscillation amplitude of normalized electric field intensity is less than 2.

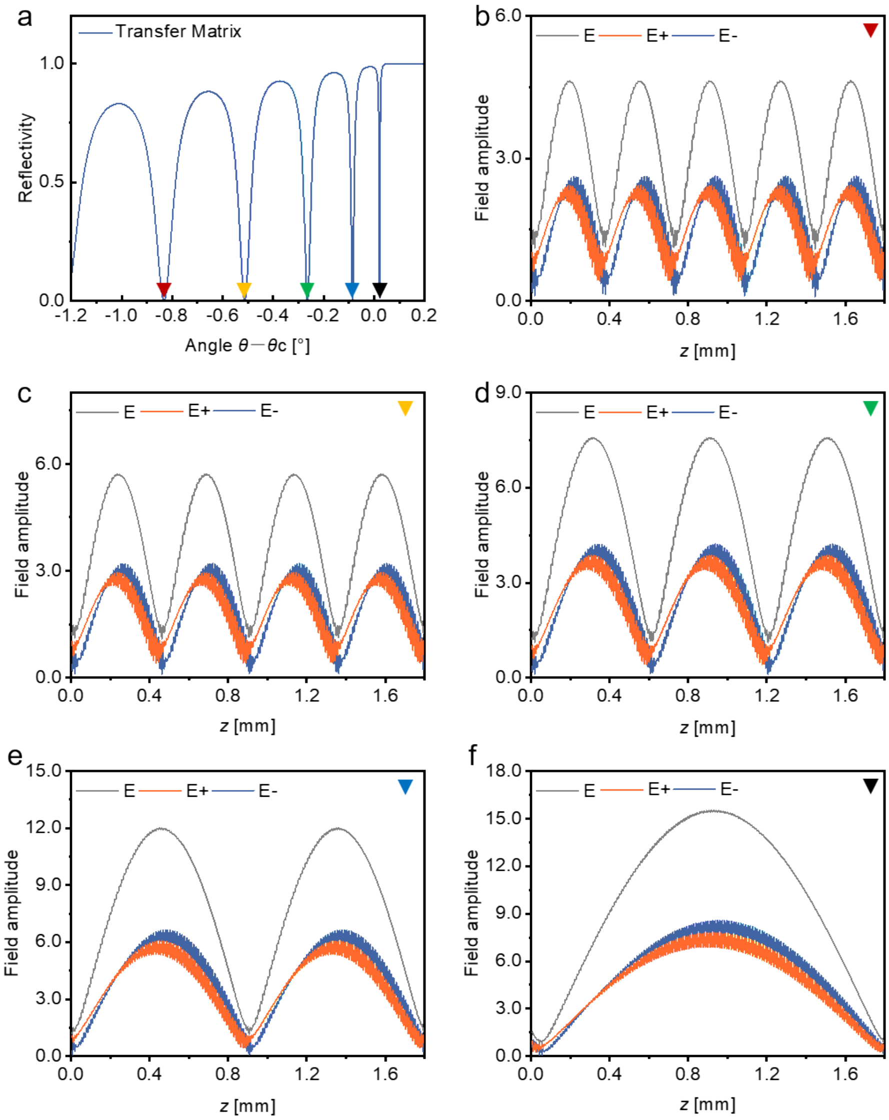

Figure S4. Electric field modes for the incidence angles corresponding to the maximum of transmittance in ordered case. (a) Same with Figure 1(c) in main text, the incidence angles corresponding to the maximum of transmittance are marked by arrows with different colors. (b) - (f) Electric field modes for the marked incidence angles in (a). Similar to the above figure, each angle corresponding to the maximum of transmittance corresponds to a change in the spatial oscillating period of EM modes. But the s coherent superposition of the forward and backward wave components results in the normalized electric field intensity higher than 2. It can be attributed to that the energy can pass through the interior of multilayers so that the multiple scattering and constructive interference occur in the interior of the EM wave at these incident angles.

