## Supporting Information: Minimal numerical ingredients describe chemical microswimmers' 3-D motion

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1

## The Supporting Information includes:

Fig. S1: Sedimentation of passive colloids under gravity

Fig. S2: Trajectory of a microswimmer which leaves the plane

Fig. S3: Velocity Distributions binned by different definitions of swimming direction

Fig. S4: Effect of thermal noise on the dynamics of a microswimmer without shape asymmetry

Fig. S5: Rendering of different shape asymmetries (1 Edge, 1 Centre, 2 Edges)

Fig. S6: Role of shape asymmetry on the dynamics of a microswimmer (1 Edge case)

Fig. S7: Role of shape asymmetry on the dynamics of a microswimmer (1 Centre case)

Fig. S8: Role of shape asymmetry on the dynamics of a microswimmer (2 Edges case)

Fig. S9: Periodic boundary conditions and finite size effects

Fig. S10: XY slices of the flow fields generated by microswimmers in the laboratory frame



Figure. S 1: Verification of the solvent parameters used to ensure a low Re regime. The gravitational force  $F_g$  applied to a sedimenting colloid is halved (from  $F_g = 1$  to  $F_g = 0.5$ , left panel and right panel respectively), with the corresponding reduction in sedimentation velocity and thus Pèclet number ( $Pe = v_g * R/D_T$ , where  $v_g$  is the velocity due to gravity, and  $R, D_T$  are constant properties of the colloid) - as expected by Stokes law. We note that the forces involved are relatively low when  $F_g = 0.5$ , leading to increasingly important role of Brownian fluctuations. As the gravitational force is increased further, dz/dt will become increasingly linear (as Pe increases).



Figure. S 2: An example trajectory of the 3-D motion of a microswimmer (right panel), with its 2-D projection onto the XY plane (left panel). We note that particle motion is characterised by long periods of motion in the 2-D plane, interdispersed with out-of-plane (z) loops.



Figure. S 3: a) The out-of-plane trajectory of a simulated CB microswimmer (z(t)). b) Using the analysis proposed in,<sup>1,2</sup> i.e. binning the instantaneous displacements of the microswimmer by the zenith angle calculated from those displacements, results in the same data structure as that found experimentally.<sup>1,2</sup> This suggests that the "orientation-dependent velocity" previously reported is in fact an inherent feature of the analytical approach used, rather than arising from e.g. the microswimmers' photo-responsiveness. c) Analysing the data using the internal orientation axis (known in simulations), we recover a more explainable structure. Specifically, when the particle swims down from the bulk towards the substrate  $(-\pi/2 < \Phi < -\pi/4)$ , it obtains the expected boost in velocity due to gravity. For  $-\pi/4 < \Phi < 0$ , the velocities are significantly lower, as this corresponds to the region where the particle is at the substrate and facing towards it due to hydrodynamic attraction (see Figure 2 h.). The particle speed here is lower as it only consists of the projection of the velocity onto the substrate. We note that this corresponds to the largest part of the trajectory, significantly reducing the overall average velocity (dashed line). As the particle leaves the substrate  $(0 < \Phi)$ , the particle swims faster than at the substrate, but as it swims against gravity is not as fast as for  $-\pi/2 < \Phi < -\pi/4$ . These results suggest that coarse-graining the effect of phoretic and light fields to hydrodynamic interactions and gravity is indeed sufficient to capture the physics of various experimental chemical microswimmer systems.



Figure. S 4: Investigation of the angle  $\theta$  between the swimming direction and internal orientation (defined by the body axis) of our microswimmers in bulk and in the absence of any shape asymmetry  $(R_{shift} = 0)$ . Left panel: The change in  $\theta$  as the swimming velocity of the microswimmer is increased (represented via  $Pe = v_p \cdot R/D_T$ . Right panel: The change in  $\theta$  as the lag-time  $\tau$  over which the subsequent displacements are evaluated (see main text for discussion of definitions). In both cases, the diamonds represent the values estimated if assuming translational diffusion occurs only perpendicular to the internal swimming axis. As no gravitational force nor substrate is present, one may expect that the internal orientation and swimming direction of a weak pusher squirmer would align perfectly. However, from our analysis we find that this is not the case. We propose that thermal fluctuations, namely rotational and translational diffusion of the particle, are the key contributing factors to this observation. On the left, we see that at lower Pe numbers, the significant divergence between the swimming direction and internal orientation can be completely explained by the expected translational diffusion of the particle. We find that as the Pe is increased, both the observed and theoretical  $\theta$  between the two vectors follow the same trend, in that the value of  $\langle \theta \rangle$ decreases. We also see on the right that by increasing lag time over which the displacements are evaluated, the observed and expected  $\langle \theta \rangle$  values also become smaller. This points to the role of the microswimmer's diffusivity in defining its motion, and thus the angle between the swimming direction and the body axis in the case of  $R_{shift} = 0$  (no shape asymmetry). Due to finite time differencing, we also expect the rotational diffusivity of the squirmer to play a further role in the difference between the particles internal orientation and swimming displacement in bulk. For example, we take the average of the particle's internal orientation between two time steps, which will introduce a component of noise due to rotational diffusion between those steps. We also expect these effects to contribute to the saturation of the effect of shape asymmetry at  $\theta \sim 35^{\circ}$ .



