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Supplementary information:
Dynamics of bubbles spontaneously entering in a tube

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1 Model

1.1 Approximation of a negligible $\Omega_{\rm cap}$

The first of the two approximations made for deriving the approximated shrinking duration considers a negligible contribution of the cap volume compared to the total volume expressed as

$$\Omega_{\rm out}^{\rm Regime1}(t) = \frac{4}{3}\pi R(t)^3 - \Omega_{\rm cap}(t), \tag{S1}$$

where

$$\Omega_{\rm cap}(t) = \frac{\pi R(t)^3}{3}\left(1-\mathcal{G}\right)^2\left(2+\mathcal{G}\right) \tag{S2}$$

with $\mathcal{G}(a, R(t)) = \sqrt{1 - a^2/R(t)^2}$.

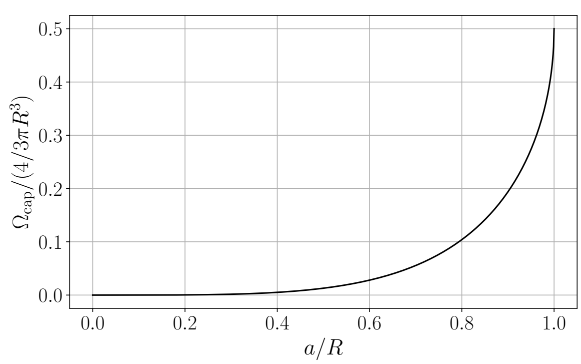

Figure S1: Ratio of the cap volume to the total volume $\Omega_{\rm cap}/(4/3\pi R^3)$ as a function of the inverse of the dimensionless bubble radius a/R.

In figure S1, we plot the ratio between the two terms on the left hand side of equation S1, *i.e.* $\Omega_{\rm cap}/(4/3\pi R^3)$ as a function of a/R. We observe that under the approximation, the volume $\Omega_{\rm out}^{\rm Regime1}(t)$ is overestimated by less than 10 % for $a/R < 0.8$ *i.e.* $R/a > 1.25$.

1.2 Effect of the number of lamellae n on the ratio $T_{\rm num}/T_{\rm approx}$

In figure S2 (a) we first plot both the numerical and approximated times presented in the model as the function of $n^{3/2}$. We notice that the approximated duration is smaller than the numerical prediction, and the difference is increasing with $n^{3/2}$.

Because both duration scale as $n^{3/2}$, we can expect a constant ratio between them. This is evidenced in figure S2 (b) for different dimensionless aspect ratios $\tilde{R}$. As presented in fig 5, this ratio depends on the aspect ratio but is constant with the number of lamellae explaining why the approximated time for large n values, which exists only for small aspect ratio, is not in good agreement with the experiment in fig 6.

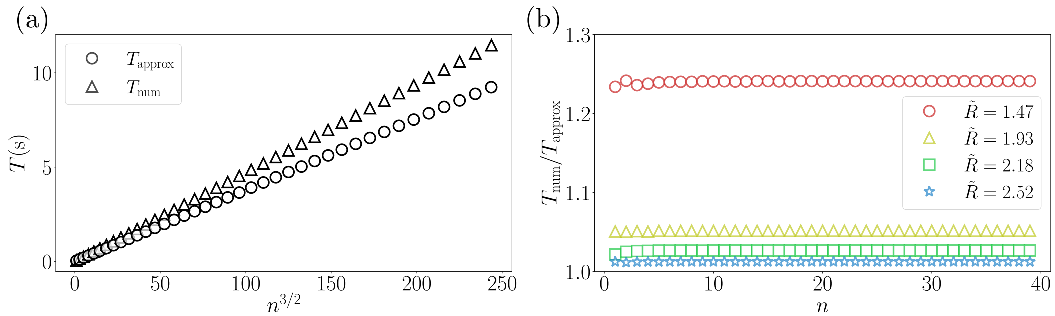

Figure S2: (a) Both numerical and approximated times needed for one bubble to empty in the tube for $\tilde{R} = 1.74$ as a function of the number of lamellae n. (b) Ratio between those two times for different aspect ratio $\tilde{R}$ as a function of n.

2 Experimental results

2.1 Reproducibility of the experiment

We perform all our data set three times and we illustrate the reproducibility of our experiment in figure S3 where we plot the results for three different experiments with $\Omega_{\text{bubble}} = 0.42 \pm 0.03$ mL and four different n value. We can see that each experiment is well reproducible. In the next figure, we show only one experimental data set for legibility.

2.2 Individual bubble dynamics

To show the agreement of the numerical model, we present the individual fitted dynamics of the bubble shrinkage for three different aspect ratio: $\tilde{R} = 1.47$ in figure S4, $\tilde{R} = 1.93$ in figure S5, $\tilde{R} = 2.18$ in figure S6. In each case, data for the different number n of lamellae are presented. We notice the disagreement for the first bubbles (typically $n < 3$) due to the time needed to put the bubble in contact with the tube that is comparable to the shrinkage duration.