S4. Correcting the permittivity of effective medium

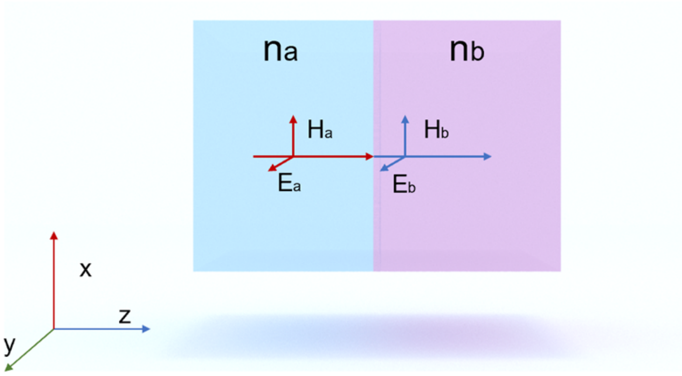

Figure S5. Schematic diagram of TE wave propagation inside the medium

We denote the two components of the electromagnetic wave [2]

$$E = U(z)e^{-jknz}$$
$$H = V(z)e^{-jknz}. \quad \text{* MERGEFORMAT (S13)}$$

We calculate only one of the a-b layers, and is approximated by a vertical wave vector k. When the wave is in a-layer,

$$U = e^{jknz}(Ae^{j\alpha z} + Be^{-j\alpha z})$$
$$V = -\frac{\alpha}{k}(Ae^{j\alpha z} - Be^{-j\alpha z}), \quad \text{* MERGEFORMAT (S14)}$$

where $\alpha = kn_a = k\sqrt{\varepsilon_a}$. The same is true for the b-layer

$$U = e^{jknz}(Ce^{j\beta z} + De^{-j\beta z})$$
$$V = -\frac{\alpha}{k}(Ce^{j\beta z} - De^{-j\beta z}), \quad \text{* MERGEFORMAT (S15)}$$

where $\beta = kn_b = k\sqrt{\varepsilon_b}$. Imposing on U and V the conditions of continuity and periodicity, we can get four homogeneous equations for A, B, C and D,

$$C + D = A + B$$
$$e^{-jknd_b}(Ce^{-j\beta d_b} + De^{j\beta d_b}) = e^{jknd_a}(Ae^{j\alpha d_a} + Be^{-j\alpha d_a})$$
$$C - D = \frac{\alpha}{\beta}(A - \mathrm{B}) \quad \text{*}$$
$$e^{-jknd_b}(Ce^{-j\beta d_b} - De^{j\beta d_b}) = \frac{\alpha}{\beta}e^{jknd_a}(Ae^{j\alpha d_a} - Be^{-j\alpha d_a}).$$

MERGEFORMAT (S16)

Setting the determinant of the system equal to zero gives the dispersion equation, which determines the corrected refractive index n

$$\cos kn(d_a + d_b) = \cos\alpha d_a \cos\beta d_b - \frac{1+x^2}{2x}\sin\alpha d_a \sin\beta d_b,$$ * MERGEFORMAT (S17)

where $x = \frac{\alpha}{\beta}$, and take the approximation of

$$\cos\alpha d_a \cos\beta d_b - \frac{1+x^2}{2x}\sin\alpha d_a \sin\beta d_b \approx \frac{1}{\sqrt{2(1-x)}},$$ * MERGEFORMAT (S18)

where $\alpha d_a, \beta d_b \approx 0$ for deep sub-wavelength structure. The other triangle functions in our other party are expanded, and only retain the lowest order, we can get

$$\varepsilon_{\text{eff}} = \overline{\varepsilon} + \Delta\overline{\varepsilon} = \overline{\varepsilon} + \frac{k_0^2 d_a^2 d_b^2}{12\left(d_a + d_b\right)^2}\left(\varepsilon_a - \varepsilon_b\right)^2.$$ * MERGEFORMAT (S19)