Figure. S 5: Overview of the different types of shape asymmetry introduced via shape particles  $P_s$  with  $R_{shift} = 0.5$  (light green) along the axes of different "cap" particles (blue) for CF microswimmers. Left panel: "1 Edge" case, where  $P_s$  is introduced at a 45° angle to the internal body axis (central blue particle). Middle panel: "1 Centre" case, where  $P_s$ is introduced along the internal body axis. Right panel: "2 Edge" case, where the two  $P_s$ particles are introduced at 45° and -45° to the internal body axis (central blue particle). In all cases, the swimming direction is with the blue cap pointing forwards. The red particles are the remaining "filler" particles constituting the rest of the microswimmer body. Solvent particles are reduced for scale and represented in purple. Graphics were generated using the visualisation software Ovito<sup>3</sup>



Figure. S 6: MSADs for the simulated bulk microswimmers for increasing values of  $R_{shift}$  for the "1 Edge" case, as described in Figure S5. The calculated angles between the swimming direction and internal orientation of bulk microswimmers, also discussed in Figure 4 of the main text, are shown in the bottom right panel. Error bars indicate the standard error of the mean from 1001 frames (sub-sampled from 50000 simulation steps).



Figure. S 7: MSADs for the simulated bulk microswimmers for increasing values of  $R_{shift}$  for the "1 Centre" case, as described in Figure S5. The calculated angles between the swimming direction and internal orientation of bulk microswimmers are shown in the bottom right panel. We note that the reduced asymmetry introduced by a  $P_{shape}$  particle along the internal microswimmer orientation has little effect on its dynamics, contrasting to the "1 Edge" case. Error bars indicate the standard error of the mean from 1001 frames (subsampled from 50000 simulation steps).



Figure. S 8: MSADs for the simulated bulk microswimmers for increasing values of  $R_{shift}$  for the "2 Edge" case, as described in Figure S5. The calculated angles between the swimming direction and internal orientation of bulk microswimmers are shown in the bottom right panel. We note that the reduced asymmetry introduced by 2  $P_{shape}$  particles on the edges of the microswimmer, resulting in mirror symmetry along its internal orientation axis, has little effect on the swimming dynamics, contrasting to the "1 Edge" case. This is despite the introduction of 2  $P_{shape}$  particles on positions each corresponding to the "1 Edge" case (opposite each other). This underlines the importance of the overall shape asymmetry of the microswimmer to its physics. Error bars indicate the standard error of the mean from 1001 frames (sub-sampled from 50000 simulation steps).



Figure. S 9: As for any simulation method that accounts for hydrodynamics, periodic boundary conditions (PBCs) introduce a finite size effect in the results, which will deviate from analytic predictions for an infinite system. The Pèclet number (*Pe*) quantifies the relevance of self-propulsion against diffusion. At small *Pe* we do not expect any significant flow profile, while the flow generated by propulsion and stresslets will be developed at high *Pe*. The DPD model recovers this behavior and generates flow profiles with the symmetries corresponding to the active motion of the microswimmer,<sup>4</sup> and finite size only affects the quantitative details of the far field hydrodynamic field. To verify that boundary effects do not qualitatively affect our key findings, we re-simulate two situations studied in Figure 3 of (a-c: Pe = 30, d-f: Pe = 100). Specifically, we increase the system size by 4x (i.e. double the dimensionality in X and Y to be 32x32x40) for these two instances, and investigate whether we observe qualitatively similar behavior (i.e. for Pe = 100, the particle cannot leave the substrate, whereas for Pe = 30 it can). We observe an almost identical behavior for the significantly larger system as that described in Figure 3, justifying our use of a smaller simulation box to reduce computational expense.



Figure. S 10: XY slices of the flow fields generated by microswimmers in the laboratory frame (through the equator of the particle). Top row: Flow fields for fixed Pe = 100 and decreasing  $\beta = \{-2.5, -5, -10\}$  to increase the *pusher* character of the squirmer. Bottom row: Using a fixed value of  $\beta = -2.5$  and varying  $Pe = \{10, 30\}$ , reproducing the findings in Figure 3 of the main text. As the  $\beta$  parameter becomes more negative, we observe the transition from a weak pusher flow field (more reminiscent of a neutral squirmer) to a strong pusher, with the development of the characteristic vortices associated with hydrodynamic pushers. These vortices retain the qualitative shape of a pusher squirmer in periodic boundary conditions with the four characteristic vortices and the saddle point, comparable to those reported previously for squirmers using other numerical schemes.<sup>5,6</sup> We note that some distortions to the flow field are noticeable due to the idiosyncrasies of the simulation (periodic boundary conditions (PBCs), the presence of the substrate, and the fact that the squirmer is being held fixed with respect to the substrate). In the absence of a gravitational force, the flow field generated by the squirmer is force free, and the main features of this class of flows are preserved irrespective of the system size. It is true that the measured flow field is affected by the flows induced by the periodic images. The magnitude of this contribution will decay with system size, and they can modify details of the flow field, but not the main features of the broken symmetry of the flow field with respect to the direction of motion of the squirmer. In our simulations the squirmer displaces in the presence of a gravitational force. This generates a Stokeslet that also contributes to the shape of the flow field (this feature is also affected by the periodic images and by the presence of the bounding walls). Nevertheless, as discussed in Figure S9, the PBCs do not affect the qualitative features of our findings. Furthermore, we note that for the lower Pe numbers primarily studied in the main text, the effect of the PBCs used are reduced.

## References

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