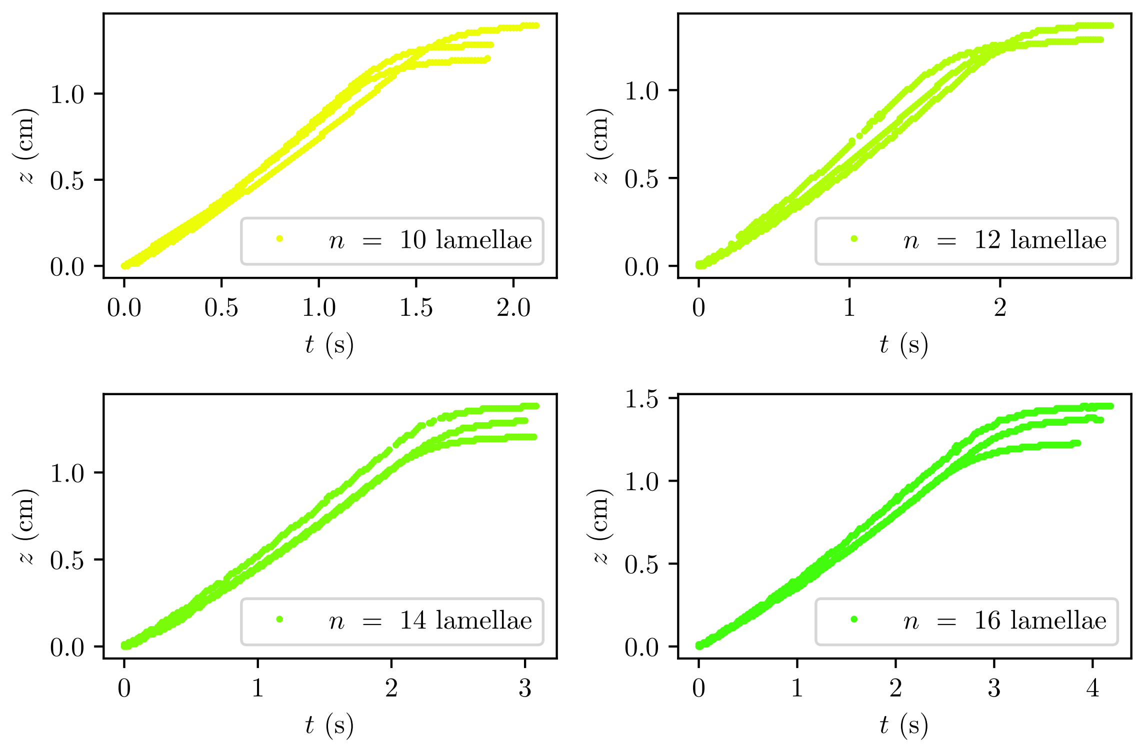

Figure S3: Dynamics of the lamella for 3 bubbles of volume $\Omega_{\text{bubble}} = 0.42 \pm 0.03$ mL. The dots are experimental data with the same color code as in figure 6(b). Each plot has a unique value of lamellae n.