S5. Reflection spectra with corrected permittivity

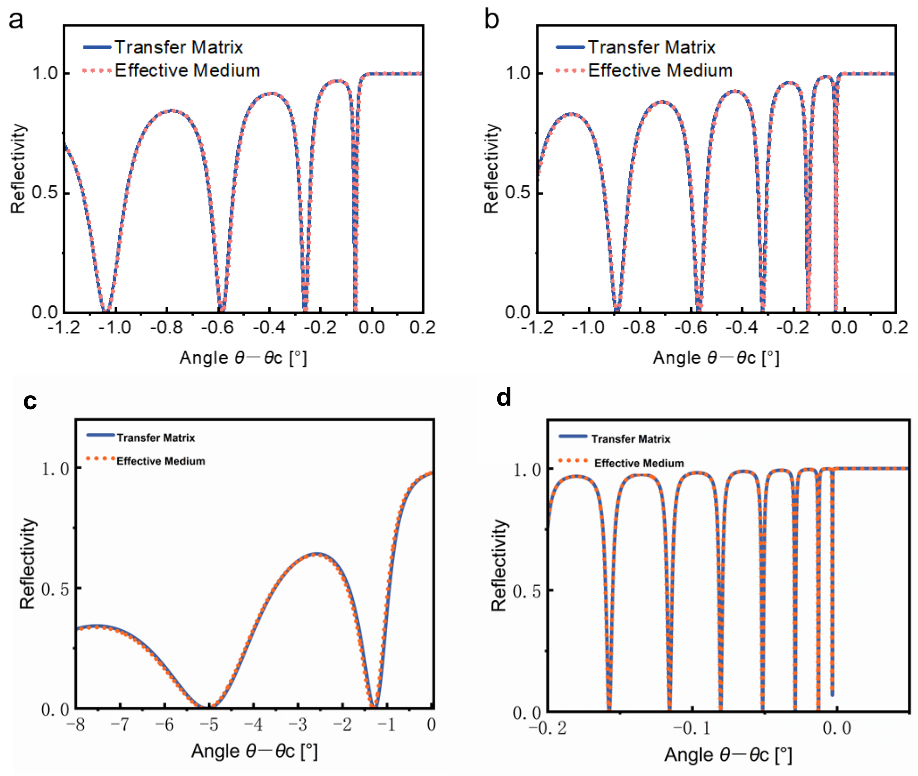

Figure S6. Reflection spectra calculated by effective medium theory with corrected permittivity in Eq. [S19]. (a) $n_{\mathrm{in}} = n_{\mathrm{out}} = 2.6$, $n_{\mathrm{a}} = 2.3$, $n_{\mathrm{b}} = 2.1$ (b) $n_{\mathrm{in}} = n_{\mathrm{out}} = 3.4$, $n_{\mathrm{a}} = 3.2$, $n_{\mathrm{b}} = 2.1$. The results of effective medium theory calculation after the correction are in perfect agreement with transfer matrix calculation, illustrating that the effective medium theory is still convincing after a simple correction. (c), (d). Reflection spectra for the 50-layer structure and the 1000-layer structure of the refractive index in (b), respectively. The results show that the Eq. [S19], although derived with the approximation of infinite-layer medium, are well applicable for multilayer deep subwavelength structures with thickness of more than one wavelength.’

S6. The distribution of the specific disorder

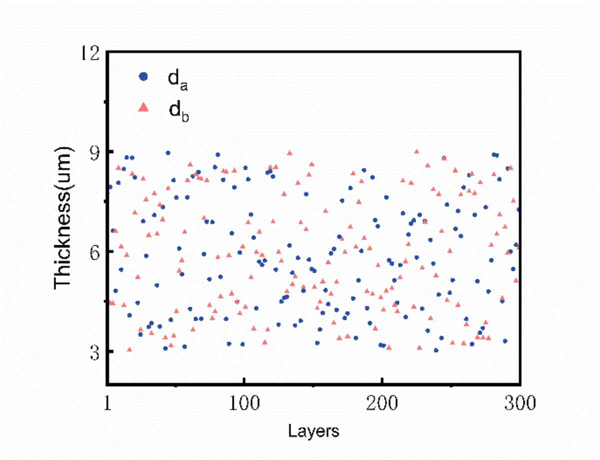

Figure S7. The distribution of the specific disorder mentioned in our main text. The blue and red points represent the thickness of each a and b layer, and the thickness of the a, b layers are generated completely randomly.

