Deforming active droplets in viscoelastic solutions

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1 Zig-zag motion of an isotropic active droplet

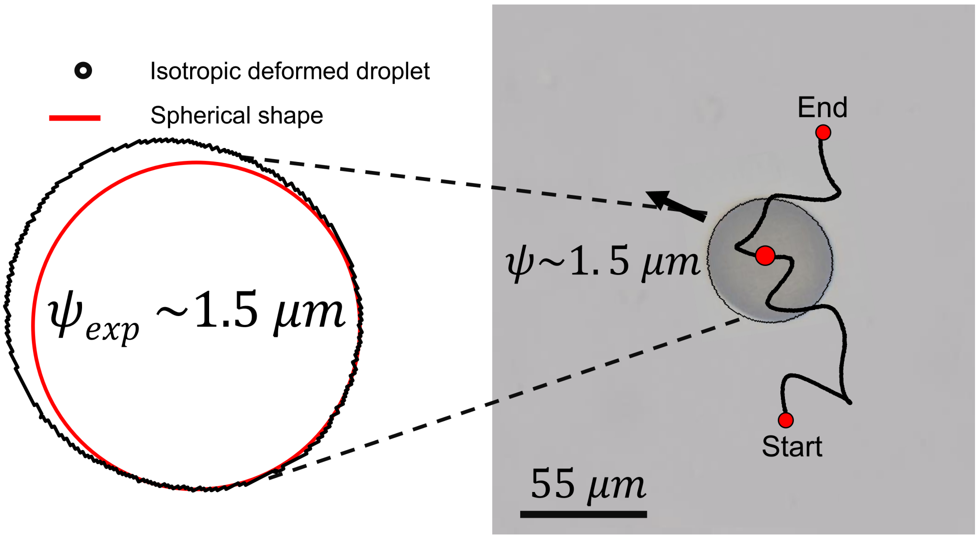

FIG. 1: Zig-zag motion of an isotropic CB15 active droplet in fluid 5 (aqueous solution with 1.25 wt.% of $8{\times}10^6$ Da PEO and 21 wt.% TTAB).

2 Theoretical derivation for droplet shape

The derivation of Eq. (8) from Eq. (7) is as follows. Let the slightly deformed interface position be given by $r(\theta) = 1+\epsilon f(\theta)$, which can then be written in the implicit form as $F = r-1-\epsilon f(\theta)$. The outward unit normal, $\mathbf{n}$, at the interface is then obtained as

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \frac{\dfrac{\partial F}{\partial r}\hat{e}_r + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}\hat{e}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial F}{\partial \phi}\hat{e}_\phi}{|\nabla F|}$$
$$= \frac{\hat{e}_r - \frac{\epsilon}{r} f'(\theta)\hat{e}_\theta}{\left(1+\frac{\epsilon^2 f'(\theta)^2}{r^2}\right)^{0.5}}.$$

Thus, retaining only the linear terms, we obtain

$$\mathbf{n} = \hat{e}_r - \frac{\epsilon}{r} f'(\theta)\hat{e}_\theta + O(\epsilon^2).$$

The interface curvature is obtained by taking the divergence of the normal, which gives

$$
\begin{aligned}
\nabla\cdot\mathbf{n} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 n_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta n_\theta)\\
&= \frac{2}{r} + \frac{n_\theta}{r}\cot\theta + \frac{1}{r}n'_\theta(\theta)\\
&= \frac{2}{r} - \epsilon\left[\frac{\cot\theta}{r^2}f'(\theta) - \frac{1}{r^2}f''(\theta)\right].
\end{aligned}
$$

Substituting for $r = 1+\epsilon f(\theta)$ in the above, we get

$$
\begin{aligned}
\nabla\cdot\mathbf{n}|_{(r=1+\epsilon f(\theta))} &= \frac{2}{1+\epsilon f(\theta)} - \frac{\epsilon\cot\theta f'(\theta)}{1+2\epsilon f(\theta)} - \frac{\epsilon f''(\theta)}{1+2\epsilon f(\theta)}\\
&= 2+\epsilon\left[-2f(\theta) - \cot\theta f'(\theta) - f''(\theta)\right].
\end{aligned}
$$

Thus, from the above, the curvature at the linear order gives

$$
\nabla\cdot\mathbf{n}|_{O(\epsilon)} = -2f(\theta) - \cot\theta f'(\theta) - f''(\theta).
$$

Expressing the interface deformation, $f(\theta)$, in terms of the Legendre basis functions as $f(\theta) = \sum_{n=2}^{\infty}\beta_n P_n(\cos\theta)$, we get

$$
\begin{aligned}
\nabla\cdot\mathbf{n}|_{O(\epsilon)} &= -2\sum_{n=2}^{\infty}\beta_n P_n(\cos\theta) - \cot\theta\sum_{n=2}^{\infty}\beta_n\frac{\partial}{\partial\theta}P_n(\cos\theta) - \sum_{n=2}^{\infty}\beta_n\frac{\partial^2}{\partial\theta^2}P_n(\cos\theta)\\
&= \sum_{n=2}^{\infty}(n+2)(n-1)\beta_n P_n(\cos\theta).
\end{aligned}
$$

Using the orthogonality property of Legendre basis functions, we get

$$
\int_1^{-1}\nabla\cdot\mathbf{n}|_{O(\epsilon)}P_n(\cos\theta)\sin\theta\,d\theta = \frac{2\beta_n(n+2)(n-1)}{2n+1}
$$

The above expression together with the interfacial normal stress balance yields

$$
\beta_n = \frac{(2n+1)}{2(n+2)(n-1)}\int_0^{\pi}\frac{f(R)}{D}A_{rr}P_n(\cos\theta)\sin\theta\,d\theta.
$$

3 Concentration parameter for the model

The polymer concentration parameter is defined as $c = \frac{\nu-\nu_s}{\nu_s}$ (Ref. 74 in manuscript) where ν and ν_s are the kinematic viscosity of polymer solution and solvent, respectively. For the present experimental system, we obtain c as 240.