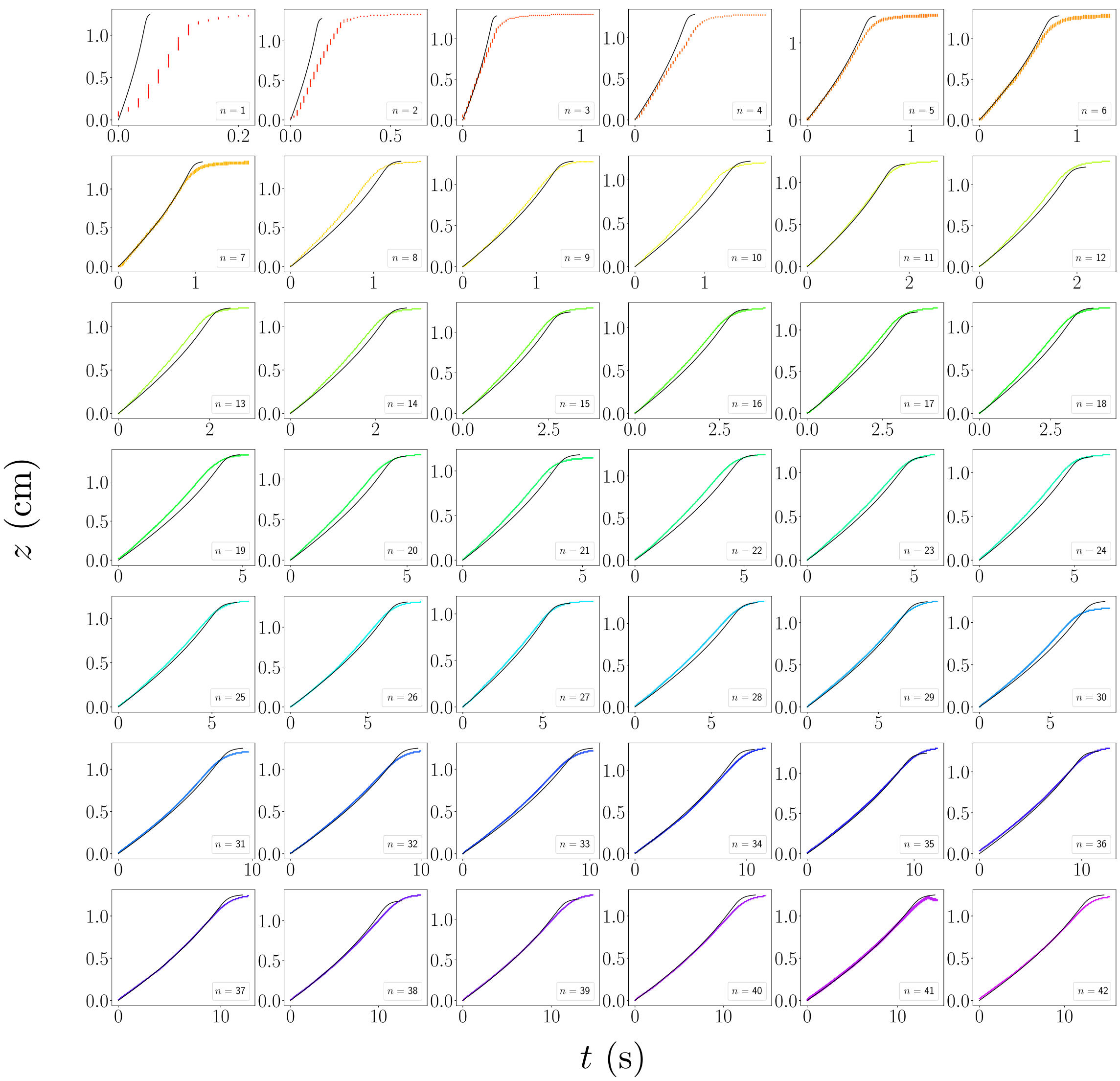

Figure S4: Dynamics of the lamella for 42 bubbles of volume $\tilde{R} = 1.47$. The colored dots are experimental data with the same correspondence as in figure 6(b) and the black lines are the numerical model.