S7. Disorder modulated electric field modes

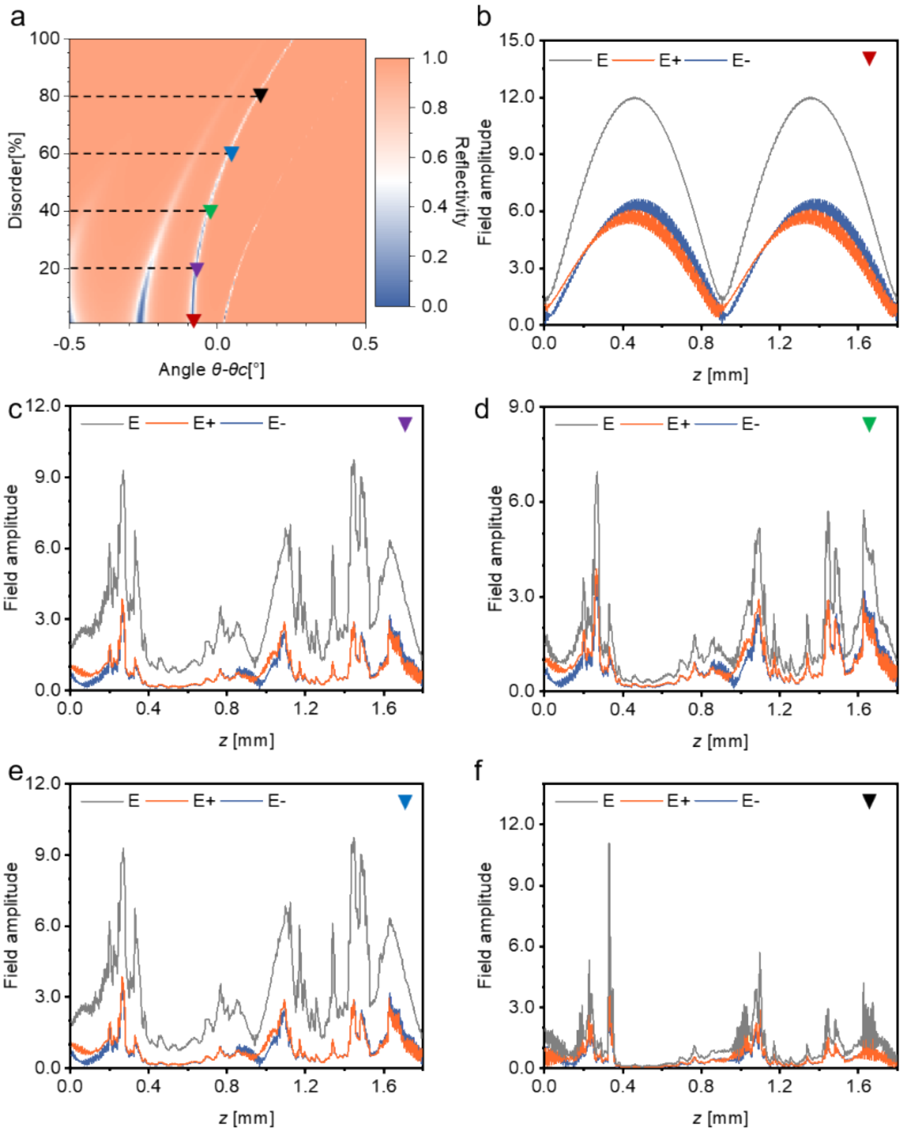

Figure S8. Electric field modes of the special points in the disordered structure. (a) Disorder modulated reflection spectra of the region near the total internal reflection angle. The five electric field distributions are disorder 0, 20%, 40%, 60%, 80%, respectively, and the angles correspond to the specific modes marked in the anomalous transmission curves in (b-f). All electric field distributions have a localized electric field inside the medium, but the intensity of the localization varies. Note that all localized maximum of field amplitude in the same branch in (a) are located at approximately z = 0.2, 1.1, 1.5 and 1.65 mm for this selected disorder.

S9. Disorder modulated angles of transmission peak

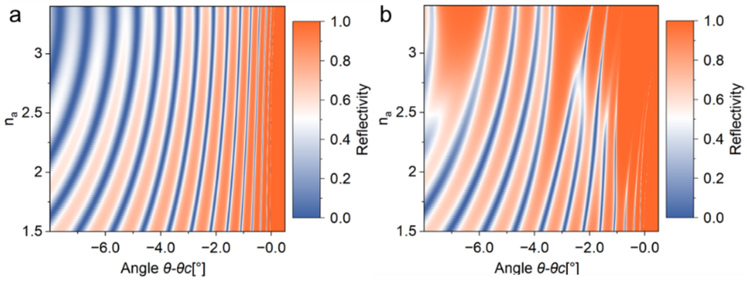

Figure S9. (a) In an ordered system, when nb is a fixed value such as 2.0 and na increases from 2.0 to 3.0, the transmittance is monotonically varying, which is because the weak modulation of the refractive index in ordered system makes the change in transmittance always characterized by the last transmittance peak near the total reflection angle. (b) In the disordered system such as Fig. 4 (a), when nb is a fixed value such as 2.0 and na increases from 2.0 to 3.0, the transmittance first decreases, then increases, which can be attributed the strong disorder modulation makes transmittance at different refractive index corresponding to two different 'anomalous' peaks.

S8. Reflectance spectra modulated by different disorder

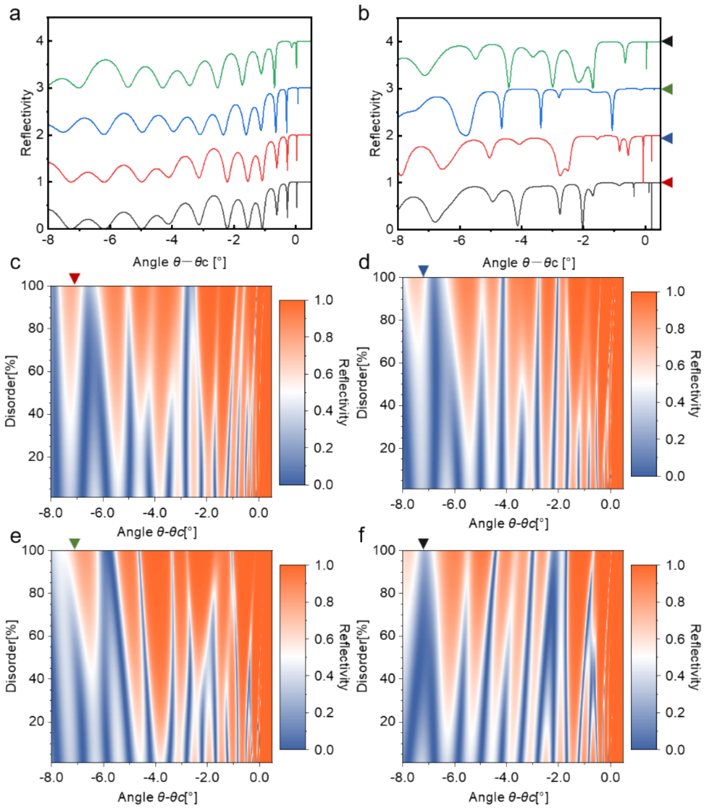

Figure S9. Reflectance spectra of the structures with different disorders in the thickness. At the same parameter of refractive index as the main text, four representative spectra with anomalous transmission peaks are sketched. (a) $n_{in} = n_{out} = 2.6$, $n_a = 2.3$, $n_b = 2.1$, (b) $n_{in} = n_{out} = 3.4$, $n_a = 3.2$. (c) - (f) Disorder modulated reflection spectra corresponding to the specific disorder marked by arrows in (b).

References

1. P. YehP., *Optical Waves in Layered Media*, John Wiley and Sons, 1988.
2. S. Rytov, *Sov. Phys. JETP*, 1956, **2**, 466.