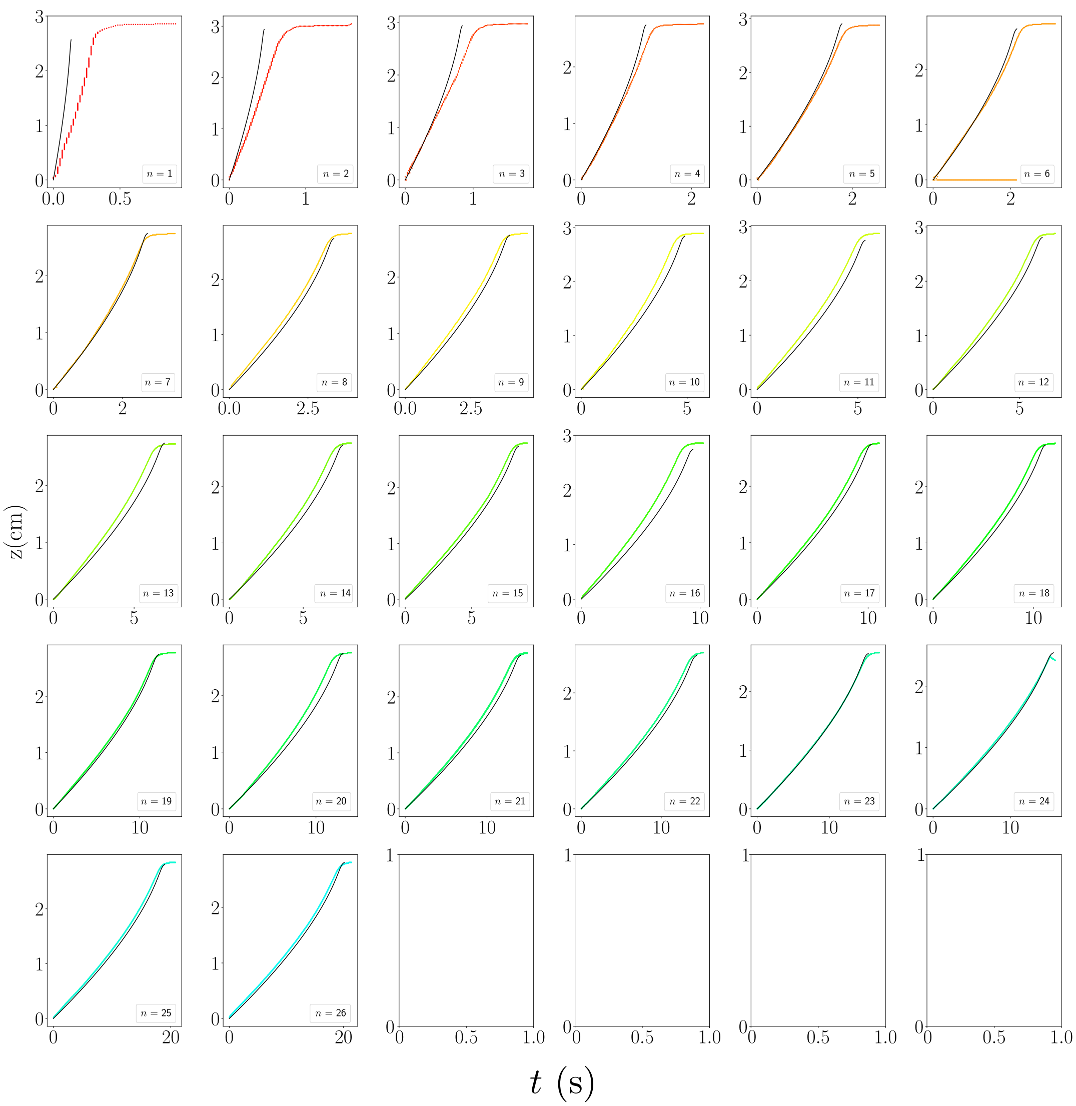

Figure S5: Dynamics of the Incompressibilitylamella for 26 bubbles of volume $\tilde{R} = 1.93$. The colored dots are experimental data with the same correspondence as in figure 6(b) and the black lines are the numerical model.

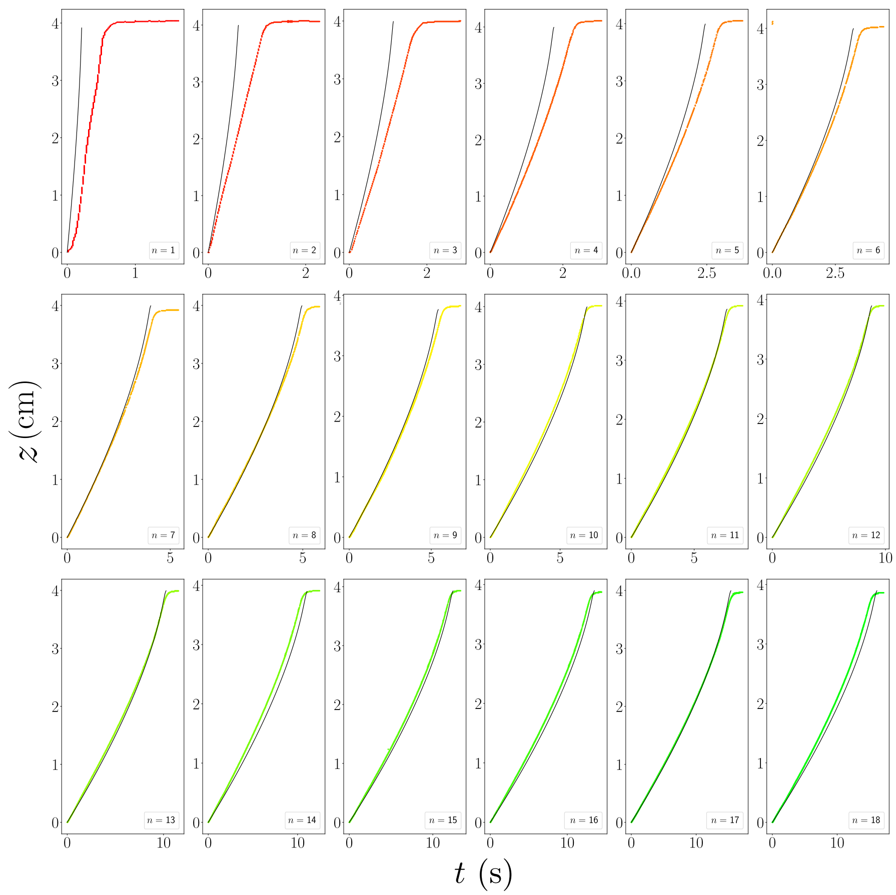

Figure S6: Dynamics of the lamella for 18 bubbles of volume $\tilde{R} = 2.18$. The colored dots are experimental data with the same correspondence as in figure 6(b) and the black lines are the numerical model.